PHILOSOPHICAL TRANSACTIONS.

Giving Some

ACCOUNT

OF THE

Present Undertakings, Studies and Labours

OF THE

INGENIOUS,

In Many

Considerable Parts of the WORLD.

VOL. XXX. For the Years 1717. 1718. 1719.

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To the Right Honourable

THOMAS

Lord PARKER,

Baron of *Macclesfield*;

Lord High Chancellor

Of *Great-Britain*:

By INCLINATION as well as OFFICE,

First PATRON of Uſeful Arts and Diſcoveries.

This THIRTIETH Volume

OF THE

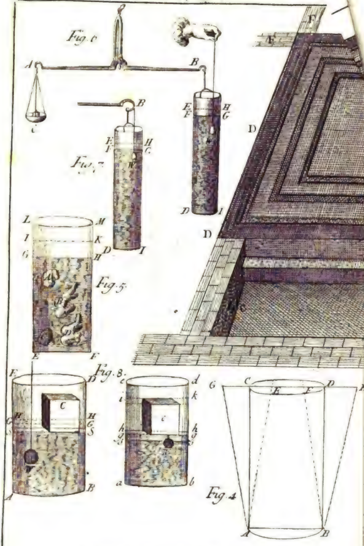
Philoſophical Tranſactions

(As a *ſmall* Acknowledgment for *very Great* Favours)

Is Humbly Inſcrib'd by

His Lordſhip's moſt obliged Servant,

Edmund Halley, R. S. Secr.



Philosoph. Transact. N.° 351

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Januar. Febr.* and *March*, 1717.

The CONTENTS.

I. *Observationes Stellæ fixæ in* Geminis *à corpore* Jovis *occultatæ,* Januarii 11*mo. St. vet.* 1717. *& Transitus arctissimi* Martis *infra Borealem in fronte* Scorpii Febr. 5. *mane.*

ANTE biennium in *Transact. Philos.* No. 344. *pag.* 294, rerum cœlestium studiosis indicavimus, *Jovem* corpore suo stellam quandam fixam obtegere debere, eosque ad observationem Phænomeni rarissimi, & magno in Astronomicis usui futuri, invitavimus, signantes diem *Januarii* hujus anni decimum. *Jove* autem pene Stationario, & paulo amplius in orientem quam per Tabulas nostras provecto, non ante undecimum incidit prædicta Occultatio; quam quidem *Londini* ob Nubes non contigit ex voto observare.

Nec tamen frustra invigilarunt Astronomi nostri. D. *Martinus Folkes Londini*, præsentibus aliis nonnullis e Societate Regia, *Jan.* undecimo 8*h.* P. M. vidit *Jovis* centrum una diametro corporis ejus Fixam sequi, quæ dicto centro Borealior erat quasi dodrante semidiametri Jovis. Postea Nubes Jovem occuparunt, sed, habita ratione motus Jovis, paulo post medium Noctis stellam Jovi conjunctam fuisse, & a Borea disci ejus parte occultatam, conclusit.

Reverendus Dominus *J. Theoph. Desaguliers*, R. S. S. & D^nus^. *Stephanus Grey, Westmonasterii*, viderunt Fixam, Horâ Sextâ vespertina, integra Jovis diametro distare a limbo ejus, Corum versus. Unde & ex sequentium dierum Observationibus, circa medium noctis incidisse conjunctionem evincitur.

Reverendus quoque D. *J. Pound*, apud *Wansted*, infrascriptas nactus est observationes, quas utique accuratissimas

tiſſimas, Tubo ſcil. prælongo & Micrometro captas, huc tranſcribere non pigebit.

Itaque *Januarii* Quinto 5^h. 6'. T. æq. Jovis centrum diſtabat a dictâ Fixâ 31'. 49". quam 5^h. 38'. ſequebatur 34' 12" Aſcenſionis rectæ: ſimulque limbus *Jovis* Auſtrinus eandem habuit Declinationem cum ſtella.

Die autem Nono ſequente 6^h. 6'. Jovis centrum diſtabat a ſtella 10'. 49"; & poſt octo minuta erat differentia Aſcenſionum rectarum 11'. 32": & tum centrum Planetæ, tantillo, ita ut vix perciperetur, erat Stellâ Auſtralius.

Die Undecimo 5^h. 30'. T. æq. erat diſtantia centrorum 1'. 24'. ſimulq; viſa eſt ſtella quaſi quadrante diametri Jovis Borealior centro ejus. Diameter autem minima *Jovis* inventa eſt 0' 43". Deinde Nubes.

Die vero Duodecimo 5^h. 17'. erat diſtantia centrorum 3'. 7"; ac 5^h. 50'. Jupiter ſtellam præcedebat 3'. 30". Aſcen. Rect. Eodemq; tempore limbus Jovis Boreus eandem habuit Declinationem quam Fixa accurate.

Collatis autem his Obſervationibus manifeſtum eſt Fixam hanc *Jovi* conjunctam, *Januarii* undecimo 13^h. circiter, non niſi 17" vel 18" centro ejus Borealiorem fuiſſe, ac proinde occultatam.

Fixa hæc, etiamſi nulli Catalogo hactenus aſcripta, Locum tunc habuit ♊ 22°. 13'. cum Lat. Auſt. 0°. 13'$\frac{1}{2}$; Comitemque habet 17 min. eam præcedentem & 7 min. Borealiorem, ſive in ♊ 21°. 56' cum Lat. Auſt. 0°. 6'$\frac{1}{2}$, cui *Jupiter* conjungi viſus eſt *Jan.* 16. 6^h. 30'. veſperi.

Sic ſpatio minus bimeſtri *Jupiter* corporaliter eclipſavit duas Fixas, cujus rei ne ſingulare quidem exemplum ab invento Teleſcopio extat: proinde hæc obſervata inter pretioſiſſima Uraniæ *κειμήλια*, in uſum Poſterorum, merito reponenda ſunt.

No-

Nostra autem stellula anno 1634. *Feb.* 6. *Jovi* Stationario conjuncta, tribus ejus diametris Australior erat, observante *Gassendo*: unde constabit, calculo rite instituto, *Jovis* Nodos quoad sensum immobiles hæsisse, per 83 annos ultimo elapsos, idque ad 2^s 8°. 35'. a 1^a ✱ ♈.

Ad alteram autem Observationem Transitûs *Martis* prope *Boream Frontis Scorpii* non minus insignem, iidem Astronomi eadem cum cura invigilarunt omnes. *Mars* autem, *Februarii* Quinto Mane, vel Quarto 16^h. visus est adeo vicinus stellæ dictæ, ut ea nudis oculis non conspiceretur; sed per Telescopium inventa est supra & ad ortum, adeoque *Mars* nondum ei conjunctus. Hora 16^h. 10'. T. app. *Mars* erat in recta cum Borea frontis & Telescopicâ quæ eam sequitur ad Boream, ad distantiam octo circiter minutorum. 16^h. 35'. *Mars* intermedius erat in recta cum Boreâ & Mediâ Frontis; & post horæ quadrantem, cum Austrinâ Frontis, ita ut 16^h. 54'. T. app. æstimabatur Conjunctio ipsa quoad Longitudinem, quo tempore *Mars* sat accurate duobus tantum minutis australior erat stellâ. Observavit etiam D. *Pound* Conjunctionem respectu Ascensionis Rectæ 17^h. 25'. T. app. cum distantiâ centrorum 2'. 07". Jucundum autem erat spectaculum, *Martem* videre stellam pedetentim aggredientem, motumque suum, lentissimum licet, manifeste prodentem.

Conferatur cum hac Observatio *Horroxii* nostri anno 1638. *Februarii* Septimo mane, quam vide in Epistolis ejus pag. 304. Tunc enim *Mars* ad eandem stellam appulsus, etiam multo propius ad eam accessit, sed ante ortum ejus præterierat Conjunctio.

His adde *Saturni* observationem *Januarii* 25to. 12^h. 25'. T. æq. à D. *Pound* habitam. Cum Planeta distabat à stella 58va. *Virginis Catal. Brit.* 13'. 16". versus Austrum, eamque sequebatur 2'. 30". Asc. Rect. Stella in ♎ 19 21'. 52". cum lat. Bor. 2°. 47'. 25".

II. An

II. *An accurate Account of a* tessellated Pavement, Bath, *and other* Roman *Antiquities, lately discover'd near* East Bourne *in* Sussex. *Being part of a Letter of* January 26. 1717. *from the learned Dr.* John Tabor *of* Lewis, *to Dr.* John Thorpe, R. S. S. *and by him communicated to the* Royal Society.

A Description of the tesserated Pavement at *East Bourne*, near *Pevensey*, must have been more imperfect than what is now given, had it come to your hands much sooner. I thought an exact Account could not be taken, unless the Ground about it was open'd: and it being part in a Meadow, and part in plough'd Ground, and under a Fence which parts two Persons Lands; by reason also the one was sow'd; I could not procure the Digging in both Places at the same time.

It was in *March* last when the Meadow was dug; and the last Week save one in *November*, before we had leave to open the Ground in the Corn Field. The Meadow in which the greatest part of the Pavement lyes, is near a Mile and half South East of *Bourne*; it contains about four Acres, and is of a triangular Form; the Southern Side is against the Sea; only a few Fishers Cottages, and a small publick House or two being between that and the Sea. On the Northern Side of the Meadow is a High-Way, which leads from *Bourne* to *Pevensey*: the West Side is by a Fence of Posts and Rails separated from a large Corn Field, in Common belonging to the Parish.

About the middle of this Fence is the Pavement, distant from High-Water-Mark a Furlong; In former times it might have been somewhat more, because from this Point to the Westward, the Sea is always gaining from the Land.

More than four Years since, *viz.* in the Summer 1712, when the Fence was repair'd; the Workman sinking a Hole to fix a Post in, was hinder'd by something Solid like a Rock; but casting out the Earth clean, found the Obstacle to be Artificial. Mr. *Thomas Willard* of *Bourne*, Owner of the Meadow, being inform'd of the Novelty, gave Order that it should be uncover'd; and sent also to *Herstmonceux*, for one *Purceglove* an ingenious Ingineer (who formerly had been imploy'd in the Mines in the Northern Counties), who with his Instruments bored through the Pavement; and in many places of the Ground about it, which he found to be full of Foundations: but this his Discovery of those Foundations, was only a Confirmation of what the Inhabitants there have always observ'd, as well in Ploughing, as in the Growth of their Corn and Grass: for in the common Corn Field, West to the Meadow, to the distance of near half a Mile, they often raise bits of Foundations with their Ploughs; and in dry Summers, by the different Growth of the Corn, they can plainly perceive all that Tract of Ground to be full of Foundations.

The Pavement was little more than a Foot below the common Surface of the Ground; what lay next it was a small Sea Gravel; the Position of it is very near due East and West (about two Foot of the West end of it reaching into the Corn Field); its length is seventeen Foot and four Inches; its breadth eleven Foot. At first it seem'd to have been bounded with a thin Brick set on Edge, about an Inch above the *Tessera*, so exactly strait and even, as if Shot with a Plane; and so well Cemented,

as

as if one entire Brick. But when the outside of the Pavement was broke up, we found, that instead of Bricks set on Edge, as was imagin'd, it was bounded with a Border of Bricks laid flat, and their ends next the *Tessera* turn'd up. The Thickness of these Bricks was an Inch and a Quarter; the Breadth not under Eleven, and not more than twelve Inches; the length full fifteen Inches; which, before they were turn'd up at their Ends, could not have been less than Seventeen. They were very firm, and not in the least Warp'd or Cast in Burning: when broke, their Substance was fine and well mixt, of as uniform and clean a Red Colour, as a piece of fine *Bole*: Except at the ends where turn'd up, they were all over cover'd with a Plaster (the same which *Vitruvius* calls the *Nucleus*, of which more afterwards), half an Inch thick; so hard, entire, and even, that it seem'd as one Stone, quite round the Pavement.

Next within the Bricks, there was a List or Border of white *Tessera*, thirteen Inches broad; within that, a List of brown *Tessera* (somewhat darker than a Whet-Stone, and somewhat lighter colour'd than the Touch-Stone) four Inches broad; then a List of the White, five Inches broad; next within that, another List of the Brown, four Inches broad: all the rest of the Pavement was set with white *Tessera*, without any Ornament or Figure; which though not Gay, lookt very Neat and Clean.

When this was first view'd, none of the Curious doubted, but that the Work was Roman, many were of opinion, that it might have been the Floor of a Temple, or place of Worship. *Pliny* indeed (*a*) informs us, that these sort of Pavements or *Lithostrota*, began to be in use in *Italy*, in the time of *Sylla*; who caus'd one of them

(*a*) Plin. Sec. Hist. Nat. Lib. XXXVI. Cap. XXV.

to be made in the Temple of *Fortune* at *Præneste*; perhaps the same which not long since was taken notice of by the Honourable Mr. *Addison* (*b*).

I was rather inclin'd to suppose, it had been that Apartment belonging to the chief Officer where Justice was administer'd; and the more, because *Pilat's* final Sentence on our Saviour was pronounced from a Throne on the *Lithostroton* (*c*); which Appellation was given to these kinds of Pavements by *Varro* (*d*) not less than sixty Years before; and by *Pliny* (*e*) not less than forty Years after our Saviours Suffering. That the Roman Generals caus'd such Pavements to be made at their Stations; we may have just reason to conclude, from that passage (*f*) in *Suetonius* cited for this purpose by Dr. (*g*) *Plot*.

When the Ground about the Pavement was dug, all these Suppositions were quash'd; for on the North Side of the Pavement, we discover'd an entire Bath, sixteen Foot long, five Foot nine Inches broad, and two Foot nine Inches deep (which the Draught sent with this represents): *Fig.* 1. It was fill'd with Rubbish of Buildings, which seem'd to have been burnt; *sc.* hard Mortar adhering to pieces of Roman Brick, squar'd Stones, and headed Flint, mingled with Ashes and Coals of Wood. From the Northwest Corner of the Pavement, was the Passage into the Bath, three Foot three Inches wide; at which place, the Bricks that bounded the Pavement, were not turn'd up at their ends, but lay even with the *Tessera*. At the distance of fifteen Inches from the *Tessera*, there was a Fall of two Inches, to the Landing-

(*b*) Remarks on several places in *Italy*, *Pag.* 377. (*c*) Ευαγγ. Ἰωαν. K. XIX. 13. (*d*) Ter. Var. de Re Rust. Lib. 3.
(*e*) Plin. Hist. Nat. Lib. XXXVI. C. XXV. (*f*) Jul. Cæs. Sect. 46.
(*g*) *Oxfordshire* Plots Nat. History, Chap. X.

place

place out of the Bath; the Landing place was alſo three Foot three Inches long, and two Foot two Inches broad: Thence by two Stairs, was the Deſcent into the Bath; the length of the Stairs, the ſame as of the Landing-place; the breadth of each Stair was eleven Inches; the height of each Step a little more than ten Inches: the loweſt Stair was twenty Inches from the farther Side of the Bath. The whole Work was very compact, and exactly well made; not in the leaſt injur'd by Time, nor the Violence it underwent when fill'd up; truly anſwering the Precepts of *Vitruvius*; which (*b*) adviſe, that for all Buildings, reſpect ſhould be had to the Strength, Conveniency, and Beauty of the Work deſign'd; and that in order thereto, a careful and judicious Proviſion ſhould be made of Materials, without Parſimony.

Although the Author and Time of theſe Works cannot as yet be diſcover'd; yet 'tis evident the Artificer near enough follow'd the directions *Vitruvius* (*i*) gave for framing ſuch like Structures.

(*b*) M. *Pollio Vitruv.* de Architectur. Lib. II. Cap. III. Hæc autem ita fieri debent, ut habeatur ratio firmitatis, utilitatis, venuſtatis. Firmitatis erit habita ratio, cum fuerit fundamentorum ad ſolidum depreſſio, & ex quaque materia copiarum ſine avaritia diligens electio.

(*i*) M. *Vitruv. Pol.* Lib. VII. Cap. I. Primumque incipiam de Ruderatione, quæ principia tenet Expolitionum, uti curioſius ſummaque providentia ſolidationis ratio habeatur. Et ſi plano pede erit ruderandum, quæratur ſolum ſi ſit perpetuo ſolidum.—Si aut omnis aut ex parte congeſtitius locus fuerit, fiſtucationibus cum magna cura ſolidetur.—Tunc inſuper ſtatuminetur ne minore ſaxo quam quod poſſit manum implere: ſtatuminibus inductis ruderetur. Rudus ſi novum erit, ad tres partes una calcis miſceatur, ſi redivivum fuerit, quinque ad duum mixtiones habeant reſponſum. Deinde Rudus inducatur, & vectibus ligneis Decuriis inductis crebriter pinſatione ſolidetur; & id non minus *poſt* pinſum abſolutum craſſitudine ſit dodrantis. Inſuper ex Teſta Nucleus inducatur, mixtionem habens ad tres partes unam Calcis; uti ne minore ſit craſſitudine pavimentum digitorum ſenum. Supra Nucleum, ad Regulam & Libellam exacta Pavimenta ſtruantur, ſive Sectilibus, ſeu Teſſeris. Cum ea extructa fuerint, & faſtigia extructiones habuerint, ita fricentur, uti, ſi Sectilia ſint, nulli gradus in ſcutulis, aut trigonis, aut quadratis, ſeu favis

First, as to the Pavement, it was ſecur'd on every Side, and the Edges of it reſted on a very firm and neat built Wall, made of Roman Brick, ſquar'd Stone and headed Flint; between five and ſix Foot deep below the Surface of the Pavement, and full twenty three Inches thick; which we may ſuppoſe to have been two Foot by the Roman Meaſure. The Bricks were not in regular courſes, as they are to be ſeen in thoſe Roman Buildings, which are in view above Ground; but without order diſpers'd about in the Wall. The Top of the Wall indeed was but fifteen Inches thick; and that was cover'd with the Bricks firſt mention'd, which bounded the Pavement: but about fourteen Inches below the Top, there was a Set-off (as our Maſons term it) in the inſide of the Wall, eight Inches broad. We did not dig up the Foundation of the Pavement to the Bottom, but opened it at one Corner only, that we might diſcover how it was Fram'd: for when it was bor'd through, they obſerv'd, next under the *Teſſeræ*, a Bed of very ſtrong Mortar, more than a Foot thick; under the Mortar a Bed of Clay two Foot thick; and under the Clay a firm Foundation of Brick. We obſerv'd the Clay (which the Ground thereabouts do not afford) to be very fine and red, and alſo cloſe; no doubt but carefully Ramm'd. The Surface of the Clay was neatly pitch'd with ſmall Flint and Stones, Pointed at their lower ends, and Headed at their upper ends. This Pitching or Paving is by *Vitruvius* call'd *Statuminatio*; and the Stones 'tis done with, he

extent. Sed coagmentorum compoſitio planam habeat inter ſe directionem. Si Teſſeris ſtructum erit, ut eæ omnes angulos habeant æquales, nullibique à fricatura extantes. Cum enim anguli non fuerint omnes æqualiter plani, non erit exacta ut oportet fricatura.

calls

calls *Statumina*. He directs them to be ſet, when the Underwork is made Sound and Firm, by well Ramming. Becauſe the firſt Chapter in his Seventh Book, treats only of the Method of making theſe kinds of Pavements, which in his time, and as may be obſerv'd from his words, were had in no ſmall eſteem by the Grandees of *Rome*; I have tranſcribed what may ſhew the accurate Methods which that great People had in Framing them.

But to return, this pitch'd Work was exactly even with the Set-off in the inſide of the Wall; on it was laid a Bed of coarſe Mortar of about nine Inches thick; the Skirts of this Mortar (which by *Vitruvius* is call'd the *Rudus*) reſted on the Set-off above-mention'd: it was compos'd of Lime, a ſharp courſe Sand, ſmall Pebbles, and bits of Brick. Upon this *Rudus* was a finer Compoſition, made, as near as I could gueſs, with Lime, a fine ſharp Sand, ſome kind of Aſhes, and (which was the greater part) ſtampt Brick and Pot-ſherds, in grains not larger than Cabbage-Seed, and the Flower or fine Powder ſeparated from it. This Bed was about half a Foot thick; and is what *Vitruvius* calls the *Nucleus*. Whether we may call it Terrace, I muſt leave it to thoſe who are better skill'd than my ſelf, in giving proper Appellations to the ſeveral parts of Maſonry. Both this *Nucleus* and the *Rudus* under it, very near equall'd the *Portland*-Stone in hardneſs and compactneſs. Upon this *Nucleus* or Terrace were the *Teſſera* ſet: they were ſet an end; but ſo exact was the Workman in ſetting them, that he us'd two ſorts of Cement to fix them withal; their lower ends ſtood in a Cement of Lyme only, well work'd; their upper halves were cemented with a fine gray Mortar, conſiſting of fine Sand (and as it ſeem'd) Aſhes and Lyme. This gray Cement every where fill'd the Intervals at their Heads; and was much harder than the *Teſſera* themſelves.

'Twas

'Twas before intimated, that the *Tesseræ* were but of two Colours, White, and of a dark Brown; they were harder than a glaz'd and well burnt Tobacco-Pipe, and of a Grit somewhat finer; the Brown seem'd to be of the same Substance with the White, but colour'd by Art, (as *Pliny* informs us (*k*) the workers in Clay of old had a Method to do): they seem'd to have been form'd in a Mould, and afterwards Burnt. Hence I am inclin'd to take the meaning of *Vitruvius*; where he makes so plain a distinction between the *Tesseræ* and the *Sectilia*; that, the one was according to the import of the name, form'd by Instruments out of Stone, Brick, and Tyle; the other shaped in a Mould and Burnt. They were not of an equal Size, none exceeding an Inch in length; the shortest were $\frac{6}{12}$ of an Inch: most of them were equally made their whole length; but of some the lower ends terminated almost as sharp as a Wedge, on purpose, as may be suppos'd, to be driven where any Interstices were left: At their Heads likewise they were not all equal and alike, some exactly Square, some oblong Square, some Semi-lunar, but none Triangular: the Diameter of those that were Square was about $\frac{4}{12}$ of an Inch; the longest Side of those that were oblong at the Head little exceeded half an Inch. It may be observ'd, that the preparations for fixing this Pavement here, go beyond those which *Vitruvius* prescribes (in the firm Wall near six Foot below the Surface, in the Bed of Clay within it two Foot thick, and in the Foundation of Brick under the Clay). But when we consider the Scituation of the Ground here is low, not many Feet higher than the Sea might be elevated at Spring Tides; and that it might as well be annoy'd by Land-Springs after great Rains, as by Water owzing through the Earth from the

(*k*) Plin. Secun. Hist. Mund. Lib. XXXVII. Cap. XII.

Sea

Sea ſo near; from which the Work in time might receive damage; we muſt allow the abovemention'd Additions to be the reſult of a very judicious Foreſight.

The Bath alſo was form'd and ſecur'd by a very compact Wall, of the ſame breadth and depth with that on which the Pavement reſted: the Wall, which ſuſtain'd the North Side of the Pavement, made the South Side of the Bath. On the South Side of the Bath, from the Eaſt end, to the ends of the Stairs, there was a ſolid Seat; twelve Foot nine Inches long, very near ten Inches broad, and fourteen Inches high. The Bottom or Floor of the Bath, was made after the ſame manner as the Pavement was made, excepting the *Teſſeræ*, and the thick Bed of Clay: for under all, there was Brick; then a Bed of the *Rudus* or coarſe Mortar ſomewhat more than a Foot thick; above that the *Nucleus* or Terrace only, half a Foot thick. The Sides of the Bath, the Seat, and the Stairs, were plaſter'd over with this Terrace about half an Inch thick; all which were throughout ſo Hard, Compact, and Smooth, that when firſt open'd, the whole ſeem'd as if it had been hew'd out of one intire Rock, and poliſh'd. At the middle of the Eaſt end, at the Bottom, there was a Sink-hole, a little more than three Inches long, and above two Inches deep: about four Inches above it, there was another paſſage through the Wall of the ſame ſize; the firſt we may ſuppoſe to let out the Water which had been uſ'd; the other to let in freſh. The Stairs and Seat were chiefly made of *Roman* Brick, between fifteen and ſeventeen Inches long, between eleven and twelve broad, and near one and a half thick. At the North Side of the Bath the Ground was not open'd; but at the Eaſt end of the Bath and Pavement, at the South Side of the Pavement, and at the Weſt end of both, there ſeem'd to have been ſeveral Vaults or Cellars: for there were

very firm 23 Inch Walls continued every way (to the farther ends of which we did not trace), whose Foundations were as low as that which supported the Pavement: so that to the depth of six Foot the Ground was fill'd with such Rubbish as was taken out of the Bath. The Bricks in this Rubbish, which were all broke, had several degrees of thickness, from three Inches to a little more than one Inch: some had one of their Sides wav'd as in *Fig*. 2; some Fretwise as in *Fig*. 3. others had Roses on them well imitated: we found also two sorts of channel'd Bricks; the one like a Trough, the Channel three Inches broad, and as many deep, the Brick it self an Inch and a half thick: The other sort, had a Cylindrical Channel; so that when two were clapt together, they form'd a hollow Cylinder of three Inches Diameter. These channell'd Bricks being all broken, their Length when whole is uncertain, as is the Use they serv'd to; whether for Passages to conveigh Water; or whether they were placed in the Walls to distribute Heat throughout the Building, as was usual in the ancient Structures at *Rome*.

'Tis farther observable, when the Ground was open'd the second time; that off from the South-West corner of the Pavement, which the Letter *G* shews; five Foot lower than the Surface of the Pavement, there was discover'd a large Space (to the end of which we did not search), paved with Brick, eleven Inches broad, almost one and a half thick, and fifteen long; substantially was it pav'd; for it had two Courses of this Brick. There was half a Foot of Mortar under the lower Course; and about an Inch of Mortar between the two Courses; these Bricks also were perfectly well made; but on the under Side of each, were two Knobs, about the size of half a Wallnut; fix'd on them as may be guess'd,

to

to keep them ſteddy, till the Mortar they were ſet in might dry. This pav'd Place was ſearcht 6 or 8 Foot every way; it was all cover'd with a Coat about two Inches thick, of Aſhes and large Coals of Wood: on that lay confuſedly large pieces of the *Rudus* or coarſe Mortar abovemention'd, and lumps of the *Teſſera* in all reſpects like thoſe on the Pavement, and cemented as They were. There were moreover mingled with the Aſhes many large Iron Nails, bigger, but not quite ſo long, as thoſe we call double Tenns; ſome Hooks for Doors to ſwing on; ſeveral ſmall pieces of earthen Ware; ſome like bits of Urns; ſome of a fine yellow Clay; ſome red, thin, neatly wrought and adorn'd with Flowers; and laſtly part of a Human Skull, and pieces of Bones near it: which Bones were not inclos'd in any Veſſel, but lay looſe; they were diſcolour'd like thoſe I have ſeen in Urns; ſo that the Body they belong'd to, might periſh by the ſame Flames, that theſe Buildings were deſtroy'd by. There was no Inſcription found either on Stone or Brick; no Statue, or other Figure, ſave thoſe on the Bricks mention'd; neither were there any Coins met with there. But ſomething more than a Furlong North-Weſt of theſe Works, near three Years ſince, there was a Malt-Houſe, and near two Years ſince a Dwelling-Houſe erected; in digging the Foundations for the firſt, there was a Coin of *Poſthumus*; and in the Ground dug for the laſt, a piece of *Conſtantine*'s found; both which I ſend with this, that the Inſcriptions and Reverſes may be incerted if neceſſary.

From the nearneſs of the Bath, it may reaſonably be concluded that the Pavement was neither a part of a Temple, nor for a place of Juſtice: the continuation of the Foundations every way to be traced from it, and what was laſt diſcover'd, are rather an Argument it was an Apartment of a magnificent Palace.

 Pliny

Pliny ſuppoſed that theſe *Lithoſtrota* (*l*) or teſſerated Pavements had their original in *Greece*; but perhaps the *Grecians* borrow'd their Patterns from *Aſia*: for from the Book of *Eſther* (*m*) we learn, there was a moſt Royal Banquet at *Suza*, on a *Lithoſtroton* (ſo the Septuagint has it) of coſtly Stones, four Hundred Years before the time of *Sylla*, who brought them firſt into *Italy*. *Joſephus* affirms (*n*), that the *Grecian* Laws, Learning and Arts were fetch'd from *Aſia*: and indeed when we reflect on the Antiquity of the *Levitick* Law; the Pyramids of *Egypt*; the Temple of *Solomon*; the Walls and Palaces of *Babylon*; and the ſumptuous remains of *Palmyra* and *Perſepolis*; we have no reaſon to eſteem the *Grecians* Authors, but as good Imitators of thoſe early Examples of Learning and Arts they had to follow.

When *Quinctus Cicero* was here with *Caſar*, the ſecond time he invaded *Britain*; his Brother the incomparable *Tully*, had the overſight of ſome Buildings he had appointed to be made in the *Villa Manliana* at *Arcano*: and in a Letter ſent into *Britain*, *Tully* informs *Quinctus*, that he was well pleas'd with the Seat, and the more, becauſe the *Pavimented Piazza* was Magnificent: that the Pavement ſeem'd (*o*) to be exactly well made: that he had directed ſome Chambers to be alter'd becauſe he did not approve of them: that in the Bathing Apartment, he had remov'd the Sweating Room into another corner of the *Apodyterium*. And afterwards in the ſame Letter makes mention of ſuch another Work which was in hand for him in the City alſo. Again, about the time *Quinctus* return'd out of *Britain*, and was fixt with the Legion he preſided over, in Winter Quarters

(*l*) Plin. Sect. Hiſt. Lib. XXXVI. Cap. XXV.
(*m*) Eſth. Chap. I v. 6. (*n*) Joſeph *againſt* Appion. Book II.
(*o*) Tull. Cic. ad Quinct. Frat. Lib. III. Ep. I.

among

among the *Nervii* (of which *Cæsar* in his Commentaries makes mention); *Tully* (*p*) takes notice of a Pavement that was making for himself also: *Expolitiones utriusque nostrum, sunt in manibus; sed tua pæne ad tectum jam perducta res est rustica Arcani & Laterii.* 'Tis hinted by *Varro* that a *Lithostroton* was one of the Members of a compleat *Villa* (*q*): *Varro* was eighty Years old when his Books *de Re rustica* were composed: *Tully* was something more than fifty when the above cited Epistles were wrote; *Cæsar* when a General, made the *Tesseræ* (*r*) and *Sectilia* for Pavements, to be part of his Baggage; and *Vitruvius*, Cotemporary with these three, calls the *Lithostrota*, *Principia Expolitionum* (*s*); which make it evident these Floors were held in esteem, by as great Men as the World has afforded, even in their riper Years. From all this, we may observe, that sometime before, and in the first Age of the Empire, the humour of these kinds of Floorings much prevail'd among the *Romans*: wherefore 'tis no wonder they are found in so many places of this Island. But, as unprofitable Inventions and Customs in time grow Stale, and are laid aside, so fared it with that of Pavements: For in the time of *Pliny* they began to be out of use on the Ground; but then he tells us, they were made above Stairs (*t*), or in his own Words in Chambers. Whether the *Lithostrota* in Chambers were usual in *Vitruvius*'s days, we have no Warrant to suppose, from any hint in his Writings; notwithstanding he gives Rules for making them, *plano pede*, on the Ground; and *sub* (*u*) *dio*, (which

(*p*) Ibid. Ep. I'I. (*q*) Ter. Varro de Re rustic. Lib. III. (*r*) Suet. Tranq. Jul. Cæs. Cap. 46. (*s*) M. *Vitruv. Pol.* Lib VII Cap. I (*t*) Plin. Hist. Lib. XXXVI. Cap. XXV. Pulsa deinde ex humo Pavimenta in cameras transiere è vitro: novitium & hoc inventum. (*u*) M. *Vitruv.* Lib. VII. Cap. I. Sub dio vero maxime idonea faciunda sunt pavimenta.

from

from the Method by him prescrib'd must be) alofet because for sustaining those *sub dio*, he orders the work underneath to be well secur'd, with two lays of Plank that should cross (*w*) each other, and be nail'd down; then the *Statuminatio* or Pitching, the Mortar, Terrace and *Tessera*, as before on the Ground. But because by *sub dio Vitruvius* could not design Chambers; and although *Pliny* informs us the *Gracians* us'd (*x*) to cover or Flat-roof their Houses with these Pavements; yet since neither *Vitruvius* nor *Pliny* mention any such Mode prevailing in their times at *Rome*; it remains, that we may imagine *Sub dio*, or the *Subdialia* of *Vitruvius*, to mean Pavements mounted on Pillars or Arches, which might afford delightful Terraces out of the upper Rooms, and shady Piazzas underneath: and in this Sense perhaps may be understood the *Porticus Pavimentata* of *Tully* above-mention'd. By the many Apartments, the Foundations about these Works point out, there seems to have been nothing wherein the Buildings that once stood there, might come short of the magnificent Structures, wherewith the *Romans* delighted to gratify their Luxury. The uses each were design'd for, is not to be determin'd: whether there was a Piazza cover'd with a *Lithostrotan*, cannot be affirm'd. But be that as it will; 'tis next to Demonstration, there was some upper Floor sustain'd by Wood, and pav'd with the *Tessera*, after the same manner as *Vitruvius* directs; and, on the Brick Pavement (last discover'd), the Coat of Ashes and Wood Coals with Nails, cover'd with large pieces of the *Rudus*, and great lumps of the *Tessera* well cemented

(*w*) Ibid. itaque si necessitas coegerit, ut minime vitiosa fiant sic erit faciundum: cum coaxatum fuerit, super altera coaxatio transversa sternatur, clavisque fixa, &c.---Statuminatione facta rudus inducatur, &c.

(*x*) Plin. Hist. Lib. XXXV. Cap. XXV. Subdialia Græci invenere talibus domus contegentes.

together

together, and the *Nucleus* adhering to them; shew there was an upper Pavement broke by its fall, when Fire had consum'd its support.

I have been thus prolix, in giving you the most exact account I could of this piece of Antiquity; because we cannot have a less Sense of the admirable Rules and Methods, the *Roman* People made use of, in framing their Buildings, and ordering other Conveniences for Enjoyment and Magnificence; than of the incomparable Management they had in their Military Preparations and Discipline; which are so to the Life represented by (*y*) *Josephus*, and so punctually described by (*z*) *Vegetius*.

As to the *Roman* Architecture, it may not be amiss here to note; that when they design'd a Building, they could not immediately begin it: their Preparations requir'd time: By their well shap'd durable Bricks, and by their Stone-like Mortar, we may plainly perceive, they built not with such hasty Materials as are now us'd. *Vitruvius* (*a*) and *Pliny* both direct, that Brick should be form'd in the Spring, and be two Years drying: And where *Pliny* speaks of their Mortar, he says, 'twas ordain'd by the old Laws (*b*) of *Rome*, that no Undertaker should Build a House with Mortar which had not been made three Years before. We find indeed, their Walls seem to bid fair for Eternity; whereas ours, from Parcimony and ill Management, are scarce able to endure one Age.

The rest of this learned Discourse, by which 'tis made more than probable that here once stood the Roman City Anderidæ, *destroyed by the* Saxons *about the Year* 500; *though very curious, yet being chiefly Historical, seems not so properly the Subject of these Transactions.*

(*y*) *Josephus*'s Wars of the *Jews*. Book III. Chap. III.
(*z*) Veget. de Re Militari. (*a*) M. *Vitruv. Pol.* Lib. II. Cap. III. Plin Hist. Lib. XXXV. Cap. XIV. (*b*) Plin Hist. Lib. XXXVI. Cap. XXIII.

III. A

III. *A short account of the Nature and Vertues of the* Pyrmont *Waters; with some Observations upon their Chalybeat Quality. Communicated by Dr.* Frederick Slare, R. S. Soc.

HAving procur'd about a dozen Quarts of *Pyrmont* Waters this last Summer, I made some Tryals with them. I found by the Taste that they contain'd a rich Chalybeat Vertue, and also made a very brisk and lively impression on the Palate, more grateful and spirituous, than the best *Spaw* Waters I ever tasted. The *Spaw* Waters are look'd upon as most excellent, if they Sparkle a little in the Glass; but these in Summer time, when pour'd into the Glass, nay sometimes even in the Bottle, as soon as the Cork was open'd and the Air was admitted, would make a notable Ebullition, somewhat like bottled Cyder, tho' this was soon over; but they did yet continue their smart and brisk Taste, and high Chalybeat Relish to the last Drop, tho' we were some Hours in Drinking them off. In the Winter time, these Waters do not Sparkle, nor Ferment, at least mine did not; but they were not carefully preserv'd, being expos'd in cold Cellars, where our Beer or Wine stood in the Winter; and yet notwithstanding, they lost not the Chalybeat Taste, and also retain'd a very pleasant brisk Gust. These Waters have been reckon'd in the Number of the *German* Saur Brunnen or *Acidulæ*, and some of my Friends to whom I gave a Glass of the Water, have ascrib'd to it a sharp Taste. and have been ready to run away with a possess'd Opinion of its being Sour: but when I requir'd them to call back that hasty Assertion, and to consider it better, whether that Taste was really

Sour or Acid, they have been forc'd to recant and confeſs, that the ſmart and brisk Taſte miſled them to call it Acid or truly Sour. Thus Cyder and ſoft Ale when Bottl'd, will give ſuch an acute Affection to the Palate, when it is far from being Sour: And even *Volatile Alkalies* of *Sal Armoniac* or of Hartshorn, may be made to give the like pungency to the Tongue.

In order to a more nice Enquiry, whether any Acidity were diſcoverable in theſe *Pyrmont* Waters, we dropt in conſiderable Quantities both of Spirit of Hart-horn, and of Spirit of *Sal Armoniac*, both juſtly prepar'd; but could not diſcover the leaſt Luctation or Motion to appear upon this Conjunction, as it uſually does with an Acid.

I made a yet more nice and certain Examen of theſe Waters, by mixing Milk with them, ſometimes in equal, ſometimes in double proportion; and in various degrees of Warmth, both in Lukewarm degrees, and alſo with a boyling Heat, but I could not perceive any Curdling. But rather on the contrary, the Water preſerv'd the Milk from Coagulation, for four or five Days, even in *September*, it being hot Weather.

Take a very little Gall in Powder, about half a Grain to a Glaſs of a quarter of a Pint; this does in a Moment render it turbid, and make a dark Purple, eſpecially if you ſtir it: but if you drop the Powder on the Surface of the ſame Water, it then cauſes a fine blew Tincture If you will make a very fine Tincture pleaſant to the Spectator, Take five Leaves of ſtrong Green Tea, put them into the bottom of a Glaſs holding a quarter of a Pint, and you will ſee thoſe Leaves unfold themſelves, and in a quarter of an Hour, tinge the Water with ſuch a Cerulous azure Blue, that few Vegetables do afford the like. We obſerve, that the longer theſe Leaves, or any other Stipticks, (which are the

Precipitators) do ſtay together, the more they degenerate into a deep Purple, or even to an Atramentarious Colour.

In reference to the internal Uſe of theſe Waters, I drank about a Quart at a time, after this manner. I firſt began with the *Spaw* Waters, which I procur'd very good, and drank them for a Week, and they agreed very well. I then drank the *Pyrmont* Waters for three or four Days, and continu'd the uſe of theſe Waters alternately, until I had drank about twenty Days. By the reſult of my Experiment it ſeem'd to me very plain, that the *Pyrmont* Water was more agreeable, gave more Strength and Spirit, and was as much or more preferable for its internal Vertue, as for its excelling the other in a brisker and more ſprightly Taſte.

There is another Excellency in theſe Waters which will make them more uſeful to us, than any Foreign *Chalybeat* Waters we yet know, becauſe theſe will keep better; they are not ſo ſoon ſpoil'd by any accidental Inſinuations of Air, as the *Spaw* are ſubject to be. The *Chalybeat* Mineral is here throughly diſſolved and well united, and mix'd in this Water, ſo that it does not eaſily precipitate: for which Reaſon it may alſo the better paſs the *vaſa lactea*, and even enter into the Maſs of Blood it ſelf, and work the more conſiderable Effects. That this is not a bare Hypotheſis may be prov'd by this Experiment.

Having ſuffer'd the *Spaw* Water to be expoſed in a Bottle which was half full, and unſtopt 12 Hours, I examin'd it, and found it taſt juſt like common Water; but the *Pyrmont* Waters that were open'd to the Air after the ſame Manner, taſted ſtrong of the Mineral, and gave their Tincture as at firſt; nay, they continued thus for full two Days, and perhaps might have done ſo longer, but I thought that Time ſuffic'd.

There

There remain ſeveral other Experiments to be made, in order to a further Search into the Excellencies of this noble Water, but this I cannot do at preſent for want of a Quantity, which I hope to obtain the next Summer; for they can with more Eaſe be brought into *England* than the *Spaw*. I may alſo fairly conclude, that ſince the *Spaw* has been very beneficial to our Patients in Chronical Diſeaſes, theſe Waters of a much ſuperior Virtue will ſurpaſs them in conquering many of our obſtinate Diſtempers.

Some Additions to the aforeſaid Account of the Pyrmont *Waters*

HAVING had lately ſome Diſcourſe about a Purging Quality contained in theſe Waters, I am now inquiring into the Truth of this Queſtion, whether they in Reality do contain any Purging Ingredients or Properties.

I evaporated about a Quart of this Water *ad ſiccitatem*; I then poured on the *reliquiæ* ſome Rain-Water, enough to diſſolve and take up the Salts, and exhal'd that Water, and had a Grane or two of the Salts, that taſted *muriatic*, ſuch as moſt River and Pump Waters give. It is well known that the Purging Waters have a very bitter Taſte, and by the moſt learned Doctor *Grew piæ Memoriæ*, and an illuſtrious Fellow of this Society, that Salt was called *Sal Catharticum amarum*, which diſtinguiſh'd it from all other Species of natural Salts: that of the *Pyrmont* Water abovementioned has no Relation to this, but to the Sea-Salt, not being in the leaſt bitter.

It is alſo well known, that unleſs our Waters be impregnated with a conſiderable Quantity of this bitter Salt, it will not purge at all: Two or three Granes ſignifie nothing, nor have the leaſt Cathartic Power. For Example, Put two Drams of the purging Salts to a Quart of common Water; and this Quantity will give but a Stool or two to one who is naturally very eaſie to work upon. I have examin'd ſeveral other *Chalybeat* Waters, and found much the like Ingredients, and never any that I could ſuſpect to carry any purging Properties.

I think we can much better demonſtrate that the *Chalybeat* Waters do contain Stiptic and Reſtringent Virtues, becauſe they owe their Birth to the *Iron Mineral*, and more particularly to the *Pyrites*, which Doctor *Liſter* ſuggeſts, (not without ſome Reaſon) to be the Parent even of all *Iron Oars*, as it is doubtleſs the Cauſe of all *Chalybeat* Waters: Thus I have often examined the Solution of the *Pyrites* by the Rain-Water at *Debtford*, and at other Places where Copperas is made, and found it a very ſtrong *Chalybeat* Water. It is from this *Minera* we have our ſtrong Stiptic and conſtringent Medicines, for external and internal uſe; we have our Powders and Salts of *Steel*, or *Vitriol* of *Mars*, from hence; nay, even thoſe obſtinate and inveterate *Diarrhæas* which have baffled the Force of all Medicines, have, by a judicious Uſe of *Tunbridge* and other Iron Waters, received a Cure.

But notwithſtanding all we can ſay, it will be retorted, that there is Matter of Fact and Experience againſt us, that the Waters really do purge at *Pyrmont*, where they are drank.

This we do allow to be true, that *Tunbridge* Waters do not only purge but ſometimes vomit, when drank haſtily and in great Quantity; but our Phyſicians have cor-

corrected this Irregularity, and we hear of no ſuch Complaints, where they obſerve a juſt *Regimen*: And we do all agree, that thoſe Waters are, in their own Nature, binding, and do oft require ſome opening Medicine. The Quantities of Water drank at *Pyrmont* are very large, often two or three *Engliſh* Quarts. It is no Wonder that their Weight forces them thorow the Bowels; for any common Water, drank haſtily, and in ſuch Quantity, will do the ſame. Whereas, if you take this Method, and will drink *Pyrmont*, or any other *Chalybeat* Waters leiſurely, *viz.* a Pint-Glaſs in an Hour, or rather two Half-Pint Glaſſes, you may drink three Pints in ſo many Hours without Danger of loſing them by Dejection. But if any one will be careful, and take this Caution with him, he will ſcarce fail of Succeſs; that is, let him be very quiet and ſtill, both in Body and Mind; the leſs he ſtirs or walks, the better he will paſs off his Waters by Urine. And tho' this will appear a Paradox, eſpecially to thoſe Phyſicians who practiſe abroad, and commend to their Patients much Action in walking, yet I know I have both Reaſon and Experience on my Side. To avoid Prolixity I ſhall not declare them at this Time, and ſhall only ask leave to mention one Obſervation I have made, that none of our *Engliſh* Steel Waters do ſtrike ſuch a Purple as the Foreign celebrated *Chalybeat* Waters do: for ours do give a more turbid and dark Colour, and the worſe the Waters are, the blacker Sediment they make: Thoſe of *Iſlington* abound with a coarſe *Oker*, the Mineral is not well diſſolved, but gives an *atramentarious* Colour; but the *Pyrmont* Waters excell all I have happened to examine, in its bright *Cærulious* Luſtre.

N. B. *Moſt of the Experiments alledg'd by Dr.* Slare, *in the foregoing Diſcourſe, were likewiſe by him ſhewn before the* Royal

Royal Society, Feb. 28. *last: and it was found that the* Pyrmont *Waters gave a much brighter Tincture with Galls and Tea, and had a much more exalted* Chalybeat *Taste than the* Spaw; *and a small Quantity of each being kept for some time in Bottles, to compare them, the* Pyrmont *was found to have retained its Virtues much better than the* Spaw. *The President, and several of the Members present, having drunk a Glass of it, found it of a very agreeable Relish, and to sit easie on the Stomach.*

IV. *Remarks on the second Paper in the History of the Royal Academy of Sciences, for the Year* 1711. *concerning the Cause of the Variation of the Barometer: to shew that the Way of accounting for it in that Paper is insufficient, and that the Experiment made use of to prove what is there asserted, does no way prove it. By* J. T. Desaguliers, *M. A. F. R. S.*

The Paper is as follows.

'IT appears by the Barometer, that when it rains, 'or a little before Rain, the Air commonly be-'comes lighter.

'That it must rain when the Air becomes lighter it 'is easie to imagine; for the imperceivable Particles of 'Water, that swim about in the Air in prodigious 'Quantity, not being sufficiently sustain'd when the 'Air has lost a certain Degree of its Weight, begin to 'fall, and several of them joining together in the Fall, 'make Drops of Rain. So when about half of the 'Air is drawn out of the Recipient of the Air-Pump,

'(and

'(and consequently the remaining *Air* is as weak again 'as at first) something like a small Rain falls. But 'why shou'd the Air become lighter? One might imagine that in the Place where it rains, it may have lost 'some of its Weight and Bulk, by means of the Winds 'carrying away some Part of it: but Monsieur *Leibnitz*, 'in a Letter to the Abbot *Bignon*, gives a more ingenious and more new Reason for it.

'He pretends that a Body, which is in a Liquid, 'weighs with that Liquid, and makes up part of its 'whole Weight, so long as it is sustained in it; but if 'it ceases to be sustain'd, and consequently falls, its 'Weight no longer makes a Part of the Weight of the 'Liquid, which thereby comes to weigh less. This 'may naturally be applied to the abovementioned Particles of Water; they encrease the Weight of the Air 'when it sustains them, which is diminished when it lets 'them fall: and as it may often happen that the Particles of Water that are highest, fall a considerable time 'before they join with those that are low, the Gravity 'of the Air diminishes before it rains, and the Barometer shews it.

'This new Principle of Monsieur *Leibnitz* is surprizing. For must not a strange Body, whether sustained in a Liquid or not, always weigh? Can it gravitate upon any other bottom than that which sustains 'the whole Liquor? Does that Bottom cease to carry a strange Body, because it falls? And is not that 'Body all the while it is falling, part of the said Liquid as to the Weight? At that rate, whilst a Chymical Precipitation is made, the whole Matter ought 'to weigh less, which has never been observed, and 'scarce appears credible.

'Notwithstanding these Objections the Principle 'holds good, when more closely examin'd. What su-

'stains

'ſtains a heavy Body is preſs'd by it. A Table, for 'Example, which ſuſtains a Pound Weight of Iron, is 'preſſed by it, and is ſo only becauſe it ſuſtains the 'whole Action and Effect of the Cauſe of Gravity, '(whatever it be) to puſh that Lump of Iron lower. 'If the Table ſhou'd yield to the Action of that Cauſe 'of the Weight (or Gravity) it would not be preſs'd, 'and therefore would carry nothing. After the ſame 'manner the Bottom of a Veſſel, which contains a 'Liquid, oppoſes it ſelf to all the Action of the Cauſe of 'Gravity againſt the ſaid Liquid: If a ſtrange Body ſwims 'in it, the bottom oppoſes it ſelf alſo to the ſaid Action 'againſt that Body, which, being in *Æquilibrio* with the 'Liquid, is in that reſpect really a Part of it. Thus the 'Bottom is preſs'd both by the Liquid and the ſtrange 'Body, and ſuſtains them both. But if the Body falls, 'it yields to the Action of Gravity and conſequently 'the Bottom does no longer ſuſtain it; neither will it 'ſuſtain it, till the ſaid Body is come down to the 'Bottom Therefore during the whole Time of the 'Fall, the Bottom is eaſed of the Weight of that Bo- 'dy, which is no longer ſuſtain'd by any thing, but 'puſh'd down by the Cauſe of Gravity, to which no- 'thing hinders it from yielding.

'Monſieur *Leibnitz* to confirm his Notion, propoſed 'an Experiment. He ſays, that two Bodies muſt be 'tied to the two Ends of a Thread, the one heavier, 'and the other lighter than Water, yet ſuch as both 'together may ſwim in Water: Put them into a Tube 'full of Water, the Tube being tied to one End of 'the Beam of a Ballance whoſe other End has a con- 'trepoiſing Weight: Then if we cut the Thread which 'ties the Bodies together (that are of unequal Weight) 'ſo that the heavieſt may preſently deſcend, He ſays, that 'in ſuch a Caſe the Tube would be no longer in *Æqui-* '*brio*, but its counterpoiſing Weight wou'd preponde-

'rate

'rate, because the Bottom of the Tube wou'd be less
'press'd. It is plain, that the Tube must be sufficiently
'long, that the falling Body may not reach the bot-
'tom before the Tube has time to rise. In Chymical
'Precipitations, the Vessels are either too short, or
'what is precipitated falls sometimes too fast and
'sometimes too slow; for then the little Bodies are
'always (as to Sense) *in Æquilibrio* with the Liquor
'that contains them.

'Monsieur *Ramazzini*, the famous Professor at *Padua*,
'to whom Monsieur *Leibnitz* had proposed his Expe-
'riment, has made it with Success, after some fruitless
'Trials. Monsieur *Reaumur* (to whom the Academy
'had recommended it) has also made it with Success:
'This is a new View in Natural Philosophy, which,
'tho' it depends upon a well known Principle, is very
'subtle and far-fetch'd; and gives us just Reason to fear
'that in Subjects that seem to be exhausted, several
'things may yet escape us.

Remarks upon Monsieur Leibnitz's *New Principle.*

Figure 4.

LET AB be the Bottom of a Vessel full of any Fluid, whose Top is either wider than the Bottom as GH, narrower as EF, or equal to it as CD. The Pressure of the Fluid upon the Base AB will be equal to the Weight of CB, or of a Cylinder or Prism of the same Fluid, made up of the Area of the Base multiplied into the perpendicular Height above it.

If the Fluid be equally dense every way as Water, or of a Density uniformly diminish'd as you go upwards, this Proposition (call'd by Mr. *Boyle* the Hydrostatical

Paradox) will hold good. This is demonſtrated by all Hydroſtatical Writers.

Figure 5.

Let *EF* repreſent part of the Surface of the Earth, and *GEFH* a Pillar of the Atmoſphere, whoſe Height is *GE* the whole Height of the Air. Let us imagine the Vapours riſing out of the Earth to form themſelves into two Clouds *A* and *B*, and to ſettle in that Place where the Air is of the ſame ſpecifick Gravity with themſelves. It is evident that they will cauſe the Air to riſe ſo much higher as their Bulk amounts to, and will therefore make the Surface which was at *GH* to riſe up to *IK*. ſo that the bottom *EF* which was preſs'd by a Pillar of Air as *GEFH*, is now preſs'd by an higher Pillar as *IEFK*. Now if the Clouds *A*, *B*, by any Cauſe ſoever, change their Place, ſo as to come downwards, (for *Exemple* to *C*, *D*) the Height of the Pillar *IEFK* will remain the ſame as it was, and therefore the Bottom *EF* will be preſs'd as before: by the foregoing Propoſition.

Corollary I.

If the Clouds *A*, *B* deſcend, and in their Deſcent keep the ſame Bulk as they had before, the Surface *IK* will remain the ſame, and therefore *EF* will be preſs'd as before.

Corollary II.

Whether a Body be ſpecifically lighter or ſpecifically heavier than a Fluid; ſo long as it is detain'd in it, it will add to the Fluid as much Weight as the Weight of an equal Bulk of that Fluid: wherefore a Body does not loſe all that Weight which it added to the whole Weight

Weight of the Fluid, when it ceafes to be fuftain'd in the faid Fluid: contrary to Monfieur *Leibnitz*'s Principle.

Scholium.

If a Cloud (by any Caufe whatfoever) becomes fpecifically heavier than that Part of the Air in which it fwims, the Excefs of its Gravity above an equal Bulk of Air will make it defcend, and accelerate its Motion downwards; and then indeed it will lofe of its Weight by the Refiftance of the *Medium*, till it comes to an uniform (or fenfibly uniform) Motion: but all the Weight that it will lofe will only be the Excefs of its Gravity above that of the Air; for with the reft of its Weight it will ftill make up part of the Weight of thé Air.

Experiment I. *Figure* 6.

Having with a Weight in the Scale *C* of the Balance *AB* counterpois'd the long Glafs of Water *EI*, with a Horfe-Hair I let down the leaden Weight *W* into the Water, which from *FG* arofe up to *EH*; and therefore the Water became heavier by the Weight of a Bulk of Water equal to the Lead. Having with another Weight in *C* made up the Counterpoife to the whole, with fine Sciffars I cut the Thread of the Plummet; and all the while the Plummet was falling, the Water defcended rather than rofe; and when the Lead was at the bottom, the Water overpois'd, becaufe it had then added to it all the Excefs of Weight of the Lead above an equal Bulk of Water, which by Experiment is about $\frac{\text{[illegible]}}{11}$ of its Weight. Had Meffieurs *Reaumur* and *Ramazzini* tri'd the Experiment thus, the Succefs had been the fame; but Mr. *Ramazzini* (as I underftood from a Gentleman who was prefent) tried it in the following Manner, as I have fince done.

Experiment II. *Figure* 7.

Making uſe of the abovemention'd Machine, after I had balanc'd the Water and Lead in it, I fix'd to the End of the Beam *B* the Thread of the Plummet, which in the former Experiment I held in my Hand. This added to the Weight hanging at *B*, and oblig'd me to put into the other Scale a Weight equal to $\frac{10}{11}$ of the Lead, to recover the *Æquilibrium*. Then cutting the Thread or Hair, the Scale with the Weights overpois'd whilſt the Lead was falling; but the *Æquilibrium* was reſtor'd when it came to the Bottom. So that the Lead even then muſt have loſt only its Exceſs of Weight above Water.

Experiment III. *Figure* 8.

I tried the Way propoſed by Monſieur *Leibnitz* in the following Manner.

I took a Cork *C* weighing an Ounce, and ſomething more than four times lighter than an equal Bulk of Water, and a Ball of Antimony *W* about four times ſpecifically heavier than Water, and of four Ounces Weight. The Cork laid upon the Water in the Veſſel *EABD* rais'd the Water from *SS* to *GG*, and added an Ounce to the Weight of the whole Water: then ſuſpending the Ball of Antimony by a String, and letting it hang in the Water at *N*, it rais'd the Water from *GG* to *HH*, and ſo added another Ounce to the Weight of the Water. Then tying the Antimony to the Cork (See the Figure of the Veſſel mark'd with little Letters) the Cork had added to it three Quarters of the Weight of the Antimony which the Hand before had ſuſtain'd, and made it ſink ſo as to be almoſt cover'd, and raiſed the Water to *ik*, adding three Ounces to its Weight. Hanging this Veſſel of Water upon the Balance, and a

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Counterpoiſe at the other End, upon cutting the String the Veſſel of Water was rais'd up, and the *Æquilibrium* was not reſtor'd till the Antimony came to the Bottom.

By obſerving that as the Cork (being freed from the Weight of the Antimony) aroſe, and that during the Fall of the Body, the Water ſunk to *h h*, it appears that this is, in effect, the ſame Experiment as the former, and concludes no more. As to the real Cauſe of the Variation of the Barometer, namely, the Accumulation of the Air by Winds over the Place where the Barometer riſes; and part of the Air being blown away where the Mercury in the Barometer ſinks, ſee Doctor *Halley*'s Account of it in the *Phil. Tranſactions.* Numb. 181.

POSTSCRIPT.

IN making the firſt Experiment before the R. Society, of a Piece of Lead ſuſpended by a Thread, whilſt it was wholly cover'd with Water in the large Tube in which it hung (whoſe Length was 4 Feet) it was obſervable, not only that the End of the Balance (to which the Tube of Water with the Lead in it was fixed) did not riſe when the Thread was cut, (to let the Lead fall from the Top to the Bottom of the Tube) as it muſt have done according to Mr. *Leibnitz*'s Principle; but that the ſaid End of the Balance began to deſcend from the Time that the Lead began to fall. Therefore to be ſure that it was not the Plummets rubbing againſt the Sides of the Tube in its Fall, which cauſed that *Phænomenon*, I hung to the Balance a long Glaſs of three Inches diameter inſtead of the Tube, and making the Experiment as before, it ſucceeded in the

the ſame manner: the End of the Balance which carried the Veſſel of Water ſunk as ſoon as the Thread of the Plummet was cut; tho' this Glaſs was not above half ſo long as the Tube.

When by holding the String I drew the Lead upwards and downwards in the Water, there was no ſenſible Alteration of the *Æquilibrium*. Neither was it alter'd by cutting the String of a Stone-Plummet, becauſe of the Shortneſs of the Glaſs, and the little Exceſs of ſpecifick Gravity in the Stone: for the greater the Difference is betwixt the Body made uſe of in this Experiment and Water, as well as the bigger the Body it ſelf is, the better the Experiment will ſucceed.

Hence it appears, that when a Body, ſpecifically heavier than a Fluid, is (by what cauſe ſoever) detain'd in any Place of the ſaid Fluid, it adds as much to the Weight of the whole Fluid as an equal Bulk of the ſaid Fluid amounts to: And when the ſaid Body, by the Action of its Exceſs of ſpecifick Gravity above the Fluid, deſcends with an accelerated Motion; ſo long as that Motion is accelerated, the Reſiſtance of the Fluid (which is as the Square of the Velocity) takes off ſomething of the whole Weight of the Body; but as much as the Body loſes, ſo much the Water gains, over and above what was given it by its riſing on Account of the immers'd Body.

A Body therefore that falls in a Fluid is ſo far from making the Fluid lighter as it falls, that it makes it preſs more upon the Bottom that ſuſtains it, when it is falling, than when it was at reſt in the Fluid.

If the Veſſel of Water be long enough for the falling Body to come to an uniform Motion before it reaches the bottom, the Force impreſs'd on the Water under the Body will make it preſs the Bottom, as much as if the Body were actually at bottom; the Body in that Caſe lo-

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ſing all its Exceſs of Gravity above that of the Water, and the Water gaining it.

Hence it follows, that a falling Cloud, when it comes to an uniform Motion, will not only add to the Weight of the Air as much as the Weight of an equal Bulk of Air; but even as much as its whole Weight amounts to, tho' it be ſpecifically heavier than the Air about it.

All the Diminution of Weight that can be allow'd in this Caſe is this. If we imagine the Air to have a ſmooth, regular Surface, as we have at firſt ſuppos'd, (or if that be not allow'd, we may take any imaginary Surface of it above the Clouds) when a falling Cloud is diminiſh'd in Bulk, (as when it is chang'd into Rain) the Surface of the Air will ſubſide in proportion to that diminution, and therefore will weigh leſs, by ſo much as is the Weight of a Quantity of Air equal to the Bulk that Cloud has loſt: But when the Drops of Rain after their Acceleration (occaſion'd by their Exceſs of Gravity above that of the Air) are come to an uniform Motion by the Reſiſtance of the Air, they reſtore to the Air the Weight that it had loſt. Now this uniform Motion being acquir'd in about two Seconds of Time, and the Diminution of Gravity in the Air being inſenſible, when compared to near three Inches of Mercury (for ſuch is the Variation of the Barometer with us) can no way be the Occaſion of thoſe ſo ſenſible Alterations in it, which happen ſome time before Rain or Fair Weather.

Add to this that the whole Quantity of Rain that falls in England *and* France, *in the Space of one Year, ſcarce ever equals two Inches of* Mercury: *And in moſt Places between the* Tropicks, *the Rains fall, at certain Seaſons, in very great Quantities, and yet the* Barometer *ſhews there very little or no Alteration.*

V. An

V. *An Account of an extraordinary Effect of the* Cholick: *communicated to the* Royal Society, *by that curious Anatomist Mr.* St. Andre, *and read* March 21. 1717.

THE *Peristaltick* Motion of the Intestins is by all Anatomists suppos'd the proper Motion of those Cylindrical Tubes.

The use of this Motion is to propel the Chyle into the *vasa lactea*, and to accelerate the grosser Parts of the Aliment downwards, in order to expel them, when all their nutritive Contents are extracted.

This Motion thus establish'd, it naturally seems to follow that an Inversion of it (call'd for that Reason an *Antiperistaltick* Motion) shou'd force the *Aliments*, *Bile*, *pancreatick Juice*, and lastly the *Faces* to ascend towards the Mouth.

The Cause of this imaginary *Antivermicular* Motion, is assigned to a Stoppage of the Intestin, or to a great length of it being ingaged in the same manner as the Fingers of a Glove are choak'd by inverting the Glove in drawing it off: Or like as a Silk-Stocking, which when 'tis not gartered, falls upon the Foot, and is in a manner strangled, so that some Force is required to bring it up again.

This suppos'd, the *Antiperistaltick Hypothesis* seems at first Sight very natural, and answers most Difficulties. For if the *Vermicular* Motion accelerates the Contents of the Intestins downwards; the *Antivermicular*, by the Law of Contraries, should force them upwards towards the Mouth.

Was

Was this Suppoſition as certain as 'tis generally receiv'd, I ſhou'd not preſume to advance that there is no ſuch thing as an *Antiperiſtaltick* Motion of the Inteſtins; nor that the *Miſerere mei* is oftner a violent Contraction of the Abdominal Muſcles, than a Stoppage or Inverſion of the Inteſtins, as 'tis ſuppos'd.

So laying aſide all Prevention, let it be granted that this Diſeaſe is a violent Contraction of the Abdominal Muſcles, as I have already ſuppos'd it, caus'd by the Redundancy of the Inteſtins or their Contents. Then comparing the Symptoms of this Diſeaſe, with thoſe of the different Kinds of *Hernias*, we ſhall find by the Analogy of the Parts, Reaſon and repeated Experience, that the *Chordapſus*, ſo call'd by *Celſus*, is a Diſeaſe in which the Inteſtins and *Omentum*; at other Times the *Pancreas* or *Spleen*; nay, even the *Meſentery* it ſelf are forc'd through the *Diaphragma* into the *Thorax*.

All theſe tender Parts being ſtrongly compreſs'd, by the continual Motion of this Muſcle, muſt by conſequence cauſe the ſame Accidents as in the *Bubonocele* or compleat *Hernia*, there being no difference in theſe two Caſes; but that the firſt is a ſtrangling of the *Inteſtin* by the *Diaphragm*, and the latter a choaking of the *Inteſtins* by the *Abdominal Muſcles*.

One Example of the many of the like Nature, that I can produce, will much confirm this Aſſertion, and may ſerve to convince any Perſon that is impartial.

The Caſe is this: A Gentleman that came to Town yeſterday was Sevennight in good Health, meeting with ſome Friends, drank a great deal of new bottled Oat-Ale, after ſome Pints of Wine. Theſe Liquors fermented ſo violently in his Stomach and Inteſtins; that he was taken with a violent *Cholick* the ſame Night.

In the morning an Apothecary was ſent for, who adminiſtred a Clyſter, and took ſome Ounces of Blood

to relieve the Patient, who complain'd of a great Pain in his left Side.

The Clysters being repeated the Night following, as also the next Morning, and the Patient growing worse; the Apothecary, without Order of any Physician, gave him a violent Vomit: which operated Eight or Nine Times: This added Fewel to the Fire; and the Patient having from that Time been in a desperate Condition, two eminent Physicians were call'd, who order'd that the Clysters shou'd be repeated: But they not prevailing, I was sent for about six Hours before the Patient died: I found him complaining of a violent Pain in all the Region of the *Abdomen*; a frequent Inclination to vomit; having a great Difficulty of breathing, together with a very slow Pulse; his Belly being as hard as a Stone, tho' not swell'd.

This last Indication made me conclude, that the Disease was a violent Contraction of the Abdominal Muscles, which had overcome the *Diaphragm*, and that probably the *Intestins* might be forc'd into the *Thorax*.

I was the more confirm'd in this Opinion from the Examples of the like Case, which I shall shortly lay before the Society; upon which I order'd a Fomentation of hot Milk, adding to every Quart a Drachm of *Liquid Laudanum*, which in these Maladies gives great Relief: But before it cou'd be got ready, the Patient expir'd in a violent Convulsion.

My Opinion having been highly censur'd by the two Physicians; I open'd this Gentleman, to justifie my self, or to own my Fault openly, if I had been mistaken: But as the thing happen'd as I conjectured, those Gentlemen will forgive me for taking the Liberty of justifying my self.

In opening this Body, I found the Abdominal Muscles so much contracted, that it was almost impossible to penetrate them with a very sharp Scalpel. Upon

Upon Examination, I found the Stomach empty, and ſome Parts of the *Duodenum*, but the *Jejunum* and *Ilium* ſo much diſtended with the fermented Oat-Ale, that the *Ilium* had four Inches of Diameter, and the *Colon* above eight.

The *Ilium* was alſo pretty much inflam'd in its inferior Part; and all the Valves of the *Colon* were obliterated, by the great Diſtention of that *Inteſtin*.

But the greateſt Diſaſter was, the Dilatation made in the *Diaphragm*, as I ſuppos'd; made juſt upon the Chink which remits the intercoſtal Nerve to the *Viſcera* of the *Abdomen*, through which a Portion of the *Colon* was forc'd, and the greateſt Part of the *Omentum* and *Pancreas*.

Theſe tender Parts being choak'd, ſoon inflamed, a Mortification of them following; and a Rupture of the *Pancreatick Vein* caus'd an internal *Hæmorrhage*, which fill'd all the left Cavity of the *Thorax*, inſomuch that the whole left lobe of the Lungs was compreſs'd almoſt under the *Muſculus Scalenus*.

The Quantity of extravas'd Blood was very great, and it was not in the leaſt coagulated.

I have brought the diſeas'd Parts with me, to ſhew the Society the Certainty of this Account, and I ſhou'd have been more particular in proving the Impoſſibility of the *Antiperiſtaltick* Motion; if Doctor *Hagumot* had not prevented me by his *Memoir*.

This Gentleman is not far from Truth, and what he ſays is certain: but I am ſurpriz'd that the like Caſe has not occurr'd in his Practice.

VI. *An*

VI. *An Account of Two late* Northern Aurora's, *as they were observ'd at the Vicarage of* Sutton at Hone *in* Kent. *By the Reverend* Edmund Barrell, *Prebend of* Rochester.

ON *February* the 5th $171\frac{6}{7}$. at Eight at Night an *Aurora Borealis* appeared. It occupied at least $\frac{1}{6}$ or near $\frac{1}{5}$ of the Horizon; it was low, and shot out bright Rays, and, I believe, would have appeared very light, had it not been that the Moon shone at the same time, being about Five Days old, and that the *Aurora* disappear'd before the Moon set.

Again, on the 30th of *March* following, there was another *Aurora Borealis*. I saw it not till past Nine: 'twas dim then, and its highest Part cover'd the lowest Star in *Cassiopea*'s *Chair*. It did not seem due *North*, but one Point to the *West*. About Ten it shot out very bright Rays, high, and tending somewhat towards one another. Near Eleven a Clock, there was (besides the *Northern* Brightness) a long Streak, not very broad, extended *East* and *West*: Which beginning in the *Serpent's-Head*, near *Hercules-Club*, and covering *Arcturus*, proceeded near *Berenices Hair*, and so went over *Cor Leonis*, and thence to the *Canicula*, and ended a little beyond that Star. It shone very bright at first, but faded away in about Eight or Nine Minutes. If it had Motion (which I am not sure of) it was Southward. I waited for the next Fit of Brightness of the *Aurora*; and in about Seven Minutes, the *Eastern* Part of the Streak, *viz.* from the *Serpent's-Head* to near *Berenices Hair*, became visible again tho' dim, and was quite effaced in Four or Five Minutes more: And I did not yet perceive any Change of its Place.

F I N I S.

Errata. Page 556. in the Note, lege *Lib.* xxxv. pag. 563. line antepenult. lege *Anderida*.

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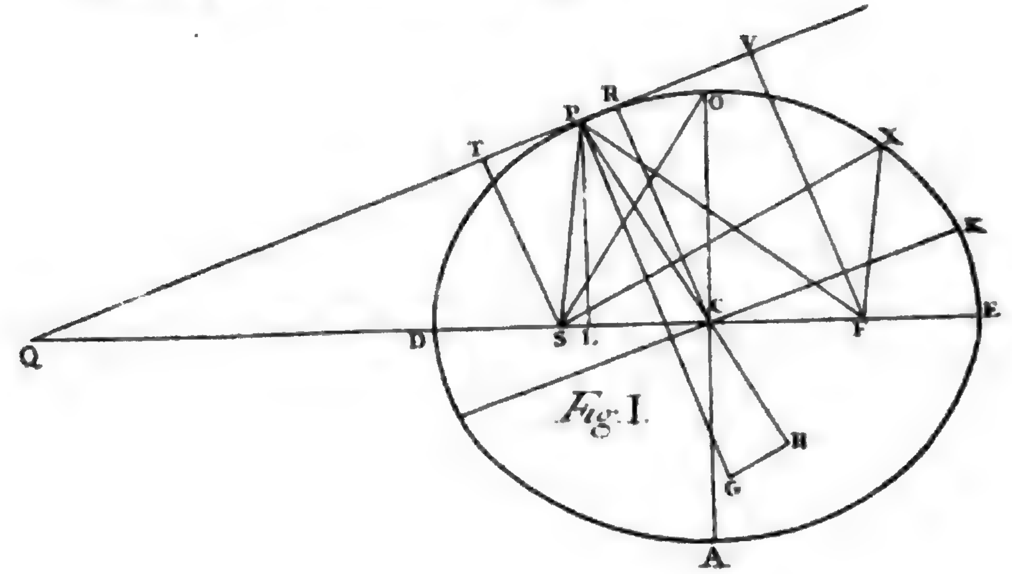


Fig. I.

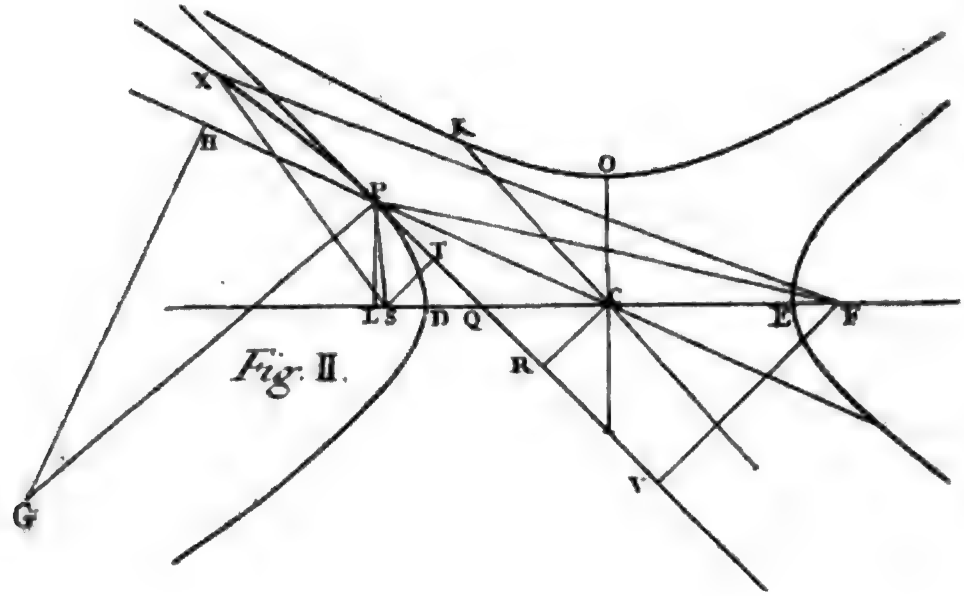
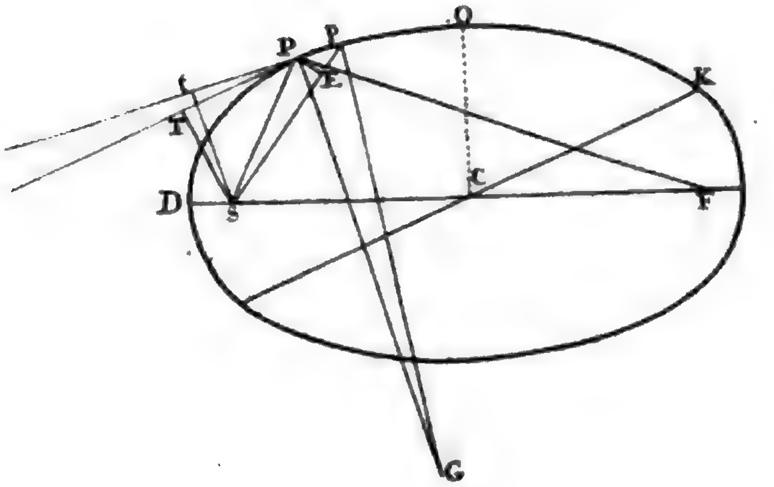


Fig. II.

Fig. III



PHILOSOPHICAL TRANSACTIONS.

For the Months of *April*, *May*, and *June*, 1717.

I. *An Account of the* Aurora Borealis, *seen at* London, *on the* 30th *of* March *last, as it was curiously Observ'd by* Martin Folkes, *Esq*; R. S. Soc.

BEing in the Street, between 8 and 9 a Clock on *Saturday* last, (30 *Martii*) I perceiv'd a Light over the Houses to the Northwards, little inferiour to that the Full Moon gives when she first rises. Upon this, suspecting some such Meteor as we saw the last Year, I made all the hast I could into the Fields, where I immediately found my Conjecture verified; and was for some time agreeably entertain'd with the sight of an *Aurora Borealis*, attended with most of the *Phænomena* that have been describ'd in that very remarkable one of the 6th of *March*, 1715-6.

The whole Northern Part of the Horizon was in the same manner cover'd with somewhat resembling a very considerable Light, whose lower part was pretty well defin'd by the common Edge of the Cloud, but the upper dy'd away more gradually. This upper Limb of the Light resembling the Arch of a Circle, whose highest Point between 9 and 10 of the Clock (when the Meteor was most considerable) was elevated about 12 Degrees, and bore, as I imagin'd, about 20 deg. Westward of the due North. It touch'd the Horizon in the West at the distance of about 65 or 70 Degrees from the North, whence the whole intercepted Arc of the Horizon would have been of near 100 Deg. had not some few Degrees in the East been hid by Clouds which lay between us and the Meteor.

The seeming black Cloud, when I first saw it, ran nearly parallel to the Horizon, and at the distance of 6 or 7 Degrees: but in about half an Hour it changed its Figure very

very much, ſinking down in the North to about half its height, and riſing in the Weſt near as much. What I principally took notice of this for, was that the Light iſſuing from behind it did not change with it, but remain'd of the ſame Figure, however the Cloud approached or receded from differing Parts of its Limb.

There aroſe at firſt ſome Streams in the *N.N.W.* but of no conſiderable Length, few of them paſſing 5 Degrees above the Arch; but beginning from behind the ſeeming Cloud, ſo as to be about 12 Degrees high in all. They were Pointed at the Ends, and nearly vertical to the Horizon. Between times there was nothing but the Arch to be ſeen, and that only reſembling a common *Aurora*; and again in an inſtant, by a ſort of tremulous Motion, ſeveral Parts of it would appear converted into a vaſt number of parallel Streams, for the moſt part very little higher than the Arch it ſelf. About 20 Minutes before Ten, a ſmall part of the Arch, almoſt due North, grew remarkably lighter than the reſt, and continued to encreaſe for about half a Minute; when there ſuddenly broke out ſome very tall Streams of at leaſt 60 Degrees high, as I found by one in particular which aroſe full North, and paſſing over the Pole Star itſelf, reach'd ſome Degrees beyond it. This was the moſt remarkable time of the Appearance, ſome ſuch Lances, though not ſo high, immediately ſhooting out of the Place that firſt of all radiated, as did ſome more a good way to the Eaſt. They were all nearly Perpendicular to the Horizon, and moſt of them did ariſe quite from the black Subſtance at Bottom, tho' I ſaw ſome few that did not reach ſo low, appearing as if their lower Parts had been broken off. Some of them were full as bright as any I ſaw the laſt Year, the Axes (if I may ſo call them) of ſome of the talleſt Streams coming up very near

near to the Colour of that pale Fire we ſee in ſome ſorts of Lightning.

About this time the Ground Weſtward was all cover'd with an odd ſort of Miſt, the ſame from which I remember laſt Year a great many People ſaid there came an ill ſmell, which I did not at all perceive; however as I remember it to be the very ſame Appearance, I thought it might not be improper juſt to take notice of it.

About 10 the *Phænomenon* very much decreas'd, and ſo continued till after 11, only ſending up now and then 2 or 3 Streams. At half an Hour after 11 it was again pretty much encreas'd, and I ſaw it again ſend out ſome Streams almoſt as conſiderable as any I had before ſeen this Evening; the Arch yet continued, but not ſo entire; and from what I could judge, its middle was ſome Degrees nearer the North than when I firſt took notice of it. Till a quarter of an Hour before 12 the light continually abated, and then I left it; but a Watchman, I order'd to bring me an Account of it next Morning, tells me it continued till towards Day-break, but never ſtream'd remarkably after I went away.

Tho' I could not this time ſee any Stars through the black Matter at Bottom, I am ſenſible it was not a Cloud, tho' it bore the reſemblance of one: for when a real Cloud (as ſeveral ſmall ones did) came over any part of it, their difference was very conſpicuous.

I have ſince receiv'd two Letters, one from *Wisbich* in the Iſle of *Ely*, the other from within 14 Miles of the *Bath*, both which take notice of it, tho' with no further Particulars, than that on *Saturday* Night, they had ſeen the ſame Light, tho' not ſo conſiderable, as in the beginning of *March* the laſt Year.

II. Guil-

II. Guilhelmi Muſgrave *Regiæ Societatis Socii, de* Britannia *quondam pœne Inſula*, *DISSERTATIO.*

CUM *Belgium* noſtrum a *Britannico* adluitur Oceano, illo latere Inſulæ hujus triquetræ, quod eſt ex adverſo *Galliæ*; viſum mihi fuit, priuſquam id deſcribere conarer, antiquam & diu agitatam movere quæſtionem, de *Britanniæ* cum *Gallia* conjunctione, &, an revera unquam eſſet, exquirere.

PRIMUM igitur, poſita Cherſoneſo *Britannica*, utrum exedi potuerit: Deinde, utrum exeſa fuerit, ediſſeram.

DE priori propterea dicendum, quod a docto & magni nominis Viro, *Voſſiorum* altero, ſtrenue negatum ſit, unquam, ubi hodie Fretum eſt, fuiſſe Cherſoneſum: & quidem ideo negatum, quoniam, illo ſentiente, nihil ei deterendæ dividendæque par inveniatur. Ut *Taprobanam* (Inſulam *Ceylon*) a vicina continente non avelli probet Vir Clariſſ. [*Otio* (a), inquit, *abundant, qui iſtiuſmodi* Ægyptiorum *fabellis, jam millies productis, totieſq; recoctis, aurem commodant. Quam conſtans & tenoris ſui obſervans ſit rerum natura, patet e Boſporis, omnibuſque omnium terrarum Fretis. Iis cum præcipue Marium & ipſius Oceani vis ſemper incubuerit, eadem tamen ubique a tot annorum millibus & ab ipſo, ut veriſimile eſt, rerum exordio, ſervaret intervalla. Currant licet, ac recurrant Undæ, allatrent undequaque Fluctus, fortius eſt Elementum quod re-*

(a) In Notis ad Melam.

ſiſtit; quam quod oppugnat. Exeſi Scopuli, ac vaſta maris antra, ſatis ubique oſtendunt, quantum Oceani impetus lapſu ſeculorum poſſit efficere: verum hæc ipſa quoque, quid non poſſit efficere Oceanus, multo clarius oſtendunt.]

Hæc *Iſaacus.*

UT diſputationem ea de re ingrediar, Vir hic Clariſſ. Naturæ, nec qua agit illa nec qua patitur, ſtatum ac conditionem, ex omni parte, ſic, ut eſt revera, animo ſatis advertiſſe videtur. Cum non de *Taprobanæ* ſolum Freto commentatur, ſed de Fretis in univerſum, & quidem Argumento *a conſtante & tenoris ſui obſervante rerum natura* accepto, videamus quam hæc cum Oceanis & Freto noſtris conveniant, & in iis quam conſtanter agat Natura.

OCEANI *Britannici*, prout nunc dierum eſt, cum latitudo tum profunditas inveſtiganda, ut ex iis de priſco ſeu Freto, ſive Sinu, poſſimus ſententiam ferre. Ut autem eas comperiamus, adeunda eſt Tabula *Halleiana*, ſui generis omnium accuratiſſima, ex juſſu Regis *Guilhelmi* ejus nominis Tertii conſtructa. Ea docemur, in Oceano *Britannico,* ubi Terrarum hiatus, hac illac ampliſſimus eſt, a veterum *Ocrino* (*Lizard-Point*) ad Inſulam ei oppoſitam *Uſhant*, unum eſſe gradum cum ſemiſſe, id eſt Leucas quaſi triginta, ſive milliaria 90. Hinc Oceanus ſe in oriente parum adducit, at multo magis ubi Promontorium in eum procurrit *Normannicum*: ibi enim eſt dimidio adductior; cum inter *Peverel-Point*, & *Cap. de Hague* e regione ſita, Leucarum *Anglicarum* quaſi 16 diſtantia ſit. Tunc ſe iterum effundit, ubi *Sequanam* recipit: at brevi in arctum agitur, inter *Beachy-Head* & *Cape St. Vallery.* Dein paulatim anguſtior, faſtigiat ſe molliter, uſque dum in Fretum contrahitur, inter *Neſs Anglorum*, & *Gallorum Blackneſs*, non amplius octo Leucis, id eſt 24 milliaribus patens. Terræ tunc

tunc aperiuntur longe lateque vaſtiſſime, & ſpatium Mari faciunt *Germanico*.

HÆ ſunt Oceani Fretique *Britannici* diverſæ latitudines; quibus apparet, eas, ſi non continuo, tamen adeo rara tamque exigua cum ampliatione minui, ut argumento noſtro nihil inde queat derogari. Ita enim Oceanus contrahitur, ut qui initio, ſeu Faucibus ejus *Britannicis*, Leucas triginta, ſive Milliaria prætcrpropter nonaginta latus ſit, poſt Leucas 153 circiter, ſive Milliaria 460, (quæ hujus Oceani longitudo eſt) ad 24 milliaria contrahatur; id eſt ad primæ latitudinis partem quaſi quartam.

PROFUNDA hujus Oceani altero jam loco ſunt expiſcanda, & quidem optime beneficio ejuſdem Tabulæ. In ea dividitur Oceanus *Britannicus* una cum Freto, in Columellas numero decem, oblongas. Harum ſingulæ latera ſunt ex circulis Meridianis accepta; quæ cum in plano ducta ſint, videntur eſſe recta. Columellæ terminantur adverſis *Galliæ Britanniaque* litoribus: hoc eſt, Lineis huc illuc curvatis in litorum morem.

INCIPIAMUS a prima in occidente, quæ & longiſſima Columella eſt: & (præmiſſo, quod *Hiberniam* & *Galliam* inter, Oceanus orgyias altus ſit in locis compluribus octoginta; uti paulo ulterius in aperto Mari, 100, 120, 140) notandæ ſunt in prima Columella *profunditates* omnium altiſſimæ; quæ decies exploratæ ſe habent, ut 58, 66, 63, 65, 58, 65, 68, 60, 60, 60. quæ *profunditatum* Summæ faciunt 623 orgyias. Eæ per decem, i. e. *profunditatum* numerum, diviſæ, mediam earum *profunditatem* oſtendunt eſſe 62.

IN Columella altera, decem altiſſimarum media *profunditas*, ſimili modo inveſtigata, eſt orgyiæ 51. In tertia 51. In quarta 40. In quinta 43. In ſexta 40. In ſeptima 36. In octava 37. In nona 33. Poſt nonam Columellam, cum Oceanus in lævam flectitur, &

 obli-

obliquus in Fretum deſinit, accipiam illud ut Terris interjacet, in *Britannia* locis appellatis *South-Foreland* & *Haſtings*, in *Gallia St. Valery* & *Eſtaples* incluſum. Hic *profunditatum* decem media eſt 30. In Freto anguſtiſſimo 16: quæ ad profunditatem mediam in prima Columella, eſt, ut 16 ad 62; id eſt, ut 1 ad 4 fere; & ad altiſſimam profunditatem *Galliam* inter *Hiberniamque*, ut 16 ad 80, i. e. ut 1 ad 5: ad altiſſimam in aperto Mari, ut 16 ad 140; i. e. ut 1 ad 9 fere.

QUA proportione minuitur altitudo Maris, ea creſcit Terræ Mari ſubjectæ acclivitas; & eſt illi in *ratione inverſa*: quæ utique propoſitio, ſi non ex omni parte vera, (propter orbis figuram minime rotundam) tamen adeo veræ proxima eſt, ut argumentationi noſtræ ſufficiat. Eſt ergo Terra, in Freto noſtro anguſtiſſimo undis ſubjecta, quam in Oceani *Britannici* faucibus, orgyias 46, id eſt pedes 276 altior; & quam Terra, *Galliam Hiberniamque* inter, Oceano ſubjecta, orgyias 64, ſive pedes 384 altior; & quam Terra, aperto Mari ſubjecta, 124 orgyias, ſive pedes 744 altior. Vide quanta ſit Terræ ab alto Mari ad Fretum acclivitas; eaque ut ex calculo prædicto patet, fere continua. Hæc eſt Oceani *Britannici*, tam in illius Latitudine, quam Profunditate contractio.

AGE, nunc tendamus ultra, velis expanſis, in Oceanum *Germanicum*: Hic Mare ſubito patentius, ſic, ut etiam profundius: quod inter Promontoria *North-foreland*, *Orfordneſs*, Oppida *Caletum* & *Oſtendam* interfluit, decem maximas habet menſuras, quarum media 24 Orgyias cum $\frac{4}{5}$ continet: quod inter *Orfordneſs*, & *Yarmouth*, *Texellam* & *Oſtendam* eſt, maximas decem menſuras habet, quarum media 25 Orgyias cum $\frac{1}{5}$. Quicquid ultra eſt, Terris, hinc ad occidentem, illinc ad orientem ſe retrahentibus, vaſtiſſimus eſt Oceanus, in quo menſuræ ſunt ab orgyiis 45 ad 50 numero quamplurimæ.

HÆC

HÆC a Freto *Britannico* tam in oriente, quam occidente Terræ declivitas (quæ a Maris altitudine utrobique aucta patefit) omnino probat, *in ipso Freto jugum esse Terræ excelsum, acutum*; quod cum hodie non multum infra Maris superficiem esse reperiatur, olim se emergere, hoc est *Cherronesum olim hic fuisse, monstrat.*

ALIA sunt duo, quæ cum in hac re momenti sint immensi, tenore tum minus certo, & natura minus sunt constanti; quandoquidem a Maris motu & ventis accepta. In refluxu Maris Aquæ nunc quiescentes incubant arenæ; nunc eam molliter præterlabuntur. In æstu mitiori, Litus & ima Rupium blandissime lambunt, tenerrime osculantur. Fervente vero æstu, res omnino alia est: Aquarum Fremitus auditur, Fluctus cernuntur, & se non parum attollunt. Nihilominus sine multa strage, terrisve aliquot annorum millibus exesis, hæc omnia posse fieri, cum *Vossio*, cogitandum est.

SIN æstuanti Mari, quod altero loco dicendum est, Ventus superveniat (*Flat* autem *Ventus sic, ut vult*, & quis ei tempus statuerit, aut modum imposuerit?) papæ! quoti, quantique Fluctus advolvuntur! *Alpes* existimare licet Crystallinos, nisi quod cito diffluant. Montem enim Mons invadit, detrudit, discutit; tantisper ejus spoliis adauctus dum in cœlum altissimus exurgit Aquæ Mons. Tanta vis Aquarum ex Oceano occidentali in *Britannicum* immittetur, & tanto impetu, quantus in universo Terrarum orbe rarus inveniatur; imo quantus ab ipso rerum initio rarissime. Oceani *Britannici* tum brevia, tum angustiæ, continuo pœne (quod ostensum est) crescentes, faciunt, ut Aquæ sic impulsæ mirum in modum eleventur, & in Isthmum (quem argumenti gratia fuisse damus) arietent, ita ut ab iis Isthmum exundari, deteri, àblui; sicque Insulam fieri *Britanniam* non videatur ἀδυνάτων ἐν; verum e contrario factu probabile. Quantæ Ventorum, at præcipue

Ze-

Zephyri Cauriq; virtutes sint, in cogendo impellendoq; hoc Oceano *Britannico*, paucis expendam, a Doctissimo viro *Rad. Bohun*, (*Novi* olim *Collegii* Socio, qui *de Ventis* omnium eruditissime scripsit) hac in re adjutus.

ETIAMSI Zephyrus a Poetis *vitam rebus ferre* dicatur.

——ejusque tepentibus auris
Laxent Arva Sinus; ——

interdum ita fit, ut

—— *Euriq; Zephyriq; tonet domus* ; & ut
— *Zephyro multæ turbentur arenæ.*

Adeo sævus, horribilis, iracundus sæpe Zephyrus, Vires in Oceano, qui *Europam* & *Americam* vastus interjacet, acquirens, & in amplissimo hocce campo recensens, vix concipi potest, quanto *Britanniæ Galliæque* oras impetu invadat. Exploratissimum enim est, in hasce oras eum communiter anni plus dimidio flare, (quod jam olim a *Julio Cæsare* notatum) & flatu eas sævissime verberare: maxime autumno, a quo sumunt originem Tempestates Idiomate nostro dictæ [*Michaelmass-Storms*] eumq; adeo interdum sævire, ut si cum æstu fervente jungi Ventus hic acciderit, tam Oceanus *Britannicus*, quam Fretum *Sabrinianum* immane quantum augeantur. *Sabrina* vastissime turget. *Uzella* longe lateque *Somersettensem* Agrum exundat: Mirum ab hisce Cataclysmis quantum mea patria perpessa est. Continuatur æstus ad usque *Tewkesbury*, id est milliaria magis ducenta. Apud *Chepstow* Aqua pedes interdum octoginta assurgit. Idem fere dicendum de Oceano *Britannico*, Venti ejusdem viribus elato: nisi quod hic, Chersoneso jam effracta, liberius Aquæ moveantur, non adeo sistantur, non tantum eleventur; quæ utique Aquarum libertas ante Chersonesum abruptam, nequaquam adeo magna esse potuit.

HAC

HAC igitur de causa [*Zephyro* nempe, *Cauro*, sive alio Ventorum aliquo, Maris æstui superveniente] Oceani *Britannici* Undam in Isthmum validissime impingi, & ab illis primum ejus superficiem, quæ ex Silice & Calce (prout hodie Terræ e regione oppositæ) constabant, ablui; deinde Isthmi quod reliquum erat, spatio bis mille annorum & eô amplius, Aquæ fluxu refluxuq; ad 16 orgyias, quæ hodierna Freti hujus (quod diximus) altitudo est, atteri credibile, verisimile est,

TANTUM abest, ut *Vossio*, Fretorum perpetuitatem a naturæ in operibus suis *constantia* tenoreque eodem arguenti, fidem habeamus, ut e contrario Fretum hocce nostrum illius *inconstantiæ* deberi, lubens agnoscerem. Vir ille clarissimus, naturæ usitatum agendi modum unice respiciens, *extraordinarium prætermisit*; qui tamen, in raris hujusmodi effectis, potissime videtur respiciendus. In Freto *Siculo* considerando, ejusque diducendi modo investigando, quis Ignis subterranei supra modum erumpentis, tamquam Causæ hac in re probabilis, non meminerit? nisi istam *Catanensium* ἀτοπίαν consecutus, qui (tradente *Alphonso* † *Borello*) post Eruptionem *Ætnæ* diu intermissam, Ignem ejus immodicum ne semel quidem unquam fuisse, satis insulse putavere. Ut Vento nihil inconstantius, sic ad Fretum hoc aperiendum (posito causarum apparatu cætero) nihil conducibilius: & cum eo res deducta est, *fortius elementum esse quod oppugnat, quam quod resistit*, (aliter quam *Vossius* statuit) omnino probabile mihi videtur. *Salmasium* ille, Virum & candidum & doctum convellit, quod de Navibus absque Costis & Interamentis, ita scripsit, ac si in *Burgundia* mansisset, nec quid rei Navis aut Mare foret, intellexisset. Nequeo satis *Isaacum* mirari, quod *de Mari & Ventis* scriptor luculentus, horum vires in Mari

† In Libro de *Incendiis Ætnæ* p. 117.

tur-

turbando agitandoque in terris obruendis abrumpendisq; prorsus omitteret; atque adeo quod *Batavus* omitteret: ea scil. Regione oriundus, quæ Mari & Vento tam obnoxia.

NON alienum erit hic Inundationum aliquot exempla, uti revera fuere, in medium proferre; quibus abunde patet, Terræ faciem frequenter obrui, & ab iis non parum mutari. Hic autem nihil necesse est, ut *Helicen* & *Burin*, *Achaiæ* Urbes adeamus: de quibus tamquam magnarum Inundationum argumentis, (1) *Ovidius*, & diu ante illum (2) *Aristoteles*. Gravissimas fuisse Oceani nostri, tam *Germanici* quam *Britannici*, satis ostendunt Historici Geographique.

IN *Zeelandia* (3) Insulæ undecem, & in iis Oppida & Pagi (quorum hodie summitates aliquæ, refluxu Maris in conspectum veniunt) numero ter centum (4) obruebantur.

ANNO 1014. [*Mare Litus egreditur III. Cal. Octob. & in* Anglia *Villas quamplurimas innumerabilemque populi multitudinem summersit.*] (5) *Simeonis Dunelmensis* Historia de Gestis Regum *Anglorum*. De hac, ut opinor, Inundatione videatur etiam Chronicon *Joh. Brompton* (6).

ANNO 1099. [*Tertio Non. Novemb. mare Litus egreditur, & villas & homines quamplures, Boves & Oves innumeras demersit*] *Sim. Dunelmensis* (7) Historia.

A D. 1176 [*Mare extra fines in* Anglia *erumpens multos in* Hollandia *homines & pecora absorbuit, & quasi*

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(1) Invenies sub Aquis, & adhuc ostendere nautæ
Inclinata solent cum mœnibus oppida versis.
1 *Metam. Lib.* 15.

(2) Τὰ δὲ ἀνάλογον συμπίπτει τούτοις καὶ ἐν θαλάσσῃ· Χάσματα γὰρ γίνεται θαλάσσης καὶ ἀναχωρήματα πολλάκις, καὶ κυμάτων ἐπιδρομαὶ ποτὲ μὲν ἀντανακοπὴν ἔχουσαι ποτὲ δὲ πρόωσιν μόνην. ὥσπερ ἱστορεῖται περὶ Ἑλίκην τε καὶ Βοῦραν. *Arist.* de Mundo. (3) *Heylin's* Geogr. L. 2. In Belgio. (4) *Laet.* Descriptione Belgii, pag. 124. (5) Apud Historiæ *Anglicanæ* Scriptores X. pag. 171. (6) Apud eosdem, pag. 892. (7) Pag. 224.

post

post biduum furore sedato in semet ipsum rediit.] Chronicon *Johannis Brompton.* (8)

INSOLITAM maris inflationem & commotionem Anno D. 1250. factam, tradit *Matthaeus* (9) *Parisiensis.* [*Unde Mare perturbatum fines solitos pertransiens, tam horribilem mugitum cum fremitu edidit, ut per remota Terrae spatia, non sine stupore audientium, reboaret. Visum est etiam sub opaca nocte ipsum Fretum quasi accensum ardere, & Fluctus Fluctibus conglomeratos dimicare. Apud* Winchelese *plusquam* 300 *domus cum quibusdam Ecclesiis per Maris violentum ascensum sunt submersae.*]

ANNO 1251. inquit idem (10) *Matthaeus* [*In* Frigia (*quae* Friselandia *appellatur*) *Aqua Diluvium fecit particulare, occupans Terrae illius spatium itineris circiter septem dierum. Post* 40 *dies ille damnosus Fluctus in locum suum remeavit.*]

ANNO 1286. [*Ingruente fortissimo Vento, flante de partibus Orientis, qui & Eurus dicitur, & fluxu Maris super provinciam* Hollandiae *terribiliter invalescente, praevaluerunt Aquae Maris, adeo ut Fossata, quae Terram ipsam & Mare disterminant, inopinatius quam credi poterat, transgrederentur; posuitque Terram fructiferam in salsuginem tam repentinus Maris impetus; qui per indigenas nullatenus poterat obviari: & maxima pars S.* Botolfi *submersa, hominumque & pecudum inaestimabilis periit multitudo.*] Ita Chronicon *Tho.* (11) *Wikes.*

DE *Hollandiae* Inundatione sic *Hadrianus Junius* (12) in *Bataviae* Historia [*Quadringentis abhinc annis inaudita* illa *Inundatione quae universam* Hollandiae *faciem longe lateque operuit, obstructo fluminis* (Rheni) *cursu, steriles arenarum colles Litus occuparunt, Mare, Terras, ipsamque Litoris oram attrivit.*]

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(8) pag. 1117. (9) Pag. 535. Ed. *Watsiana.* (10) Pag. 549. (11) Pag. 114. (12) Pag. 196.

ANNO 1404. quo beatissimus noster *Wiccamus* obiit, [*Tanta repente ruptis limitibus irrupit Aquarum influentia in* Cantio, *quanta nunquam fuerat illic ante visa, qua submersa sunt animalia numero & pretio excessiva: nec solummodo deflevit* Anglia *damna talia, sed ut fertur,* Selandia, Flandria & Hollandia, *per Undarum excrementa, innumerabilia sensit dispendia eo anno.*] Hypodigmate *Neustriæ* per *Tho.* (13) *Walsingham.*

[REGNANTE Edwardo I. *cum Oceanus ventorum violentia exasperatus, hunc* (Cantii) *tractum operuisset, lateq; hominum, pecorum ædificiorumque stragem dedisset, &* Bromhill *viculo frequente pessundato, etiam* Rother, *qui hic prius se in Oceanum exoneravit, alveo emovit, ostiumque obstruxit, novo in Mare aditu compendio per* Rhiam *aperto.*] *Camdenus* in *Britanniæ Cantio.*

QUID quod *Tungros,* oppidulum *Leodiense,* a Mari pene centum milliaria jam remotum, Mare quondam adluere opinati sunt Viri doctissimi, argumento non uno persuasi (14).

NEQUE nostra ætas caruit hujusmodi Exundationibus: Narrant Noveliæ *Feb.* 27. 1713. in *Essexia,* plura Terræ jugerum millia, per milliaria aliquot, inter *Barking* & *Purfleet,* eversis obstaculis, Maris influxu obrui.

HÆC de Oceani *Germanici* exundationibus: *Britannici* nostri, & *Sabrinæ,* neque pauciores, neque minores sunt. Enimvero vidimus ætate nostra Isthmum, uno eodemque *Pedredi* fluxu & refluxu (Terræ superficie prius ab Agricolis semota) dilui, fluviumque veteri cursu relicto novum acquirere: hoc, inquam, unico fluvii istius æstu factum vidimus, nullo auxilium præbente vel Undas adigente Vento.

(13) Pag. 564. Ed. *Francofurt.* MDCIII. (14) *Verstegan*'s Antiquities, pag. 102. *Ray*'s Physico-Theological Discourses, 1693, pag. 169.

SA-

SABRINÆ fluminis impulsu fieri probabile est Exundationem illam, qua in Agro *Monumetensi*, Parœciæ N°. 26. A. D. 1607. mense *Januario* Aquâ obruerentur: Cujus eodem Anno publicata fuit (15) Historiola.

JOHANNE rerum *Anglicarum* potito [*Subita & improvisa Aquarum Inundatio pluribus in locis per* Angliam *facta est, unde plures homines submersi sunt, & domus eversæ, maxime apud* Excestre *& Sanctum* Ivonem] Imagines Historiarum Autore (16) *Radulfo* de *Diceto*. [Post diutinam malaciam, mare *Vergivium*, adeo per totam hyemem, regnante tunc *Henrico* secundo, tempestatibus agitabatur, ut toto illo temporis spatio, Navicula nulla ad *Hiberniam* adpulsa, de reliquo terrarum orbe nihil apud eam inauditum. Terror hinc universus, tamquam malo impendente quodam gravi, de cælis missc. Arenarum aggeres, in Australi *Cambria*, quasi Cataclysmo abluebantur, Litora subvertebantur] *Giraldi Cambrensis* (17) *Hibernia expugnata*.

AT ô quam terribilis illa tempestas, qua Maris Undam in Oceanum *Britannicum* impellente *Africo*, ita ille turgebat, ut Pharos illa celeberrima, *Eddistone* appellata, quæ fuit e regione *Plimuthæ*, tamquam in contemptum *Æoli Neptunique* fabricata, quasi ludibrio habita simul cum ædificatore dirueretur. Cujus utique tempestatis in hoc nostro Oceano si non eadem vis & potestas esse videatur, atque illarum in prædictis Oceani *Germanici* Exundationibus, propter majorem ab hisce stragem & damnum in *Hollandia Zeelandiaq*; factam; Litoribus hoc nostris rupibus munitis, quæ & duriores & altiores quam apud *Batavos* sunt, deberi judico.

TEMPESTATES (ut argumentum hoc conficiam) quæ in Oceanis *Germanico* & *Britannico*, nostra

(15) *Lamentable News from* Monmouthshire. (16) Pag. 710.
(17) Cap. XXXV.

& patrum memoria sæviere, & de quibus omnino constat, adeo fuere turbulentæ, ut si earum aliquæ in Chersonesum, ad hosce dies usque manentem, recta fuissent collineatæ, nullus (ut opinor) esset dubitandi locus, quin a tanta vi auferretur Isthmus: & tot annorum sæculis, quot illico dicentur, revera hoc accidisse nequaquam improbabile videtur.

SIN ex *Americâ* turbo maris æstui superveniat, mare, cœlum, omnia miscens, omnia confundens, ac si naturæ instaret dissolutio, (posse vero hæc concurrere nemo sanæ mentis ibit inficias) En causam huic negotio parem!

ALTERUM hujus Dissertationis membrum jam aggrediamur, & speciatim inquiramus, utrum excesa fuerit revera hæc Chersonesus; annon. Si de ejus dividendæ modo, qui fieri posset, conveniat, magna inde lux emanabit, unde argumenta, quæ a Viris doctis passim afferuntur, ad divisionis hujus probabilitatem arguendam, egregie confirmabuntur: præmisso nempe (quod hactenus fuit desideratum) Vento, ejusque in elidendo hoc Isthmo virtute omnium causarum maxima, tantoque negotio (cum cæteris) pari. Horum ego argumentorum nonnulla persequar; sed leviter tangam, utpote ab aliis fusius antehac tractata.

PRIMO, Terræ jugum illud notabile, quod Freto subjicitur, & de quo supra; quid aliud sibi vult, quam quod eo loci Terra olim multo altior esset; at Maris per aliquot annorum millia reciprocationibus, ad eum in quo nunc est statum, abluta & detrita? Præcipue, si advertamus Regulam hanc constantem & perpetuæ veritatis esse, *Maris* scil. *imum, quo magis Oceano præterlabente tritum, (quantum patitur ejus durities) eo magis planum & æquale reddi.*

QUID deinde Rupes in Freti Litoribus oppositis, sive Montes prærupti, albi, ex eadem materia, Calce nimirum & silice compositi, ad sex utrobique milliaria, sibi

sibi invicem, tamquam Tesseræ respondentes; quid, inquam, volunt, nisi olim interfringi se, & ablutione Terræ interpositæ disrumpi?

TERTIUM, apprime convenit cum isthac Chersonesi *Britannicæ* opinione, tractus illius, qui hodie *Rumney-Marsh* appellatur, ratio & ingenium. Durante enim Isthmo, cum Oceani fluxus eo tamquam obice sisteretur, æstuare eum necesse erat, atque adeo Terram illam *Rumneiensem*, utpote planam & humilem, in propinquo exundare. Hoc ostendunt Oceani *Britannici* Fluxus, hodieque hac planitie altiores, aggere fortissimo & magnis sumtibus aspulsi: Dentes item ostendunt, atque Ossa, sive Hippopotami sive alius cujusdam marini animalis (18), anno 1668, *Charthami*, altitudine pedum 17, dum puteus aperiretur, eruta: at luce clarius ostendit Anchora, non ita pridem ex alto loca hæc circiter effossa. Perrupto autem Isthmo obiceque jam remoto, Oceani unda subsidit, a Terra illa recessit, in alveum se contraxit; unde quæ olim Æstuarium, hodie Planities, longa viginti milliaria, lata octo, eaque fertilissima bobus saginandis aptissima reperitur.

NOVISSIME, fac Chersonesum olim fuisse, Lupos, aliaque animalia, generi humano inimica, posse huc migrare, conceptu facillimum est: at si illa non fuit, navigiis ea, tamquam ad tuendas & conservandas eorum Species, advehi, stulte cogitabimus?

NEQUE me moratur, quod nulla sive *Latinorum*, sive *Græcorum*, sive alius cujusvis populi Historia Chersonesi hujus abruptæ mentionem fecisse perhibeatur; (quamvis hoc nequaquam ex omni parte verum:) Dic sodes, Historiæ quam brevis sit ætas, si ad ætatem mundi comparetur. A rerum initio ad primam, quæ

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(18) Vide Clariss. *Somneri* Diatribam *Chartham-News* appellatum: Actorum Philosoph. *No.* 272. Et Clariss. *Wallisii* de hac Chersoneso Dissertationem. Act. Phil. *No.* 275.

nunc

nunc exstat (i. e. *Herodoti*) Historiam, 3500 circiter anni sunt; & a *Noa* Diluvio, 1800. At tam immenso temporis spatio (quod supra innuimus) quæ Causarum accidere possint συζυγίαι; quæque ex iis in orbe nostro fieri mutationes, nemo tam cito statuere debebit.

DIXI *hoc nequaquam ex omni parte verum:* quid enim planius illo *Virgilii*,

— *Penitus toto divisos orbe* Britannos.

[*Nonne putatis* (inquit eruditissimus & Antiquitatum *Britannicarum* scientissimus (19) *Joh. Twinus*) *vocabulum* [divisos] *habere eam vim ut significat abscissionem alicujus ab aliquo? Et Auctorem mire gnarum significationis fuisse, & rerum antiquarum maxime peritum, & bene memorem sui? Ad hac verba* Servius [*Quia olim juncta fuit Orbi* Britannia.] Nihil clarius esse potest ad demonstrandum Isthmi hujus divisionem veteribus fuisse notam. Ut omnino frustra esse *Vossius*, & nimio plus ὑποθέσει δουλεύειν existimetur, cum in Fabellis *Ægyptiacis* (quo nempe suæ opinioni habeatur honos) eam poni voluerit.

CONCLUDIMUS ergo e prædictis simul acceptis, *Britanniam non jam inde ab initio fuisse Insulam, sed ex Pæne-Insula factam: idque ut videtur, a Vento e sævioribus aliquo, cum Maris æstu concurrente & Isthmum perrumpente.*

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(19) De rebus *Albionicis*, pag. 22.

Extracts

III. *Extracts from Mr.* Gascoigne's *and Mr.* Crabtrie's Letters, *proving Mr.* Gascoigne *to have been the Inventor of the* Telescopick Sights *of Mathematical Instruments, and not the* French. *By* W. Derham, *Prebend of* Windsor, *and* R. Soc. Soc.

IN *Monsieur de la Hire*'s first Part of his *Tabulæ Astron.* published in 1687. I find an Invention, which was undoubtedly our Countryman Mr. *Gascoigne*'s ascribed to *Monsieur Picard*, and that is, the *Application of Telescopick Sights to Astronomical Instruments.* Mr. *de la Hire*'s Words are, *Paucis abhinc annis D. Picard insignis Astronomus, atque in eadem Academiâ* [*Regiâ Scientiarum*] *Socius, Dioptrarum crenas ab instrumentis sustulit, eorumque loco substituit Telescopia; quæ res Presbytis & Myopibus*, &c. In which Words it is not indeed expresly said that Mr. *Picard* was the Inventor of this way, but only that he applied Telescopes. But by reason it implies that it was that curious and ingenious Gentleman Mr. *Picard*'s Invention, and it is in effect claimed as such, in *Monsieur Auzout*'s Account of the *Telescopick Micrometer*, in the *Philos. Trans. No.* 21. therefore I think my self in Duty bound, to do that young but ingenious Gentleman, Mr. *Gascoigne*, the Justice, to assert his Invention to him; by reason all his Papers, that by the late ingenious Mr. *Towneley*'s Diligence could be picked up, are now (together with Mr. *Towneley*'s own Papers) in my Hands.

As

As for the Invention of the Micrometer, which Mr. *Auzout* claims as his and *Monsieur Picard*'s, I shall say little to it, Mr. *Towneley* having sufficiently prov'd it to be Mr. *Gascoigne*'s, in the *Philos. Transact.* No. 25. And the Descriptions and Draughts of that, and some other Instruments of that kind, are now by me, in Mr. *Gascoigne*'s own Hand, to confirm Mr. *Towneley*'s Account, if occasion were.

And as Mr. *Gascoigne* was the first that measured the Diameters of the Planets, *&c.* by a Micrometer; so I shall prove that he was the first that applied *Telescopick Sights* to Astronomical Instruments. In a long Letter to his sagacious Friend Mr *Crabtrie*, of *Jan.* 25. 164$\frac{0}{1}$. (wherein he describes his Micrometer, and shews his way of finding the Refractions, the Moon's Parallax, and how he measured the Diameters of the Planets) Mr. *Gascoigne* tells him how the *measuring Glasses*, which he had been speaking of, might be applied to a *Quadrant*. *If*, saith he, *here* (that is in the Distinct-Base) *you place the Scale that measures* —, *or if here an Hair be set, that it appear perfectly through the Glass* —, *you may use it in a Quadrant, for the finding of the Altitude of the least Star visible by the Perspective wherein it is. If the Night be so dark, that the Hair or the Pointers of the Scale be not to be seen, I place a Candle in a Lanthorn, so as it cast Light sufficient into the Glass; which I find very helpful when the Moon appeareth not, or it is not otherwise light enough.*

In another Letter, dated on *Christmas Eve* 1641. (wherein he describes the Wheel Work of his Micrometer, and shews how he could apply it to the taking of three Points; and specifies his Observations of the Diameters of the Sun and Moon; and mentions a Theory he had contriv'd of the Sun; *&c.* and saith what pains he had taken in the Anatomy of the Eye) he tells Mr.

Crabtrie how he had applied his Telescopick Sights to a Sextant. Saith he, *Mr.* Horrox *his Theory of the Moon I shall be shortly furnished to try. For I am fitting my Sextant for all manner of Observations, by two Perspicills with Threads. And also I am consulting my Workman about the making of Wheels like β, γ, δ, ε, of † Diagr. 3, to use two Glasses like a Sector. If I once have my Tools in readiness to my Desire, I shall use them every Night. I have fitted my Sextant by the Help of the Cane, two Glasses in it, and a Thread, so as to be a pleasant Instrument, could Wood and a Country-Joiner or Workman please me.*

In another Letter (the Date of which is worn out, but is, in Mr. *Crabtrie*'s Hand, called his 10th Letter to him) he saith, *I have given order for an Iron Quadrant of Five Foot, which will give me the 1000th Part of One Degree, which shall be furnished like my first Scale; only my Workman is so * throng for my Father, that I fear it will not be finished before the Eclipse. I have caused a very strong Ruler to be exactly made, and intend to fit it with Cursors of Iron, with Glasses in them and a Thread, for my Sextant.*

To these I could have added many other Passages of the like Nature: but these may be sufficient, to shew that *Mr. Gascoigne*, as early as 1640, made use of Telescopes on Quadrants and Sextants, as well as in his Invention of the Micrometer.

What Commendations these Contrivances got him, and what Expectations they raised in some of the Astronomers of that Time, particularly in two of the most acute of that Age, Mr. *Horrox*, and Mr. *Crabtrie*, may be seen in the same Mr. *Crabtrie*'s Letters to Mr. *Gascoigne*, which are also in my Hands. Some Passages of which I shall recite, and at the same time give the Society a Taste of what those curious Letters do contain.

† This Diagram is wanting in the Letter.

* A *Yorkshire Phrase* for *fully employed.*

In Mr. *Crabtrie's* second Letter, which is of *October* 30. 1640; after a very clear Demonstration that the Solar Spots are not Planets at a Distance from the Sun, but something adhering to, or very near the Sun's Body: and also after a no less clear Demonstration of the Errors of *Lansberges Hipparchian Diagram*, his *Lunar Parallax*, his Doctrine of *Eclipses*, and indeed his whole *Lunar Astronomy*, together with divers other curious Matters, too many to be specified: after this, I say, Mr. *Crabtrie* saith thus, *Something I am sure you were telling me concerning a way of observing the Places of the Planets by your Glasses. But I have not a little lamented that my Time cut me so short, when I was with you, that I could not more fully ruminate and digest those strange Inventions which you shewed me, and told me of. My Lassitude after an unexpected and unacquainted Journey; my unpreparedness for those Cogitations (not intending that Journey the Day before) and the Multiplicity and Variety of the Novelties you shewed me, so wholly distracted my Thoughts into Admiration, that I cannot now give my Meditations any reasonable Account of what I saw: but must intreat you, in a few Lines, to rub up my Memory, and tell me again what you shewed me, and the Extent of those your Inventions. Which I desire, that I might consider, and rejoice to consider, how much and wherein* Urania's *Structure will grow to Perfection by your Assistance: and that (what in me lies) I may help you to remember when and wherein your Inventions and Observations will be of most use. I should also desire you to inform me what Bigness of a Quadrant you conceive to be large enough for Observation with your Devices. For I am e're long going to* Wigan, *12 Miles from hence, where much Brass is cast; and then I could see whether I could procure such an one cast. You told me (as I remember) you doubted not in time to be able to make Observations to Seconds. I cannot but admire it and yet, by what I saw, believe it: but long*

to have some farther Hints of your Conceit for that Purpose. One Means, I think, you told me was, by a single Glass in a Cane, upon the Index of your Sextant, by which (as I remember) you find the exact Point of the Sun's Rays. But the way how, I have quite forgotten, and much desire. Your Device for the exact Division of a Quadrant, by dividing 11 Degrees into 10 Parts, I did then understand, but do not now fully remember. If it might not be too much Trouble to you, I should intreat you to give me such a Paper-Demonstration thereof as you shewed me, and two or three Lines plainly of the Use thereof, how to find those small Parts. I lost the little Paper, wherein I noted the Moon's Diameter, which we observed when I was with you: I pray you send it me, if, &c.

I cannot conceal how much I am transported beyond my self with the Remembrance (of that little I do remember) of those admirable Inventions which you shewed me when I was with you. I should not have believed the World could have afforded such exquisite Rarities, and I know not how to stint my longing Desires, without some further Taste of these selected Dainties. Happier had I been, had I never known there had been such Secrets, than to know no more, but only that there are such. Of all Desires the Desire of Knowledge is most vehement, most impatient: and of all kinds of Knowledge, this of the Mathematicks affects the Mind with most intense Agitations. I doubt not but you can experimentally witness the Truth hereof, and one time or other have been no Stranger to such Thoughts as mine. And therefore although Modesty would forbid me to request any thing (until you give me leave) but what you please voluntarily to impart, yet the Vehemence of my Desire forceth me to let you know how much I desire, and how highly I should prize any thing that you should be pleased to communicate to me in those Optick Practices. Could I purchase it with Travel, or procure it for Gold, I would not long be without a Telescope for ob-

ſerving ſmall Angles in the Heavens; nor want the Uſe of your other Device of a Glaſs in a Cane upon the moveable Ruler of your Sextant (as I remember) for helping to the exact Point of the Sun's Rays. But ſeeing Urania *is*, &c.

Thus was the moſt ingenious Mr. *Crabtrie* tranſported with Mr. *Gaſcoigne*'s Devices, although at that time far leſs perfect than they were in a ſhort Time after. And no leſs affected was the incomparable *Horrox*, as Mr. *Crabtrie* ſets forth, in his third long Letter of *Dec.* 28. 1640. which hath theſe Words, *My Friend Mr.* Horrox *profeſſeth, that little Touch which I gave him of your Inventions, hath raviſhed his Mind quite from it ſelf, and left him in an Extaſie between Admiration and Amazement. I beſeech you, Sir, ſlack not your Intentions for the perfecting of your begun Wonders. We travel with Deſire till we hear of your full Delivery. You have our Votes, our Hearts, and our Hands ſhould not be wanting, if we could further you.* And then after many curious Matters (which would take up too much of the Societies time to relate) he thus proceeds, *Your Diagrams for Perſpectives I have viewed again and again, and cannot ſufficiently admire your indefatigable Induſtry, and profound Ingenuity therein. I am much affected with the Symbolical Expreſſions of your Demonſtrations. I never uſed them before (but I will do) yet I underſtand them all at the firſt Sight, and ſee well the Truth of your Demonſtrations.*

To theſe I ſhall only add one Paſſage more, and this becauſe it ſhews ſome other of Mr. *Gaſcoigne*'s exquiſite Contrivances, or at leaſt the Accuracy of what are mentioned; and that is in Mr. *Crabtrie*'s Letter of *Dec:* 6. 1641. at the Beginning of which he ſaith, *That which you give me a full Projection of was above my Hope: and if the Screws keep an exact Equality of Motion forward in each Revolve, it is a moſt admirable Invention; and with the other Accommodations, I had almoſt ſaid without*

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Compare. But that the Divisions of a Circle should be measured to Seconds, without the Limb of an Instrument, or that Distances, Altitudes, Inclinations, and Azimuths should be taken all at one Moment, without the Limb of an Instrument likewise, and each to any required Number of Parts; or that the Diameter of Jupiter *should be projected in such prodigious Measures as you speak of,* &c. *were enough to amuse and amaze all the Mathematicians in* Europe, *and may indeed be rather a Subject of Admiration than Belief, to any that hath not known your former Inventions to exceed Vulgar (I had almost said Humane) Abilities. And for my Part, I must confess Modesty so checks my ambitious Desires, that I dare scarce hope such Miracles should ever be produced in real Practice to such Exactness.* Then (to give the Society a further Taste of those Letters) follows an Account of the Agreement of Mr. *Horrox*'s Theory of the Moon with Mr. *Gascoigne*'s Observations; and also very curious Ratiocinations, and a Disquisition about finding the Parallax of the Sun and Moon, and their Distance from the Earth. In which he censures *Morinus*'s Braggs, *&c.* and then saith, that *no Man that hath written of the Diagram* [of Hipparchus] *understood it fully, or described it rightly, but only* Kepler *and our* Horrox; *for whose immature Death* [which was suddenly, and about the Age of 25.] *there is yet scarce a Day which I pass without some Pang of Sorrow.*

Thus, among many, I have related some of the Passages of Mr. *Gascoigne*'s and Mr. *Crabtrie*'s Letters relating to *Telescopick Sights.* From whence it is very manifest, that long before the *French* Gentleman's Claims, our Countryman Mr. *Gascoigne* had made use of those Sights in his Astronomical Instruments; particularly in two or more Sorts of *Micrometers* (as I plainly find) and in his *Quadrant* and *Sextant.* And had it pleased God to have given him a longer Life, we might have ex-

pected

pected greater things from his pregnant and sagacious Wit. For he was scarce 20 Years of Age when he held these Correspondencies with Mr. *Crabtrie*. And at the Age of 23. he was killed at *Marston-Moor-Battle*, on *July* 2. 1644. fighting for King *Charles* I. His Father was *Henry Gascoigne* Esq; of *Middleton*, between *Leeds* and *Wakefield*.

IV. *An Attempt towards the Improvement of the Method of approximating, in the Extraction of the Roots of Equations in Numbers. By* Brook Taylor, *Secretary to the Royal Society.*

IN *Phil. Tran.* No. 210. Dr. *Halley*, now Secretary of the Royal Society, has publish'd a very compendious and useful Method of extracting the Roots of affected Equations of the common Form, in Numbers. This Method proceeds by assuming the Root desired nearly true to one or two Places in Decimals (which is done by a Geometrical Construction, or by some other convenient way) and correcting the Assumption by comparing the Difference between the true Root and the assumed, by means of a new Equation whose Root is that Difference, and which he shews how to form from the Equation proposed, by Substitution of the Value of the Root sought, partly in known and partly in unknown Terms.

In doing this he makes use of a Table of Products (which he calls *Speculum Analyticum*,) by which he computes the Coefficients in the new Equation for finding the Difference mentioned. This Table, I observed, was formed in the same Manner from the Equation pro-

propos'd, as the Fluxions are, taking the Root sought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product successively by the Numbers 1, 2, 3, 4, *&c.* Hence I soon found that this Method might easily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further degree of Generality; it being Applicable, not only to Equations of the common Form, (*viz.* such as consist of Terms wherein the Powers of the Root sought are positive and integral, without any Radical Sign) but also to all Expressions in general, wherein any thing is proposed as given which by any known Method might be computed; if *vice versâ*, the Root were consider'd as given: such as are all Radical Expressions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given, at least by Logarithms, whatever be the Index of the Power of that Nomial; as likewise Expressions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abscissa's* or any other Fluents, or Roots of Fluxional Equations, *&c.*

For the sake of this great Generality, it may not be improper to shew how this Method is derived from the foresaid *Corollary.* Therefore z and x being two flowing Quantities (whose Relation to one another may be exprest by any Equation whatsoever) by this *Corollary*, while z by flowing uniformly becomes $z + v$, x will become $x + \frac{\dot{x}}{1 \cdot \dot{z}} v + \frac{\ddot{x}}{1 . 2 \dot{z}^2} v \mp \frac{\dddot{x}}{1 . 2 . 3 \dot{z}^3} v^3 + \&c.$ or $x + \frac{\dot{x} v}{1} + \frac{\ddot{x} v^2}{1 \times 2} + \frac{\dddot{x} v^3}{1 . 2 . 3} + \&c.$ for $\dot{z}$ putting 1.

Hence if y be the Root of any Expression formed of y and known Quantities, and supposed equal to nothing, and

and z be a part of y, and x be formed of z and the known Quantities, in the ſame manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z+v$: the difference v will be found by Extracting the Root of this expreſſion $x+\frac{\dot{x}v}{1}+\frac{\ddot{x}v^2}{1.2}+\frac{\dddot{x}v^3}{1.2.3}$ $+$ &c. $=0$: For in this Caſe z being become $z+v=y$, x, which is now become $x+\dot{x}v+\frac{\ddot{x}v^2}{2}+$ &c. muſt become equal to nothing.

The Root v in the Equation $x+\frac{\dot{x}v}{1}+\frac{\ddot{x}v^2}{1.2}+\frac{\dddot{x}v^3}{1.2.3}$ $+$ &c. $=0$, is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which means the Terms $\frac{\dddot{x}v^3}{1.2.3}+\frac{\ddddot{x}v^4}{1.2.3.4}+$ &c. may be neglected, upon account of their ſmallneſs with reſpect to the other Terms, ſo as to leave the Equation $x+\frac{\dot{x}v}{1}+\frac{\ddot{x}v^2}{1.2}=0$, for finding the firſt approximation of v.

By extracting the Root of this Equation, we have $v=\sqrt{\frac{\dot{x}^2}{\ddot{x}^2}-\frac{2x}{\ddot{x}}}-\frac{\dot{x}}{\ddot{x}}$. That is,

First, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2}-\frac{2x}{\ddot{x}}}-\frac{\dot{x}}{\ddot{x}}$, if $x+\dot{x}v+\frac{\ddot{x}v^2}{2}=0$.

Sec. $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2}+\frac{2x}{\ddot{x}}}-\frac{\dot{x}}{\ddot{x}}$, if $-x+\dot{x}v+\frac{\ddot{x}v^2}{2}=0$.

Thirdly

3. $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

4. $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}}$, if $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

This approximation gives v exact to twice as many places as there are true Figures in z, and therefore trebles the number of true Figures in the Expreſſion of y by $z + v$, which may be taken for a new Value of z, for computing a ſecond v, ſeeking other Values of x, $\dot{x}$, $\ddot{x}$, &c. Tho' when z is tolerably exact (which it may be eſteem'd when it contains two or three or more true Figures in the Value of y, according to the Number of Figures the Root is propoſed to be computed to,) the Calculation may be reſtor'd without ſo much trouble, only by taking $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{1.2.3.4\ddot{x}}v^4}$ &c. inſtead of $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}}}$ taking every time for v its Value laſt computed.

From the ſame Equation $x + \dot{x}v + \frac{\ddot{x}v^2}{2} + \frac{\dddot{x}v^3}{1.2.3}$ $+$ &c. $= 0$, may be gather'd alſo a rational Form, *viz.* $v = \frac{-x}{\dot{x} - \frac{x\ddot{x}}{2\dot{x}}}$ For neglecting the Terms $\frac{\dddot{x}v^3}{1.2.3}$, &c. we have $v = \frac{-x}{\dot{x} + \frac{\ddot{x}}{2}v}$ which is nearly $= \frac{-x}{\dot{x}}$. Therefore in the Diviſor inſtead of v writing $\frac{-x}{\dot{x}}$ we have

more exactly $v = \frac{-x}{\dot{x} - \frac{\ddot{x}\,x}{2\,\dot{x}}}$, that is

1. $\frac{-x}{\dot{x} - \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $x + \dot{x}\,v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

2. $\frac{x}{\dot{x} + \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $-x + \dot{x}\,v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

3. $\frac{x}{\dot{x} - \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $x - \dot{x}\,v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

4. $\frac{-x}{\dot{x} + \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $-x - \dot{x}\,v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

This *Formula* will also triplicate the number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for a Divisor $\dot{x} \pm \frac{\ddot{x}}{2}\,v + \frac{\dddot{x}\,v^2}{1.2.3} + \frac{\ddddot{x}\,v^3}{1.2.3.4}$ + &c. instead of $x + \frac{\ddot{x}\,x}{2\,\dot{x}}$.

Dr. *Halley* has fully explain'd the manner of using both these *Formula*'s in Æquations of the common Form; wherefore I shall be the shorter in explaining two or three Examples of another sort.

Ex. I. Let it be proposed to find the Root of this Equation $\overline{y^2 + 1}|^{\surd 2} + y - 16 = 0$. In this Case, for y writing z, and for o writing x, we have $\overline{z^2 + 1}|^{\surd 2}$
$+ z$

$+ z - 16 = x$. Whence by taking the Fluxions, we have $\dot{x} = 2\sqrt{2} \times z \times \overline{z^2 + 1}|^{\sqrt{2}-1} + 1$, and $\ddot{x} = 2\sqrt{2} \times 8 - 4\sqrt{2}\, z^2 \times \overline{z^2 + 1}|^{\sqrt{2}-2}$. For finding the first Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2 + 1}|^{\frac{3}{2}} + y - 16 = 0$, which being expanded gives $y^6 + 3y^4 + 2y^2 + 32y - 255 = 0$.

By this Equation I find that for the first supposition we may take $z = 2$. Therefore in order to find v, let us now make $\sqrt{2} = \frac{7}{5}$, (which is nearer than before) and we have $x = \overline{z^2 + 1}|^{\frac{7}{5}} + z - 16 = \overline{2^2 + 1}|^{\frac{7}{5}} - 14 = 5^{\frac{7}{5}} - 14 = -4,48$; $\dot{x} = 10,66$; $\ddot{x} = 4,72$. Whence by the second rational Form $v = \dfrac{4,48}{10,66 + \dfrac{4,72 \times 4,48}{2 \times 10,66}} = 0,38$; which must be too big, because $\frac{7}{5} < \sqrt{2}$, and therefore will require a larger Value of y to exhaust the Equation, than where $\sqrt{2}$ is exact. For the second supposition therefore, let us take $z = 2, 3$, and make $\sqrt{2} = 1,4142136$, and by help of the Logarithms we shall have $\overline{z^2 + 1}|^{\sqrt{2}} = 13,47294$, whence $x = -0,22706$; $\dot{x} = 14,93429$, and $\ddot{x} = 5,18419$. Hence by the 2*d*. irrational *Formula* $v = \sqrt{\dfrac{14,93429^2}{5,18419^2} + \dfrac{0,45412}{5,18419}} - \dfrac{14,93429}{5,18419} = 0,01516$, which gives $y = z + v = 2,31516$, which is true to six Places. If you desire it more exact than to the extent of the Tables of Logarithms, taking $z = 2,31516$ for the next supposition, the Calculation must be repeated by computing of $\overline{zz + 1}|^{\sqrt{2}}$ to a sufficient number of Places; which must be done by the Binomial Series, or by making a Loga-

 rithm

rithm on purpose, true to as many places as are necessary.

Ex. II. For another Example, let it be required to find the Number whose Logarithm is 0, 29, supposing we had no other Table of Logarithms but Mr. *Sharps* of 200 Logarithms to a great many places. This amounts to the resolving this Equation $ly = 0,29$, or $ly - 0,29 = 0$. Hence therefore we have $x = lx - 0,29$, $\dot{x} = \frac{a}{z}$ (a being the *Modulus* belonging to the Table we use, *viz.* 0, 4342944819, *&c.*) $\ddot{x} = \frac{-a}{z^2}$, $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} = \frac{-6a}{z^4}$ *&c.* In this Case because $\ddot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Case, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{2.3.4\ddot{x}}v^4 \text{ \&c.}} = z - \sqrt{z^2 + \frac{2lz - 0,58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2} + \frac{2v^5}{5z^3}}$ *&c.* In this Case to avoid often dividing by z, it will be most convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0,58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}}$, *&c.* The nearest Logarithm, in the Tables proposed, to the proposed Logarithm 0, 29 is 0, 2900346114, its Number being 1, 95. Therefore for the first supposition taking $z = 1,95$, we have x $(= lz - 0,29 = 0,2900346114 - 0,29) =$ 0,0000

0,0000346114, and $\frac{2lz - 0,58}{a} = \frac{0,0000692228}{0,4342944819} =$ 0,000159 39139, and $1 + \frac{2lz - 0,58}{a} = 1,000159$-39139. Whence for the first approximation we have $\frac{v}{z} = 1 - \sqrt{1,00015939139} = -0,00007969247$, and $v = -0,00015540032$, and $y = z + v =$ 1,94984459968. Which is true to eleven places, and may easily be corrected by the Terms $\frac{2v^3}{3z}$ &c. which I leave to the Readers curiosity.

Being upon the Subject of Approximations, it may not be amiss to set down here two Approximations I have formerly hit upon. The one is a Series of Terms for expressing the Root of any Quadratick Equation: and the other is a particular Method of Approximating in the invention of Logarithms, which has no occasion for any of the Transcendental Methods, and is expeditious enough for making the Tables without much trouble.

A general Series for expressing the Root of any Quadratick Equation.

Any Quadratick Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be exprest by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2}$$

$$+ D \times \frac{1}{c^2 - 2} \text{ \&c.}$$ Which must be thus interpreted.

1. The Capital Letters A, B, C, &c. stand for the whole Terms with their Signs, preceding those wherein

in they are found, as $B = A \times \frac{1}{\frac{mq^2}{y} - 2}$.

2. The little Letters a, b, c, &c. in the Divisors, are equal to the whole Divisors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\sqrt{2}$. Putting $\sqrt{2} = x + 1$, we have $x^2 + 2x - 1 = 0$, which being compared with the general *Formula*, gives $mq = -2$, and $my = -1$: therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values substituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$, &c. The Fractions here wrote down giving the Root true to twenty three Places.

A new Method of computing Logarithms.

This Method is founded upon these Considerations.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of those two Numbers Multiplied together.

2. That the Logarithm of Unite is nothing; and consequently that the nearer any Number is to Unite, the nearer will its Logarithm be to 0. 3*dly*. That the Product by Multiplication of two Numbers, whereof one is bigger, and the other less than Unite, is nearer to Unite than that of the two Numbers which is on the same side of Unite with its self; for Example the two Numbers being $\frac{2}{3}$ and $\frac{4}{3}$, the Product $\frac{8}{9}$ is less than Unite, but nearer to it than $\frac{2}{3}$, which is also less than Unite. Upon these Considerations, I found the present Approximation;

proximation; which will be best explain'd by an Example. Let it therefore be proposed to find the Relation of the Logarithms of 2 and of 10. In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{\overline{2}|^7}{\overline{10}|^2}$ and $\frac{\overline{2}|^3}{\overline{10}|^1}$, whose Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other less than 1. Having set these down in Decimal Fractions in the first Column of the Table annext, against them in the second Column I set A and B for their Logarithms, expressing by an Equation the manner how they are Compounded of the Logarithms of 2 and 10, for which I write $l\,2$ and $l\,10$. Then Multiplying the two Numbers in the first Column together, I have a third Number 1,024, against which I write C for its Logarithm, expressing likewise by an Equation in what manner C is formed of the foregoing Logarithms A and B. And in the same manner the Calculation is continued; only observing this *Compendium*, that before I Multiply the two last Numbers already got in the Table, I consider what Power of one of them must be used to bring the Product the nearest to Unite that can be. This is found, after we have gone a little way in the Table, only by Dividing the Differences of the Numbers from Unite one by the other, and taking the Quotient with the nearest, for the Index of the Power wanted. Thus the two last Numbers in the Table being 0,8 and 1,024, their Differences from Unit are 0,200 and 0,024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore Multiplying the ninth Power of 1,024 by 0,8, I have the next Number 0,990352031429, whose Logarithm is $D = 9C + B$. In seeking the Index in this manner by Division of the Differences, the Quotient ought generally to be taken with

with the least: but in the present case it happens to be the most, because instead of the Difference between 0, 8 and 1, we ought strictly to have taken the difference between the reciprocal 1, 25 and 1, which would have given the Index 105; and that would be too big, because the Product by that means would have been bigger than 1, as 1,024 is. Whereas this Approximation requires that the Numbers in the first Column be alternately greater and less than 1, as may be seen in the Table.

When I have in this manner continued the Calculation, till I have got the Numbers small enough, I suppose the last Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by means of the foregoing Equations, I have the relation of the Logarithms proposed. In this manner if I suppose $G = 0$, I have $2136\, l\, 2 - 643\, l\, 10 = 0$. Which gives the Logarithm of 2 true in seven Figures, and too big in the Eighth; which happens because the Number corresponding with G is bigger than Unite.

There is another Expedient which renders this Calculation still shorter. It is founded upon this Consideration, that when x is very small $\overline{1+x}|^{n}$ is very nearly $1+nx$. Hence if $1+x$, and $1-z$ are the two last Numbers already got in the first Column of the Table, and their Powers $\overline{1+x}|^{m}$ and $\overline{1-z}|^{n}$ are such as will make the Product $\overline{1+x}|^{m} \times \overline{1+z}|^{n}$ very near to Unite, m and n may be found thus: $\overline{1+x}|^{m} = 1+mx$, and $\overline{1-z}|^{n} = 1-nz$, and consequently $\overline{1+x}|^{m} \times \overline{1-z}|^{n} = 1+mx-nz-mnzx$, or (neglecting $mnzx$) $1+mx-nz$. Make this equal to 1, and we have $m:n :: z:x :: l\,\overline{1-z} : l\,\overline{1+x}$. Whence $x\,l\,\overline{1-z} + z\,l\,\overline{1+x} = 0$. To give an Example of the Application of this, let 1, 024 and 0, 990352 be the last Numbers in the Table, their Logarithms being C and D. Then we have

1,024

$1,024 = 1 + x$, and $0,990352 = 1 - z$, and consequently $x = 0,024$, and $z = 0,009648$. Whence the Ratio $\frac{z}{x}$ in the least Numbers is $\frac{201}{500}$. So that for finding the Logarithms proposed we may have $500\,D + 201\,C = 48510\,l2 - 14603\,l10 = 0$, which gives $l2 = 0,3010307$, which is too big in the last Figure; but it is nearer the truth, than what is got from the Logarithm F supposed equal to nothing. So that by this means we have saved four Multiplications, which were necessary to find the Number 9989595&c. correspondent to F, and which must have been had if we would make the Logarithm true to the same Number of places without this *Compendium*.

1, 28000000000	$A = 7\,l2 - 2\,l10$	$l2 > 0,28$
0, 80000000000	$B = 3\,l2 - l10$	$< 0,33$
1, 02400000000	$C = B + A = 10\,l2 - 3\,l10$	$> 0,300$
0, 990352031429	$D = 9\,C + B = 93\,l2 - 28\,l10$	$< 0,30107$
1, 0043362777664	$E = 2\,D + C = 196\,l2 - 59\,l10$	$> 0,301020$
0, 998959536107	$F = 2\,E + D = 485\,l2 - 146\,l10$	$< 0,3010307$
1, 000162894165	$G = 4\,F + E = 2136\,l2 - 643\,l10$	$> 0,30102996$
0, 999936281874	$H = 6\,G + F = 13301\,l2 - 4004\,l10$	$< 0,301029997$
1, 000035441215	$I = 2\,H + G = 28738\,l2 - 8651\,l10$	$> 0,3010299951$
0, 999971720830	$K = I + H = 42039\,l2 - 12655\,l10$	$< 0,3010299959$
1, 000007161046	$L = K + I = 70777\,l2 - 21306\,l10$	$> 0,30102999562$
0, 999993203514	$M = 3\,L + K = 254370\,l2 - 76573\,l10$	$< 0,30102999567$
1, 000000364511	$N = M + L = 325147\,l2 - 97879\,l10$	$> 0,3010299956635$
0, 999999764687	$O = 18\,N + M = 6107016\,l2 - 1838395\,l10$	$< 0,3010299956640$
Com. Ar. 235313		
$0 = 364511\,O + 235313\,N = 2302585825187\,l2 - 693147400972\,l10$		$> 0,3010299956663987$

I have computed this Table so far, that the Reader may see in what manner this Method Approximates; this whole Work, as it appears, costing a little more than three Hours time.

V. *Proprietates quædam simplices* Sectionum Conicarum *ex natura* Focorum *deductæ; cum Theoremate generali de Viribus Centripetis; quorum ope Lex Virium Centripetarum ad Focos Sectionum tendentium, Velocitates Corporum in illis revolventium, & Descriptio Orbium facillime determinantur. Per* Abr. *de* Moivre. R. S. Soc.

SIt DE Axis Transversus Ellipseos, AO Axis alter, & C centrum Sectionis. Sit P punctum quodvis in circumferentia ejus; PQ Tangens curvæ ad P, occurrens Axi Transverso ad Q; puncta S, F Foci; CP, CK semidiametri Conjugatæ; PH Semilatus rectum ad diametrum PC; PG normalis ad Tangentem, cui occurrat HG, perpendicularis ipsi PCH, in puncto G, ut fiat PG radius Curvaturæ Ellipseos in puncto P: sint etiam ST, CR, FV perpendiculares in Tangentem PQ demissæ: Jungatur SO, & demittatur in Axem normalis PL. His positis, Dico quod,

I. *Rectangulum sub distantiis ab utroque Ellipseos Foco, sive* $SP \times PF$ *æquale est quadrato Semidiametri* CK.

Demonstratio.

$PSq = PCq + CSq - 2CS \times CL$ *per* 13. II. Elem.
$PFq = PCq + CSq + 2CS \times CL$ *per* 12. II. Elem.
Unde $PSq + PFq = 2PCq + 2CSq$.
Jam $PS + PF = DE = 2CD$; ac propterea
$PSq + PFq + 2PS \times PF = 4CDq$. Quare

Quare transponendo, $2\,PS \times PF = 4\,CDq - 2\,PCq - 2\,CSq$.

Ac Dimidiando $PS \times PF = 2\,CDq - PCq - CSq$.

Est autem CS *quad.* $= CD$ *quad.* $- CO$ *quad*, atque adeo $PS \times PF = CDq + COq - PCq$.

Sed $CDq + COq = PCq + CKq$. *per* 12. VII. *Conic. Apollonii.*

Quare $PS \times PF = CKq$. *Q. E. D.*

II. *Distantia à Foco SP est ad perpendicularem in Tangentem demissam, ut Semidiameter Conjugata CK ad Semiaxem minorem CO.*

Demonstratio.

Ob similia Triangula SPT, FPV, erit $PS : PF :: ST : FV$; ac componendo $PS + PF$ erit ad $ST + FV$, & earundem dimidia CD ad CR, ut PS ad ST. Unde $CD \times CK$ erit ad $CR \times CK$ ut PS ad ST. Sed $CR \times CK$ æquale est rectangulo sub Semiaxibus CD in CO, per 31. VII. *Conic.* Proinde PS est ad ST ut CD in CK ad $CD \times CO$, sive ut CK ad CO. Ac pari argumento demonstrabitur PF esse ad FV in eadem ratione. *Q. E. D.*

III. *In eadem etiam est ratione Semiaxis Transversus CD ad normalem è centro C ad Tangentem demissam, sive ad CR.*

Etenim cum rectangulum $CR \times CK$ æquale sit rectangulo $CD \times CO$, uti jam dictum est, erit ἀνάλογον CD ad CR ut CK ad CO. *Q. E. D.*

IV. *Semidiameter quævis PC est ad distantiam puncti P à foco S, sive ad SP, ut distantia ab altero Foco FP ad dimidium lateris recti ad Verticem P pertinentis, sive ad PH.*

Hoc autem manifestum est ob *Propr.* I. cum nempe quadratum ex CK æquale sit rectangulo sub $SP \times PF$.

V. *Rectangulum Semiaxium $CD \times CO$ est ad quadratum semidiametri conjugatæ CK, ut CK ad Radium Curvaturæ in puncto P, sive ad PG.*

Sunt

Sunt enim Triangula PCR, PGH inter se similia, unde CR est ad PC, ut semilatus rectum PH ad PG: hoc est, per præmissam Proprietatem III, $\frac{CD \times CO}{CK} =$ CR est ad PC ut $\frac{CK^2}{PC} = PH$ ad $\frac{CK^3}{CD \times CO} = PG$. proinde ἀνάλογον $CD \times CO : CK^2 :: CK : PG$. *Q.E.D.*

THEOREMA GENERALE I.

Vis centripeta ad idem punctum S tendens, in Curvis omnibus, est semper proportionalis Quantitati $\frac{SP}{PG \times ST^3}$

Hoc Theorema ante plures annos à me investigatum & cum amicis communicatum, propriis demonstrationibus firmavere Geometræ Clarissimi D. *J. Bernoullius* in *Act. Lipsiæ*; D. *J. Keillius* in harum *Transact.* N. 317. & D. *Jac. Hermannus* in *Phoronomiâ* suâ pag. 70. quos vide.

Scribendo autem $\frac{CK^3}{}$ pro PG, per *Propr.* V; & $\frac{SP}{CK}$ juxta *Propr.* II, pro ST; (ob datas scilicet CD, CO) erit Vis centripeta tendens ad focum Ellipseos S, semper ut $\frac{SP \times CK^3}{CK^3 \times SP^3}$, hoc est ut $\frac{SP}{SP^3}$ vel $\frac{1}{SP^2}$, nempe reciprocè ut quadratum ex SP. Unde patet quod si Sectio fuerit Ellipsis motu corporis descripta, erit Vis Centripeta ut quadratum distantiæ à centro Virium reciprocè. Ex his Proprietatibus consequuntur Corollaria nonnulla notatu non indigna.

Coroll 1. *Velocitas Corporis in Ellipsi revolventis, ad punctum quodlibet P, est ad Velocitatem revolventis in circulo ad eandem distantiam SP à centro Virium, in subdupla ratione distantiæ ab altero foco PF, ad Semiaxem transversum Sectionis, sive ut media proportionalis inter PF & CD ad CD.*

Est enim velocitas revolventis in Ellipsi ad distantiam SP, ad Velocitatem revolventis in Circulo vel Ellipsi ad dist-

diſtantiam Semiaxis *C D* vel *S O*, ut *C O* ad *S T*; hoc eſt per *Propr.* II. ut $\sqrt{P F}$ ad $\sqrt{S P}$. Velocitas autem revolventis in Circulo ad diſtantiam *C D* eſt ad velocitatem revolventis in Circulo ad diſtantiam *S P*, ut $\sqrt{S P}$ ad $\sqrt{C D}$. Ex æquo igitur, Velocitas revolventis in Ellipſi ad diſtantiam *S P*, eſt ad Velocitatem revolventis in Circulo ad eandem diſtantiam ut $\sqrt{P F}$ ad $\sqrt{C D}$.

Coroll. 2. *Ex datis Velocitate in Ellipſi, poſitione Tangentis, & centro Virium ſeu Foco, facile eſt determinare Focum alterum.*

Sit enim Velocitas Data *R*; ea autem Velocitas quâ deſcriberetur Circulus ad datam à centro diſtantiam *S P* ſit *Q*; ac per *Coroll.* præcedens, *R* eſt ad *Q* ut $\sqrt{P F}$ ad $\sqrt{C D}$, adeoque *Q Q* eſt ad *R R* ut *C D* ad *P F*, & 2 *Q Q* — *R R* erit ad *R R* ut *S P* ad *P F*: Datur autem *S P*; data eſt igitur *P F* magnitudine. Datur etiam poſitione, ob angulum *V P F* angulo *S P T* æqualem. Datur igitur punctum *F* alter Focorum: Quo invento pronum eſt Sectionem deſcribere.

Si vero $\frac{1}{2}$ *R R* majus fuerit quadrato ex *Q*, 2 *Q Q* — *R R* fit quantitas Negativa, & loco Ellipſeos Trajectoria deſcribenda in Hyperbolam tranſit. Eritque *R R* — 2 *Q Q* ad *R R* ut *S P* ad *P F* diſtantiam alterius Foci, ad alterum Tangentis latus ponendam, ut habeatur Focus *F*. Proprietates autem omnes quas in Ellipſi demonſtravimus; mutatis mutandis etiam Hyperbolæ competunt. *Fig.* II.

Quod ſi acciderit *Q Q* æquale eſſe dimidio quadrati ex *R*; evaneſcente quantitate 2 *Q Q* — *R R* = 0, quarta proportionalis *P F* fit infinita: proinde Trajectoria decribenda Parabolica eſt, Foco ſcilicet altero in infinitum abeunte. Axis autem Trajectoriæ poſitione datur; eſt enim ipſi *P F* parallelus, exiſtente ſcilicet angulo *F P V* angulo dato *S P T* æquali.

Coroll. 3. *Velocitas revolventis in data Sectione Conica ad diſtantiam S P eſt ad Velocitatem ejuſdem ad diſtantiam aliam S X, ut media proportionalis inter F P & S X ad mediam proportionalem inter S P & F X.*

Velo-

Velocitas enim in P est ut $\sqrt{\frac{FP}{SP}}$ (per *propr.* II.) & per eandem, Velocitas in X est ut $\sqrt{\frac{FX}{SX}}$. Unde manifesta est propositio.

Coroll. 4. *Ratio etiam Velocitatum duorum Corporum in eodem Systemate, sed in datis Conisectionibus diversis, revolventium, datis utrinsque à communi Orbium Foco distantiis, ope Corollarii* 1^{mi}. *statim obtinebitur.*

Cum enim Velocitas corporis in P sit ad Velocitatem in Circulo ad eandem distantiam SP, ut $\sqrt{PF}$ ad $\sqrt{CD}$; & in alia supposita Conisectione, cujus Semiaxis cd & Foci S, f, ad distantiam Sp Velocitates illæ sint ut $\sqrt{pf}$ ad $\sqrt{cd}$: Velocitas autem revolventis in circulo ad distantiam SP sit ad Velocitatem in Circulo ad distantiam Sp ut $\sqrt{Sp}$ ad $\sqrt{SP}$; Compositis rationibus, erit Velocitas in P ad Velocitatem in p, ut $\sqrt{PF \times cd \times Sp}$ ad $\sqrt{pf \times CD \times SP}$. Quod si Sectio illa altera fuerit Parabola; erunt cd, pf infinitæ, sed in ratione 1 ad 2; proinde ratio Velocitatum erit ut $\sqrt{PF \times Sp}$ ad $\sqrt{2CD \times SP}$.

Coroll. 5. *Quod si in Hyperbola punctum P abeat in infinitum, ex præcedentibus manifestum est, Velocitatem ultimam ac minimam, qua cum corpus in æternum ascenderet, æqualem esse ei qua, ad distantiam CD Semiaxi transverso æqualem, Circulum describeret.*

Coroll. 6. *Ex data distantia à Foco, datur quoque Positio Tangentis, sive angulus SPT, sub distantia SP & Tangente PT contentus.*

Est enim (per *propr.* II.) PS ad ST ut CK ad CO sive ut $\sqrt{SP \times PF}$ ad CO, atque ita Radius ad Sinum anguli SPT. At in Ellipsibus Circulis affinibus præstaret angulum PST, ejusdem complementum ad quadrantem, inquirere: Hujus autem Sinus est ad Radium ut $\sqrt{SP \times PF - COq}$ ad $\sqrt{SP \times PF}$.

Coroll.

Coroll. 7. Atque hinc consequuntur Velocitates quibuscum distantiæ SP crescunt vel decrescunt.

Nam cum, ex Corollario præcedente, $\sqrt{SP \times PF}$ sit ad $\sqrt{SP \times PF - COq}$ ut Radius ad sinum anguli PST, ac in eadem sit ratione Velocitas Corporis in P ad Velocitatem momenti ipsius SP; Velocitas autem illa in P sit (per *propr.* II.) ut $\sqrt{\frac{PF}{SP}}$; elisis superfluis, erit $\sqrt{\frac{SP \times PF - COq}{SP}}$ Velocitati, qua crescit vel decrescit distantia SP, semper proportionalis.

THEOREMA GENERALE II.

In omni Trajectoria Curvilinea Velocitates angulares circa centrum Virium sunt reciprocè proportionales quadratis distantiarum à centro.

Nam ob Sectorum minimorum Areas æquales, arcus angulis minimis subtensi sive Bases, sunt reciprocè ut Radii: Anguli autem minimi quibus Bases æquales subtenduntur sunt etiam reciproce ut Radii. Proinde anguli Sectorum minimorum Areâ æqualium, sunt inter se reciprocè in dupla rationeRadiorum, sive ut quadrata distantiarum.

Coroll. 8. Hinc Velocitates angulares revolventium in diversis Ellipsibus datis comparantur inter se.

Velocitates enim angulares quibuscum ad distantias Semiaxibus Transversis æquales circuli describerentur, sunt reciproce in ratione sesquialtera Axium, sive ut $\frac{1}{CD\sqrt{CD}}$. Velocitates autem angulares has medias habent Corpora revolventia, cum quadrata distantiarum æquantur rectangulis sub semiaxibus Ellipseôn. Ideo (per *Theor.* II.) erit SPq ad $CD \times CO$ ut $\frac{1}{CD\sqrt{CD}}$ ad $\frac{CO}{SPq \times \sqrt{CD}}$: quæ quidem Quantitas est ut Velocitas anguli ad centrum S, motu rectæ SP, tempore quam minimo dato, descripti.

Coroll. 9. Velocitas angularis qua circumgyratur Tangens TT, sive recta in Tangentem perpendicularis ST, est ad Velocitatem

locitatem angularem rectæ SP, ut Semiaxis transversus CD ad distantiam ab altero Foco PF.

Demonstratio.

In *Fig.* III. Sint puncta P, p, quamproxima inter se; ductisque $SP . Sp$, sint PT, pt duæ Tangentes, ad quas demittantur normales ST, St; iisque parallelæ ducantur radii Curvaturæ PG, pG coeuntes in G: ac describatur, centro S & radio SP, arcus minimus PE occurrens ipsi Sp in E. Manifestum est angulum PGp æqualem esse angulo TSt, sive angulari Velocitati normalis ST. Est autem angulus PSp angularis velocitas rectæ SP; quare angulus PGp est ad angulum PSp ut angularis Velocitas ipsius ST ad angularem velocitatem rectæ SP; hoc est, ut $\frac{Pp}{PG}$ ad $\frac{PE}{PS}$. Sed $Pp . PE :: SP . ST :: CK : CO$ (per *propr.* II). Hæ igitur Velocitates sunt ut $\frac{CK}{PG}$ ad $\frac{CO}{PS}$. Pro PG scribe $\frac{CK^3}{CD \times CO}$ (per *propr.* V.) ac $\frac{CK}{PG}$ fiet $\frac{CD \times CO}{CKq} = \frac{CD \times CO}{PS \times PF}$. Hinc $\frac{CD \times CO}{PS \times PF}$ erit ad $\frac{CO}{PS}$, sive, deletis superfluis, CD ad PF, ut angulus TSt ad angulum PSp, sive Velocitas angularis Tangentis ad angularem Velocitatem distantiæ SP: proinde Velocitas qua circumgyratur Tangens, semper proportionalis est quantitati $\frac{CO \times \sqrt{CD}}{PF \times SPq}$.

Pleraque horum Corollariorum ex aliis Conicarum Sectionum Proprietatibus deducta, vel facile deducenda, inveniet Lector in Sect III. Lib. I. Princip. Nat. Philosophiæ.

FINIS.

LONDON, Printed for *W. Innys*, at the *Princes Arms* in St. *Paul's-Church-Yard*, 1717.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *July*, *August*, and *Sep.* 1717.

I. *An Account of the Dissection of a Child. Communicated in a Letter to Dr.* Brook Taylor, *R. S. Secr. By Dr.* Patrick Blair, *R. S. S.*

AS nothing is more apt to lead us to the Knowledge of the several Distempers which affect the Human Body, and to acquaint us with the just Prognosticks of the like Cases, than the opening of distemper'd Persons. I hop'd it would not be an unacceptable instance of my Zeal and Readiness to serve the Most Honourable the *Royal Society* upon all occasions, to desire you to present them with the following account of the Dissection I lately made of a Child.

This Child was five Months old, and was so emaciated, that he appear'd rather to have decreased, than to have encreased in Bulk, from the time of his Birth; his whole Body not weighing above five Pounds. The Skin and Muscles of the Abdomen were very thin, but the Peritoneum was preternaturally thick. The Ventriculus was more like to an Intestin than to a Stomach, its length being five Inches, and its breadth but one Inch. The Coats of it were thick and fleshy, and the Cavity very inconsiderable. The Pylorus, and almost half of the Duodenum were Cartilaginous, and something inclin'd to an Ossification, so that no Nourishment could have passed into the Intestins, tho' the Stomach had been capable of containing it, which makes it no Wonder that the Body was so emaciated. There were scarce any foot-steps of the Omentum to be seen, even at the Bottom of the Stomach, to which it usually adheres.

The

The right Lobe of the Lungs adhered firmly to the Ribs and had three Exulcerations, which contain'd purulent Matter. It was ſo very thin and compact, that it ſeem'd as if that Lobe had never been of uſe in Reſpiration. The left Lobe was of a more florid Red, ſpongy, and free from any Adheſion.

Upon enquiring after the Symptoms this Child had been affected with, his Mother told me, he ſeem'd to be healthy till he was about a Month old, when he was ſeized with a violent Vomiting, and a Stoppage of Urine and Stool. Some time after, both theſe became more regular, but the Vomiting ſtill continued. He ſeem'd to have a great Appetite, taking what Suck, Drink, or other Food was offer'd him, with a kind of eagerneſs; but he immediately threw it all up again. He had all along breathed freely, and had no Cough, notwithſtanding the Exulcerations above mention'd. This confirm'd me in the Opinion that he had never Breath'd, by the right Lobe of the Lungs.

There could be nothing more emaciated than this Child was; and it ſeems to be worth conſidering, whether his Illneſs might not be owing in a great meaſure to the want of the Omentum, (for he ſeem'd never to have had any); as alſo, whence it is that this Part is generally conſum'd in an Atrophy, and in moſt Hydropical Caſes, except where it ſelf is more eſpecially concerned.

II. *De Seriebus infinitis Tractatus. Pars Prima.* *Auctore* Petro Remundo *de* Monmort. R. S. S.

Prop. I. *Prob.*

INvenire summam terminorum quot libuerit Seriei hujus $a \times \overline{a+n} \times \overline{a+2n} \times \&c. \times \overline{a+p-1n}$ $+ \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times \&c. \times \overline{a+pn}$ $+ \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n} \times \&c. \times \overline{a+p+1n}$ $+ \overline{a+3n} \times \&c.$ Ubi est n differentia data, tam inter Factores continuos, a, $a+n$, $a+2n$, *&c.* ejusdem cujusvis termini, quàm inter Factores homologos terminorum diversorum in Serie continuatâ; atque designat p numerum factorum hujusmodi in quovis termino.

Solutio. Per x designetur primus Factorum in ultimo terminorum quorum summa requiritur, atque summa illa erit

$$\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \&c. \times \overline{a+p-1n}}{\overline{p+1n}}$$

Q. E. I.

Ex. 1. Proponatur Series numerorum naturalium $1+2+3+4+\&c.$ & invenienda sit summa tot terminorum quot sunt unitates in numero z, qui in hoc casu est etiam ultimus terminorum quorum summa requiritur. In hoc itaque casu sunt $a=1$, $n=1$, $p=1$, & $x=z$. Unde fit $x \times \overline{x+n} \times \&c. \times \overline{x+pn} = z \times \overline{z+1}$, $\overline{a-n} \times a \times \&c. \times \overline{a+p-1n} = 0 \times 1$, atque $\overline{p+1n} = 2 \times 1$; adeoque summa quæsita est $\frac{z \times \overline{z+1}}{2}$.

Ex. 2. Invenienda sit summa tot terminorum, quot sunt unitates in numero z, Seriei $1+3+6+10+\&c.$ Numerorum Triangularium. Numeri 1, 3, 6, 10, *&c.* in hac

 Serie

Serie sic scribi possunt $\frac{1\times2}{2}$, $\frac{2\times3}{2}$, $\frac{3\times4}{2}$, $\frac{4\times5}{2}$, &c. Hoc pacto, seposito divisore dato 2, Series revocatur ad formam Propositionis, existentibus $a=1$, $n=1$, & $p=2$. $x=z$. Unde summa Seriei duplicata est $\frac{x\times\overline{x+1}\times\overline{x+2}-0\times1\times2}{3}=\frac{x\times\overline{x+1}\times\overline{x+2}}{3}$; adeoque habitâ ratione divisoris 2, Summa Seriei ipsius est $\frac{x\times\overline{x+1}\times\overline{x+2}}{2\times3}$, vel $\frac{z\times\overline{z+1}\times\overline{z+2}}{2\times3}$, in hoc casu existente x eodem ac z. Ad eundem modum inveniuntur summæ cæterorum numerorum figuratorum, quorum formulæ jam vulgò innotescunt.

Ex. 3. Sint $a=1$, $n=2$, $p=3$. ut sit Series proposita $1\times3\times5+3\times5\times7+5\times7\times9+$ &c. In hoc itaque casu formula summæ fit $\frac{x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6}-\overline{1-2}\times1\times3\times5}{4\times2}=\frac{x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6}+15}{8}$. Verbi gratiâ si quæratur summa decem terminorum, fit $x=19$ (nempe terminus decimus in Serie Arithmeticè proportionalium, 1, 3, 5, 7, &c.) adeoque summa est $\frac{19\times21\times23\times25+15}{8}$ $=28680$. Propositio vero sic demonstratur.

Demonstratio. Sit Series quantitatum A, B, C, D, E, &c. quarum differentiæ constituant Seriem a, b, c, d, &c. (nempe ut sint $a=B-A$, $b=C-B$, $c=D-C$, &c.) Hinc statim colligitur esse $a+b=C-A$, $a+b+c=D-A$, $a+b+c+d=E-A$: & in genere aggregatum quotlibet terminorum Seriei a, b, c, d, &c. æquale est termino proximè insequenti Seriei A, B, C, D, E, &c. mulctato termino primo A. Pro A, B, C, &c. sume terminos

$a-n$

$\frac{\overline{a-n}\times a\times \&c.\times \overline{a+p-1\,n}}{\overline{p+1}\,n}$, $\frac{a\times \overline{a+n}\times \&c.\times \overline{a+pn}}{\overline{p+1}\,n}$,
$\frac{\overline{a+n}\times \overline{a+2n}\times \&c.\times \overline{a+p+1\,n}}{\overline{p+1}\,n}$, *&c.* hoc est, valores successivos ipsius $\frac{x\times \overline{x+n}\times \&c.\times \overline{x+pn}}{\overline{p+1}\,n}$; & eorum differentiæ, pro a, b, c, d, *&c* sumendæ, erunt $a\times \overline{a+n}\times \&c.\times \overline{a+p-1\,n}$, $\overline{a+n}\times \overline{a+2n}\times \&c.\times \overline{a+pn}$, *&c.* qui sunt ipsissimi termini Seriei propositæ. Sed comparando has Series, si terminus aliquis Seriei posterioris sit $x\times \overline{x+n}\times \&c.\times \overline{x+p-1\,n}$, constat terminum uno ulteriorem in Serie priori fore $\frac{x\times \overline{x+n}\times \&c.\times \overline{x+pn}}{\overline{p+1}\,n}$. Summa itaque Seriei posterioris usque terminum $x\times \overline{x+n}\times \&c.\times \overline{x+p-1\,n}$ inclusivè est $\frac{x\times \overline{x+n}\times \&c.\times \overline{x+pn} - \overline{a-n}\times a\times \&c.\times \overline{a+p-1\,n}}{\overline{p+1}\,n}$

Q. E. D.

Scholium 1. In hâc propositione continetur particula quædam Methodi incrementorum, de quâ ante biennium librum edidit D. *Brook Taylor Soc. Reg. Lond. Secr.* mihi amicitiâ conjunctissimus. Librum ipsum adeat qui de eâ methodo plura scire velit: ad institutum nostrum sufficit observare quanta intersit affinitas inter Methodum hanc & Methodum Fluxionum seu differentialem Nam ut in Methodo differentiali, ad inveniendum differentiale ipsius x dignitatis x^m, unum latus x convertendum est in differentiam dx; & ortum ducendum est in dignitatis Indicem m, ut sit $m\,dx\,x^{m-1}$ differentiale quæsitum; sic in Methodo Incrementorum *Ad inveniendum Incrementum facti hujusmodi* $x\times \overline{x+n}\times \overline{x+2n}$, (*ubi factores* x, $x+n$, $x+2n$,

$\overline{x+2n}$, sunt in progressione Arithmeticâ, cujus differentia communis est ipsius x Incrmentum datum n,) Factorum minimus x convertendus est in Incrementum, & ortum ducendum est in numerum Factorum, ut fit $3n \times \overline{x+n} \times \overline{x+2n}$ Incrementum quæsitum, numero Factorum in casu exposito existente 3. Sic etiam ipsius $x \times \overline{x+n}$ Incrementum fit $2n \times \overline{x+n}$.

2. Incrementa etiam Reciprocorum hujusmodi Factorum inveniuntur per eandem regulam; hoc nempe observato, quòd cum sit Divisio contrarium Multiplicationis, vice ablationis minimi Factorum, sit jam addendus alius factor adhuc uno Incremento major; item quòd Factorum numerus sit scribendus cum signo negativo. Hoc pacto ipsius $\frac{1}{x}$ Incrementum fit $\frac{-1 \times n}{x \times \overline{x+n}}$; ipsius $\frac{1}{x \times \overline{x+n}}$ Incrementum fit $\frac{-2 \times n}{x \times \overline{x+n} \times \overline{x+2n}}$; & sic de aliis hujusmodi. Hoc facile probatur sumendo differentias inter Integralium valores duos continuos.

3. Insistendo vestigiis Methodi directæ, hinc colliguntur præcepta Methodi inversæ, quibus inveniuntur Integralia Incrementorum oblatorum. *Applicetur enim Incrementum oblatum ad lateris Incrementum datum; addatur Factor adhuc uno Incremento minor, & applicetur ortum ad numerum Factorum sic auctorum.* Sic *e. g.* oblato Incremento $n \times x \times \overline{x+n} \times \overline{x+2n}$, fit primò $x \times \overline{x+n} \times \overline{x+2n}$; deinde $\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}$, addito Factore $\overline{x-n}$; denique $\frac{\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}}{4}$, quod est Integrale quæsitum. Hoc quidem ubi Factores sunt Multiplicantes; Ubi vero Factores occupant locum divisoris, mutatis mutandis, regula hæc est, *Applicetur Incrementum oblatum ad lateris incrementum datum; rejiciatur*

Factorum

Factorum maximus, & applicetur ortum ad numerum Factorum relictorum cum signo negativo. Exempli gratiâ oblato Incremento $\frac{n}{x \times \overline{x+n} \times \overline{x+2n}}$, fit primò $\frac{1}{x \times \overline{x+n} \times \overline{x+2n}}$, deinde $\frac{1}{x \times \overline{x+n}}$, denique $\frac{1}{-2 \times x \times \overline{x+n}}$, seu $\frac{-1}{2x \times \overline{x+n}}$, quod est Integrale quæsitum.

4. In casu hoc novissimo Integrale inventum, cum signo contrario, æquale est summæ omnium Incrementorum in Serie in infinitum continuatâ; *v. g.* est $\frac{1}{2x \times \overline{x+n}}$ $= \frac{n}{x \times \overline{x+n} \times \overline{x+2n}} + \frac{n}{\overline{x+n} \times \overline{x+2n} \times \overline{x+3n}}$ $+ \frac{n}{\overline{x+2n} \times \overline{x+3n} \times \overline{x+4n}} +$ *&c.* Nam in hoc casu, facto x tandem infinito, evanescit $\frac{1}{2x \times \overline{x+n}}$, hoc est, ultimus terminorum A, B, C, *&c.* fit nihil; & ob contrarietatem signorum Integralis & Incrementi, vice $-A$ exprimitur aggregatum per $+A$.

Lemma 1.

Per X designetur terminus quilibet in Serie quâvis numerorum M, N, O, P, *&c*; per x designetur locus termini istius X in Serie illâ (*v. g.* ut sit $x = 1$, quando designat X terminum primum M, sit $x = 2$, quando designat X terminum secundum N, & sic de cæteris) & sint terminorum M, N, O, P prima differentiarum primarum b, c prima differentiarum secundarum, d prima tertiarum, e prima quartarum, & sic porrò. Tum erit

 $X = M$

$X = M + b \times \frac{x-1}{1} + c \times \frac{x-1}{1} \times \frac{x-2}{2} + d \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} + e \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} \times \frac{x-4}{4} +$ &c. Sequitur hoc ex tabulâ æquationum *pag.* 66. tractatûs nostri *Essay d'Analyse, &c.*

Lemma 2.

Iisdem positis, per z designetur terminus quilibet in Serie Arithmeticè proportionalium a, $\overline{a+n}$, $a+2n$, &c & sit jam $X = A + Bz + Cz \times \overline{z+n} + Dz \times \overline{z+n} \times \overline{z+2n} + Ez \times \overline{z+n} \times \overline{z+2n} \times \overline{z+3n} +$ &c. Tum ipsorum A, B, C, D, E, &c. valores erunt.

$$A = M + b \times \frac{-a}{n} + c \times \frac{-a}{n} \times \frac{-a-n}{2n} +$$

$$+ d \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} +$$

$$+ e \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} \times \frac{-a-3n}{4n} + \&c.$$

$$B = \frac{1}{n} \times b + c \times \frac{-a-n}{n} + d \times \frac{-a-n}{n} \times \frac{-a-2n}{2n}$$

$$+ e \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} \times \frac{-a-3n}{3n} \ \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times c + d \times \frac{-a-2n}{n} + e \times \frac{-a-2n}{n} \times \frac{-a-3n}{2n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} d + e \times \frac{-a-3n}{n} + \&c.$$

$$E = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \frac{1}{4n} e + \&c.$$

Ordo

Ordo formandi coefficientes ipsorum $b, c, d, e,$ &c. in his valoribus, per se est satis manifestus.

Demonstratio. Quoniam per x & z designantur termini correspondentes progressionum Arithmeticarum $1, 2, 3, 4,$ &c. & $a, a+n, a+2n, a+3n,$ &c. indicabit $x-1$ numerum differentiarum n qui in z continetur, ut sit $z=a+\overline{x-1}\,n$. Hinc fit $x-1=\frac{z-a}{n}$, $x-2=\frac{z-n-a}{n}$, $x-3=\frac{z-2n-a}{n}$, &c. Substituendo itaque hos valores $x-1, x-2, x-3,$ &c. in Serie Lemmatis præcedentis, & terminis in ordinem redactis, prodeunt ipsorum $A, B, C,$ &c. valores exhibiti.

Cor. Ubi $a=n$, prodeunt $A, B, C, D,$ &c. per formulas simpliciores, nempe

$$A=M-b+c-d+e \text{ \&c.}$$

$$B=\frac{1}{n}\times\overline{b-2c+3d-4e \text{ \&c.}}$$

$$C=\frac{1}{n}\times\frac{1}{2n}\times\overline{c-3d+6e \text{ \&c.}}$$

$$D=\frac{1}{n}\times\frac{1}{2n}\times\frac{1}{3n}\times\overline{d+4e \text{ \&c.}}$$

Lemma 3.

Symbolis X & x eodem modo interpretatis ac in Lemmate primo; sint $q, r, s, t, u,$ &c. generatores Trianguli Arithmetici cujus lineam transversam, occupat Series M, N, O, P, Q &c in ordine nempe inverso, ut sit $q\,(=M)$ generator ultimus, r penultimus, s antepenultimus, & sic porrò. Tum erit

$$X=q+r\times\frac{x-1}{1}+s\times\frac{x-1}{1}\times\frac{x}{2}+t\times\frac{x-1}{1}\times\frac{x}{2}\times\frac{x+1}{3}$$
$$+\text{ \&c.}$$

Constat

Conſtat ex contemplatione ipſius Trianguli Arithmetici, quam exhibuimus *pag.* 63 tractatûs *Eſſay d'Analyſe, &c.* ubi idem fuſius explicatur.

Lemma 4.

Iiſdem poſitis, & Symbolo z eodem modo interpretato ac in *Lem.* 2. ſi ſit $X = A + Bz + Cz \times \overline{z+n} + \&c.$ ut in *Lem.* 2. erunt coefficientium $A, B, C, D,$ *&c.* valores.

$$A = q + r \times \frac{-a}{n} + s \times \frac{-a}{n} \times \frac{-a+n}{2n}$$

$$+ t \times \frac{-a}{n} \times \frac{-a+n}{2n} \times \frac{-a+2n}{3n} + \&c.$$

$$B = \frac{1}{n} \times \overline{r + s \times \frac{-a}{n} + t \times \frac{-a}{a} \times \frac{-a+n}{2n} + \&c.}$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times \overline{s + t \times \frac{-a}{n} + \&c.}$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \overline{t + \&c.}$$

Ordo coefficientium in his valoribus eſt manifeſtus, & demonſtratur Lemma ad modum *Lemmatis* 2.

Cor. 1. Ubi $a = n$, coefficientes, $A, B, C, D,$ *&c.* prodeunt per formulas ſimpliciores, nempe

$$A = q - r, \qquad C = \frac{1}{n} \times \frac{1}{2n} \times \overline{s - t}$$

$$B = \frac{1}{n} \times \overline{r - s}, \qquad D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \, \overline{t - u} \qquad \&c.$$

Cor. 2. Unde ſi generatorum $q, r, s, t, u,$ *&c.* aliquot ſint inter ſe æquales, exhibebitur X per formulam ſimpliciorem, evaneſcentibus aliquot coefficientium $A, B, C, D,$ *&c.*

Sic

Sic exempli gratiâ, propositâ Serie numerorum 4, 69, 530, 2676, 10350, &c. qui constituunt lineam decimam transversam in Triangulo Arithmetico cujus generatores tres priores sunt 54, -18, 5, & septem posteriores sunt æquales 4; existente $a = 1 = n$. Terminus X exhibetur per formulam quatuor tantùm terminorum.

$$-\frac{z}{1}\cdot\frac{z+1}{2}\cdot\frac{z+2}{3}\ \&c.\times\frac{z+6}{7}+23\frac{z}{1}\cdot\frac{z+1}{2}\cdot\&c.\times\frac{z+6}{7}-72\frac{z}{1}\cdot\frac{z+1}{2}\cdot\&c.\times\frac{z+7}{8}+54\frac{z}{1}\cdot\frac{z+1}{2}\cdot\&c.\times\frac{z+8}{9}.$$

evanescentibus coefficientibus sex primis A, B, C, D, E, F.

Prop. II. *Prob.*

Invenire summam quotlibet terminorum Seriei

$$\frac{M}{a\times\overline{a+n}\times\&c.\times\overline{a+p-1n}}+\frac{N}{\overline{a+n}\times\&c.\times\overline{a+pn}}+\frac{O}{\overline{a+2n}\times\&c.\times\overline{a+p+1n}}+\&c.$$

ubi numeratores M, N, O, &c. constituunt Seriem quamlibet terminorum, quorum differentiæ, vel primæ, vel secundæ, vel aliæ quædam dantur; vel quod perinde est, qui constituunt lineam quamvis transversam in dato quovis triangulo Arithmetico; Denominatores autem constituunt Seriem in *Prop.* I. exhibitam.

Solutio. Per X designetur primus factorum a, $a+n$, $a+2n$, &c. in denominatore ejusdem termini, ut sint X & z iidem ac in *Lemm*: præmissis, adeoque designetur terminus quilibet Seriei per $\frac{X}{z\times\overline{z+n}\times\&c.\times\overline{z+p-n}}$

Per *Lem.* 2, vel per *Lem.* 4. (prout magis commodum videatur

videatur vel differentias, vel generatores trianguli Arithmetici adhibere,) resolvatur X in Multinomium $A + B\times z + Cz\times\overline{z+n} + Dz\times\overline{z+n}\times\overline{z+2n} + \&c.$ Hoc pacto (terminis multinomii ad denominatorem $z\times\overline{z+n}\times\&c.\times\overline{z+p-n}$ applicatis) terminus quilibet Seriei revocabitur ad formulam $\frac{A}{z\times\overline{z+n}\times\&c.\times\overline{z+\overline{p-1}\,n}}$

$$+\frac{B}{\overline{z+n}\times\&c.\times\overline{z+\overline{p-1}\,n}}+\frac{C}{\overline{z+2n}\times\&c.\times\overline{z+\overline{p-1}\,n}}$$

$+ \&c.$

Unde (per Scholium 4 *Prop.* I.) aggregatum totius Seriei, à termino $\frac{X}{z\times\overline{z+n}\times\&c.\times\overline{z+\overline{p-1}\,n}}$ inclusive in infinitum continuatæ, est

$$\frac{A}{\overline{p-1}\times n\times z\times\overline{z+n}\times\&c.\times\overline{z+\overline{p-2}\,n}}$$

$$+\frac{B}{\overline{p-2}\times n\times\overline{z+n}\times\&c.\times\overline{z+\overline{p-2}\,n}}$$

$$+\frac{C}{\overline{p-3}\times n\times\overline{z+2n}\times\&c.\times\overline{z+\overline{p-2}\,n}}+\&c.$$

re si dematur hoc aggregatum ab ejusdem aggregati valore quando $z=a$, residuum erit summa omnium terminorum ante terminum $\frac{X}{z+\&c.}$, hoc est, tot terminorum quot sunt unitates in $\frac{z-a}{n}$. *Q. E. I.*

Ex. 1. Sit primum exemplum in Serie $\frac{5}{3\cdot5\cdot7\cdot9\cdot11\cdot13}$ +

$$+\frac{41}{5\cdot7\cdot9\cdot11\cdot13\cdot15}+\frac{131}{7\cdot9\cdot11\cdot13\cdot15\cdot17}$$

$$+\frac{275}{9\cdot11\cdot13\cdot15\cdot17\cdot19}+\frac{473}{11\cdot13\cdot15\cdot17\cdot19\cdot21}$$

+ &c. Sunt hic $a=3$, $n=2$, $p=6$, $M=5$, & capiendo differentias numeratorum inveniuntur $b=36$, $c=54$, $d=0=e=$ &c. Hinc in Lemmate secundo sunt $A=5+36\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-5}{4}=\frac{209}{4}$, $B=\frac{1}{2}\times36+54\times\frac{-5}{2}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times54=\frac{27}{4}$, $D=0=E=$ &c. Summa itaque totius Seriei est $\frac{209}{4\times5\times2\times3\cdot5\cdot7\cdot9\cdot11}+\frac{-99}{2\times4\times2\times5\cdot7\cdot9\cdot11}+\frac{27}{4\times3\times2\times7\cdot9\cdot11}=\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}$, atque summa terminorum numero $\frac{z-3}{2}$ $\left(=\frac{z-a}{n}\right)$ est

$$\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}-\frac{209}{40\times z.\overline{z+2}.\overline{z+4}.\overline{z+6}.\overline{z+8}}$$

$$+\frac{99}{16\times\overline{z+2}\times\overline{z+4}.\overline{z+6}.\overline{z+8}}-\frac{27}{24\times\overline{z+4}.\overline{z+6}.\overline{z+8}}.$$

Quærantur *v. g.* octo termini; tum existente $\frac{z-3}{2}=8$ fit $z=19$, quo valore in formulâ adhibito, prodit summa $\frac{155891}{2\cdot3\cdot3\cdot3\cdot3\cdot5\cdot5\cdot5\cdot7\cdot11\cdot19\cdot23}$.

Iidem Numeratores occupant lineam tertiam transversam in Triangulo Arithmetico

54 . 54 . 54 . 54 . 54 . 54 . &c.
= 18 . 36 . 90 . 144 . 198 . &c.
5 . 41 . 131 . 275 . &c.

Unde

Unde in formula *Lem.* 4. sunt generatores $q = 5$, $r = -18$; $s = 54$, $t = 0 = \&c.$ & prodeunt coefficientes $A = 5 - 18 \times \frac{-3}{2} + 54 \times \frac{-3}{2} \times \frac{-3+2}{4} = \frac{209}{4}$, $B = \frac{1}{2} \times \overline{-18 + 54 \times \frac{-3}{2}} = \frac{-99}{2}$, $C = \frac{1}{2} \times \frac{1}{4} \times 54 = \frac{27}{4}$, $D = 0 = E = \&c.$ iidem ac supra.

Ex. 2. Sit Series $\frac{4}{1.2.3.4.5.6.7.8.9.10.11} + \frac{69}{2.3.\&c.12} + \frac{530}{3.4.\&c.13} + \frac{2676}{4.5.\&c.14} + \frac{10350}{5.6.\&c.15} + \&c.$ Ubi sunt $a = 1$, $n = 1$, $p = 11$, atque Numeratores constituant Seriem in *Corol.* 20. *Lem.* 4. exhibitam. Applicando itaque valorem X in *Corol.* illo ad denominatorem $z \times \overline{z+1} \times \&c. \times \overline{z+10}$, fit Seriei propositæ Terminus

$$\frac{-1}{1.2.3.4.5.6 \times \overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$+ \frac{23}{1.2.3.4.5.6.7 \times \overline{z+7}.\overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$- \frac{72}{1.2.3.4.5.6.7.8 \times \overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$+ \frac{54}{1.2.3.4.5.6.7.8.9 \times \overline{z+9} \times \overline{z+10}}.$$ Adeoque per hanc *Prop.* summa Seriei à termino illo in infinitum continuatæ est

$$\frac{-1}{4 \times 1.2.3.4.5.6 \times \overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}}$$

+

$$+\frac{23}{3\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\times z\overline{+7}\cdot\overline{z+8}\cdot\overline{z+9}}$$

$$-\frac{72}{2\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\times z\overline{+8}\cdot\overline{z+9}}$$

$$+\frac{54}{1\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\times z\overline{+9}}.$$

Itaque pro z ſumpto 1, fit ſumma totius Seriei $\frac{305}{12\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10}$. Et in genere ſumma terminorum numero $\frac{z-1}{1}$, eſt $\frac{305}{12\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10}$

$$+\frac{1}{4\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\times z\overline{+6}\cdot\overline{z+7}\cdot\overline{z+8}\cdot\overline{z+9}}$$

$$-\frac{23}{3\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\times z\overline{+7}\cdot\overline{z+8}\cdot\overline{z+9}}$$

$$+\frac{72}{2\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\times z\overline{+8}\times\overline{z+9}}$$

$$-\frac{54}{1\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\times z\overline{+9}}.$$

Scholium 1. In computandis ſummis hujuſmodi Serierum, calculus plerumque levior eſt adhibitis generatoribus trianguli Arithmetici, quam ſi adhibeantur differentiæ. Libet itaque hac occaſione oſtendere quomodo ex datis differentiis inveniri poſſunt generatores Trianguli Arithmetici.

Sunto itaque ω primus Seriei terminus, a differentia ultima data, b prima differentiarum penultimarum, c prima antepenultimarum, & ſic porrò d, e, *&c.* atque ſint t, u, x, y, *&c* generatores quæſiti Trianguli Arithmetici, cujus lineam tranſverſam ordine p occupet Series

 pro-

proposita. Tum (quod ex contemplatione Trianguli Arithmetici facile constat) sunt

$$a = t$$

$$b = \frac{p-1}{1}t + u$$

$$c = \frac{p-1}{1} \times \frac{p-2}{2}t + \frac{p-2}{1}u + x$$

$$d = \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3}t + \frac{p-2}{1} \times \frac{p-3}{2}u$$

$$+ \frac{p-3}{1}x + y \ \&c.$$

Unde colliguntur generatorum valores

$$t = a$$

$$u = b - \frac{p-1}{1}t$$

$$x = c - \frac{p-1}{1} \times \frac{p-2}{2}t - \frac{p-2}{1}u$$

$$y = d - \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3}t - \frac{p-2}{1} \times \frac{p-3}{2}u$$

$$- \frac{p-3}{1}x \ \&c.$$

Ultimus autem generator æqualis est Seriei termino primo a.

2. D[nus] de *Monsoury Abbas Orbacensis* mihi amicissimus, & ruri vicinus, postquam cum eo hæc communicaveram, aliam invenit hujus Problematis Solutionem, cujus formulam ob ejus miram simplicitatem hic referre juvat. Itaque in Serie numeratorum sint a terminus primus, b prima differentiarum primarum, c prima secundarum, d prima tertiarum, & sic porrò; atque sit termini primi Denominator $z \times \overline{z+n} \times \&c. \times \overline{z+p-1\,n}$; Tum summa

summa totius Seriei in infinitum continuatæ exhibebitur per formulam $\frac{\omega}{n \times \overline{p-1} \times z \times \overline{z+n} \times \&c. \times \overline{z+p-2n}}$ $+ \frac{b}{n^2 \times \overline{p-1} \times \overline{p-2} \times \overline{z+n} \times \&c. \times \overline{z+p-2n}} +$ $\frac{c}{n^3 \times \overline{p-1} \times \overline{p-2} \times \overline{p-3} \times \overline{z+2n} \times \&c. \times \overline{z+p-2n}}$ $+ \&c.$

Sit exemplum in Serie $\frac{5}{3 \cdot 5 \cdot \&c. 13} + \frac{41}{5 \cdot 7 \cdot \&c. 15} + \frac{131}{7 \cdot 9 \cdot \&c. 17} + \frac{275}{9 \cdot 11 \cdot \&c. 19} + \&c.$ cujus summam jam exhibuimus. In hoc casu sunt $\omega = 5$, $b = 36$, $c = 54$, $d = 0 = e = \&c.$ Unde per formulam summa Seriei integræ fit $\frac{5}{2 \cdot 5 \times 3 \cdot 5 \ldots 11} + \frac{36}{4 \cdot 5 \cdot 4 \times 5 \ldots 11}$ $+ \frac{54}{8 \cdot 5 \cdot 4 \cdot 3 \times 7 \ldots 11} = \frac{283}{80 \times 3 \cdot 5 \ldots 11}$, ut per formulam nostram exhibetur. Si quæratur summa ejusdem Seriei incipientis à termino decimo $\frac{2273}{21 \ldots 31}$, in eo casu $\omega = 2273$, $b = 522$, $c = 54$, & summa esset $\frac{2273}{2 \cdot 5 \times 21 \ldots 29} + \frac{522}{4 \cdot 5 \cdot 4 \times 23 \ldots 29} + \frac{54}{8 \cdot 5 \cdot 4 \cdot 3 \times 25 \ldots 29}$.

Hæc formula est commodissima, & summam exhibet nullo ferè negotio, quoties quæritur summa Seriei integræ, & differentiæ non sunt nimis multæ. Sed ubi plures sunt differentiæ, & quæritur non Series integra, sed termini tantùm initiales aliquammulti, formulæ nostræ sunt commodiores.

3. Quando

3. Quando Serierum termini formantur tantùm per Multiplicationem, nec afficiuntur divisoribus variabilibus, summæ semper exhiberi possunt per Methodum in *Prop.* I. traditam, sint licet formulæ quantumlibet compositæ. Nam possunt semper revocari ad terminos in formâ quam postulat Propositio illa. Sic si differentiæ ipsorum z & x sint m & n, & designetur terminus Seriei per $z\,x$; hic terminus revocabitur ad formam $\overline{a-n}\,z + \frac{n}{m}\,z \times \overline{z+m}$; cujus Integrale datur *per Prop.* I; nempe quoniam $dx = n$, & $dz = m$, est $dx = dz \times \frac{n}{m}$; unde regrediendo ad integralia fit $x = \frac{n}{m}\,z + a$ (adjecto invariabili a, ut habeatur ratio relationis inter z & x in Seriei termino primo,) quod sic scribi potest $\overline{a-n} + \frac{n}{m} \times \overline{z+m}$, ut deinde in z ductum induat formam requisitam. Et ad eundem modum procedere licet in aliis casibus ejusmodi. Sed ubi formulæ oblatæ divisoribus afficiuntur, eædem ac in Calculo integrali, ut vocant, difficultates occurrunt, eâdem industriâ superandæ. Nec tamen semper superari possunt. Nam præterquam quod vix certò sciri possit quæ debeat relatio intercedere inter Numeratorem fractionis & Denominatorem, ut formula oblata ad Integrale revocari possit; sæpe etiam difficillimum est explorare an adsit jam talis relatio in formulâ istâ, aut si desit, an introduci possit. Quicquid ego in hâc materiâ potissimum inveni, continetur in tribus sequentibus propositionibus.

Prop. III. *Prob.*

Crescentibus, z, u, y, x, *&c.* per differentias datas n, m, l, o, *&c.* invenire valorem numeratoris integri

tegri N, ut existente Denominatore $z . \overline{z+n} . \&c. \overline{z+pn} \times u . \overline{u+m} . \&c. \overline{u+qm} \times y . \overline{y+l} . \&c. \overline{y+rl} \times x . \overline{x+o}$ $\&c. \overline{x+so} . \&c.$ Fractio ad Integrale revocari possit.

Solutio. Fiat $N = \overline{z+pn} \times \overline{u+qm} \times \overline{y+rl} \times \overline{x+so} \times \&c. - zuyx \&c.$ atque Integrale erit fractio, cujus Denominator $z . \overline{z+n} . \&c. \overline{z+\overline{p-1}\,n} \times u . \overline{u+m} . \&c. \overline{u+\overline{q-1}\,m} \times y . \overline{y+l} . \&c. \overline{y-\overline{x-1}\,l} \times x . \overline{x+o} . \&c. \overline{x+\overline{s-1}\,o} \times \&c.$ existente 1 Numeratore.

Differentia enim hujus fractionis est fractio cujus numerator est ipsius N valor exhibitus, & denominator idem est ac denominator propositus, ut fieri debuit.

Ex. 1. Sit denominator propositus $z \times \overline{z+2} \times u \times \overline{u+3}$. In hoc casu sunt $n=2$, $m=3$, $p=1$, $q=1$; adeoque est $N = \overline{z+2} \times \overline{u+3} - zu = 3z+2u+6$, & per $\frac{3z+2u+6}{z.\overline{z+2} \times u.\overline{u+3}}$ representatur terminus Seriei summabilis, cujus nempe in infinitum continuatæ summa exhibetur per $\frac{1}{zu}$. *Sint* verbi gratiâ, ipsorum z & u primus valor communis 1, atque Series summabilis erit $\frac{11}{1.3 \times 1.4} + \frac{23}{3.5 \times 4.7} + \frac{35}{5.7 \times 7.10} + \&c.$ quippe cujus totius summa est 1. Per p designetur ordo termini cujusvis in hâc Serie, erit $p = \frac{z-1+2}{2} = \frac{u-1+3}{3}$, adeoque $z = 2p-1$, & $u = 3p-2$; quibus valoribus pro z & u scriptis, designabitur terminus per formulam $\frac{12p-1}{\overline{2p-1} \times \overline{2p+1} \times \overline{3p-2} \times \overline{3p+1}}$. Summa autem terminorum omnium ante terminum illum, hoc est terminorum initialium numero $\frac{z-1}{2} = p-1$, est

$1 - \frac{1}{zu}, = \frac{zu-1}{zu}$, hoc est $\frac{6pp-7p+1}{\overline{2p-1}\times\overline{3p-2}}$. Quare pro p scripto $p+1$, erit $\frac{p\times\overline{6p+5}}{\overline{2p+1}\times\overline{3p+1}}$ aggregatum tot terminorum initialium quot sunt unitates in p.

Ex. 2. Iisdem manentibus z, u, n, m, sit denominator $z.\overline{z+2}.\overline{z+4}\times u.\overline{u+3}$. Tum per formulam numerator erit $\overline{z+4}\times\overline{u+3} - zu = 3z+4u+12$, & summa Seriei exhibebitur per formulam $\frac{1}{z.\overline{z+2}\times u}$. Sit ipsorum z & u primus valor communis 1, & hinc elicietur Series $\frac{19}{1.3.5\times 1.4} + \frac{37}{3.5.7\times 4.7} + \frac{55}{5.7.9\times 7.10}$ + &c. $= \frac{1}{3}$.

Scholium. In Seriebus jam expositis eadem ubique est differentia inter factores continuos ejusdem cujusvis termini, ac inter factores homologos terminorum continuorum. In sequentibus exempla quædam sunt Serierum, quarum summæ in terminis numero finitis exhiberi possunt, quamvis ea regula non observetur.

Prop. IV. *Prob.*

Crescente z per differentias datas qn, invenire numeratorem integrum N, ut ad Integrale revocari possit fractio, cujus Denominator fit ex certo numero p terminorum z, $z+n$, $z+2n$, &c. Arithmeticè proportionalium in invicem ductorum. Debet autem esse q numerus integer minor quam factorum numerus p.

Solutio. Erit $N = \overline{z+\overline{p-1}\,n}\times\overline{z+\overline{p-2}\,n}\times$ &c. $\times\overline{z+\overline{p-q}\,n} - z\times\overline{z+n}\times$ &c. $\times\overline{z+\overline{q-1}\,n}$, Integrale

tegrale existente $\frac{1}{z \times \overline{z+n} \times \&c. \times \overline{z+p-q-1n}}$ Demonstratur ad modum propositionis præcedentis.

Sumptis ad libitum n, p, q, & primo valore z, hinc oriuntur infinitæ Series summabiles, cujusmodi sunt Series tres sequentes.

$$A = \frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \frac{17}{7.8.9.10} \&c.$$

$$B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11}$$
$$+ \frac{16}{10.11.12.13.14} + \&c.$$

$$C = \frac{1}{1.2.3.4.5} + \frac{14}{5.6.7.8.9} + \frac{55}{9.10.11.12.13}$$
$$+ \frac{140}{13.14.15.16.17} + \&c.$$

Has Series jampridem communicavi cum primariis quibusdam Geometris, à quibus minimè contemni videntur. Sic ad me scribit peritissimus Geometra D *Nicolaus Bernoulli* in epistolâ datâ 25 *Julii* 1716. " Vous " me ferez un extreme plaisir, *Monsieur*, de me com- " muniquer la Solution de vostre probleme, *Etant donnée* " *une suitte des Fractions dont les Numerateurs soient des* " *nombres figurés quelconque, & dont les Denominateurs* " *soient formés du produit d'un nombre egal de Facteurs* " *qui soient en Progression Arithmetique, trouver la som-* " *me*; & principalement comment vous avez trouvé " ces deux formules $\frac{p}{24 \times \overline{4p+1}}$, $\frac{p.\overline{p+1}}{12 \times \overline{3p+1} \times \overline{3p+2}}$.

Hæ formulæ spectant ad Series C & B, designante p numerum terminorum, quorum summa requiritur. Sic etiam ad me scribit D. *Taylor* in epistola datâ 22 *Aug.* 1716. " Ut & quâ ratione incidisti in summationem " Serierum à te exhibitarum, præsertim loquor de

Serie

" Serie $\frac{1}{1.2.3.4.5}+\frac{4}{4.5.6.7.8}+\frac{9}{7.8.9.10.11}+$ &c.
" quæ videtur esse altioris indaginis.

Sed ut ad exempla jam redeamus. In Serie *A* sunt $p=4$, $q=2$, $n=1$, primo valore z existente 1. Est itaque $\overline{z+3}\times\overline{z+2}-z\times\overline{z+1}=2\times\overline{2z+3}$ formula, unde (rejecto dato numero 2) derivantur numeratores 5, 9, 13, 17, &c. Formula etiam summæ est $\frac{1}{z\times\overline{z+1}}$. Quare habitâ ratione numeri 2, quem ex numeratoribus rejecimus, summa totius Seriei, à termino in quo est z in infinitum continuatæ, exhibetur per formulam $\frac{1}{2z\times\overline{z+1}}$; adeoque summa Seriei integræ est $\frac{1}{2\times1\times2}=\frac{1}{4}$.

In Serie *B* sunt $n=1$, $p=5$, $q=3$, primo valore z existente 1. Est itaque $N=\overline{z+4}\times\overline{z+3}\times\overline{z+2}-z\times\overline{z+1}\times\overline{z+2}=6\times\overline{z+2}|^2$. Ipsius autem $\overline{z+2}$ valores continui sunt 3, 6, 9, &c. qui quoniam omnes sunt divisibiles per 3, ponendo $z+2=3x$, fit $N=6\times\overline{3x}|^2=6\times9x^2=54x^2$, ipsius x valoribus continuis existentibus 1, 2, 3, &c. Rejecto itaque numero dato 54, hinc prodeunt numeratores 1, 2^2, 3^2, &c. hoc est 1, 4, 9, &c. Formula etiam Integralis est $\frac{1}{z\times\overline{z+1}}$; quare habitâ ratione numeri 54 quem ex numeratoribus rejecimus, summa Seriei à termino in quo est z in infinitum continuatæ est $\frac{1}{54z\times\overline{z+1}}$. Unde summa Seriei integræ est $\frac{1}{108}$.

In Serie denique *C* sunt $n=1$, $p=5$, $q=4$, & primus valor $z=1$. Unde fit $N=\overline{z+4}\times\overline{z+3}\times\overline{z+2}\times\overline{z+1}-z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}=4\times\overline{z+1}$

×

$\overline{z+2} \times \overline{z+3}$. Valores autem N per hanc formulam prodeuntes semper possunt dividi per $4 \times 2 \times 3 \times 4 = 96$. Ergò hoc divisore rejecto prodeunt numeratores 1, 14, 55, 140, *&c.* Et formula Summæ, habitâ ratione numeri 96, est $\frac{1}{96z}$. Adeoque Summa Seriei integræ est $\frac{1}{96}$.

Scholium 1. Per Propositiones has duas novissimas nullo negotio inveniri possunt Series quot libuerit summabiles. Et vicissim oblatâ Serie hujus speciei, si summari potest, ejus summa plerumque revocatur ad alterutram ex his Propositionibus. In examine tamen solertiâ est opus. Optime autem procedit si termini Seriei oblatæ revocentur ad formulam *Prop.* III. Sic *e. gr.* propositâ Serie $\frac{7}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11} + \frac{11}{7 \cdot 9 \cdot 11 \cdot 13 \cdot 15} + \frac{15}{11 \cdot 13 \cdot 15 \cdot 17 \cdot 19} +$ *&c.* Denominatores sic scribi possunt $3 \cdot 7 \cdot 11 \times 5 \cdot 9$, $7 \cdot 11 \cdot 15 \times 9 \cdot 13$, $11 \cdot 15 \cdot 19 \times 13 \cdot 17$ · *&c.*

Unde juxta *Prop.* III. fit $n = 4$, $m = 4$, $p = 2$, $q = 1$, primus valor $z = 3$, primus valor $u = 5$. Hinc formula Numeratoris invenitur $4 \times \overline{z + 2u + 8}$, Est autem $z + 2u + 8$ semper divisibile per 3; quare rejectis divisoribus datis 4 & 3, per hanc formulam prodeunt Numeratores 7, 11, 15, *&c.* iidem ac Numeratores in Serie proposita, quæ proinde summabitur per illam propositionem.

2. Cùm Series illas *A*, *B*, *C*, communicaveram cum D. *Taylor*, rescripsit se earum summas invenisse primam quidem *A* & tertiam *C*, eas revocando ad casus simplices Methodi Incrementorum, tertiam *C*, *e g.* revocavit ad hanc formam $\frac{1}{24} \times \frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \frac{1}{13 \cdot 17}$ *&c*, ut habeatur summa per præcepta tradita in *Scholio Prop* 1.

In Serie autem secundâ B, cùm hoc non æquè successit, sequenti usus est Analysi, quam, ipsius venia jam impetratâ, ob ejus eximiam elegantiam huc transferre non piget. " Seriei istius terminus [in Stylo ejus] ex-
" hibetur per formulam $\frac{\overline{z+2} \times z}{27 z \times \overline{z+1} \times z \times \overline{z+1}}$; pro
$z+3$ in denominatore scripto z, quoniam est $z=3$.
" Pone $\frac{B}{27 C}$ æquale esse Integrali quæsito, hoc est $\frac{B}{C}$
" esse Integrale ipsius $\frac{\overline{z+2} \times z}{z \cdot \overline{z+1} \times z \cdot \overline{z+1}}$, seposito divi-
" sore dato 27. Ipsius autem $\frac{B}{C}$ incrementum est
" $\frac{B\dot{C} - BC}{C\dot{C}}$. Debet ergo $\frac{B\dot{C} - BC}{C\dot{C}}$ idem esse ac
$\frac{\overline{z+2} \times z}{z \cdot \overline{z+1} \times z \cdot \overline{z+1}}$. Comparando denominatores inveni-
" tur $C = z \times \overline{z+1}$. Hinc itaque sumendo incremen-
" ta fit $\dot{C} = 2 z\dot{z} + \dot{z}^2 + \dot{z}$ ($= 2 z\dot{z} + 4\dot{z}$, quoniam
" est $\dot{z} = 3$.) His valoribus in locum C & $\dot{C}$ substitu-
" tis prodit $B\dot{C} - B\dot{C} = \overline{zz + z}\, \dot{B} - 2\dot{z} \times \overline{z+2}\, B$,
" quod debet esse idem ac $\overline{z+2} \times \dot{z}$. Sit $B = a + v$,
" existente a ipsius B parte invariabili, & v parte va-
" riabili. Tum sumendo incrementa fit $\dot{B} = \dot{v}$. Unde
" ad invenienda a & v habetur æquatio $zz + z\dot{v}$
" $- 2\dot{z} \times \overline{z+2} \times \overline{a+v} = \overline{z+2} \times \dot{z}$, quæ sic scribi
" potest $\overline{zz+z}\,\dot{v} - 2\dot{z} \times \overline{z+2}\, v = \dot{z} \times \overline{z+2} \times \overline{1+2a}$,
" vel etiam $C\dot{v} - \dot{C}v = \dot{z} \times \overline{z+2} \times \overline{1+2a}$. Pone
" $1 + 2a = 0$ (unde fit $a = \frac{-1}{2}$,) & fit $C\dot{v} - \dot{C}v = 0$;
" ubi

" ubi fieri poteſt $\dot{v}=0$, (quoniam æquationis termini
" ſinguli afficiuntur vel ab v, vel ab $\dot{v}$) Hinc ergò fit $B=$
" $a=\frac{-1}{2}$, adeoque $\frac{B}{C}=\frac{-1}{2z\times\overline{z+1}}$. Unde habitâ ra-
" tione diviſoris 27, Integrale quæſitum fit $\frac{-1}{54\times z\times\overline{z+1}}$.
" Sed & comparando æquationem $C\dot{v}-\dot{C}v=0$ cum
" formulâ generali $\frac{B\dot{C}-\dot{B}C}{C\dot{C}}=0$, inde etiam conclude-
" re licet eſſe $\frac{v}{C}=$ quantitati datæ, (quoniam ipſius
" incrementum eſt 0.) Unde pro n ſumpto quovis
" numero dato, fit $v=nC$, atque $B=-\frac{1}{2}+nC$.
" Quo pacto Integrale quæſitum fit $\frac{B}{C}=\frac{-\frac{1}{2}+nC}{C}=\frac{-1}{2C}$
" $+n$, quod ab Integrali prius invento differt quan-
" titate datâ n. Hoc inde fit, quòd, ut in quadraturâ
" Curvarum Area inventa augeri poteſt vel minui areâ
" datâ, ſic in Methodo incrementorum Integrale inven-
" tum augeri poteſt vel minui quantitate datâ. Per
" Integrale autem primum, ubi deeſt n, exhibetur
" ſumma Seriei in infinitum continuatæ.

Prop. V.

Creſcente z per unitates, & exiſtentibus a, b, c, *&c.* numeris datis integris, quorum nullæ inter ſe æquantur; invenire Integrale ipſius $\frac{1}{z\times\overline{z+a}\times\overline{z+b}\times\overline{z+c}\times\&c.}$.

Solutio. Ducendo tam numeratorem quam denominatorem fractionis in terminos $z+1$, $z+2$, *&c.* $z+a+1$, $z+a+2$, *&c.* $z+b+1$, $z+b+2$, *&c.* $z+c+1$, $z+c+2$, *&c.* in denominatore deficientes, revocetur Denominator ad formulam $z\times\overline{z+1}$

×

$\times\overline{z+2}\times$ &c. denominatoris in *Prop.* I. *Schol. n.* 3. Deinde revocetur Numerator ad formam $A+Bz+Cz \times\overline{z+1}+Dz\times\overline{z+1}\times\overline{z+2}+$ &c. Tum applicando terminos ad Denominatorem novum $z\times\overline{z+1}\times\overline{z+2}\times$ &c. revocetur fractio ad hanc formam

$$\frac{A}{z\times\overline{z+1}\times \&c.}+\frac{B}{\overline{z+1}\times\overline{z+2}\times \&c.}+\frac{C}{\overline{z+2}\times\overline{z+3}\times \&c.}$$

$$+\frac{D}{\overline{z+3}\times\overline{z+4}\times \&c.}\ \&c.$$

Unde denique quæratur Integrale per *Schol. Prop.* I. *n.* 3.

Ratio Solutionis per se satis est manifesta.

Scholium 1. Hujus Solutionis tota difficultas latet in revocatione numeratoris ad formam requisitam, quod tamen quomodo sit faciendum uno exemplo patebit. Proponatur itaque factum $\overline{z+2}\times\overline{z+3}\times\overline{z+7}$, quod ad formam propositam sit revocandum. Terminos itaque evolvo gradatim ut sequitur. Factorem primum $z+2$ sic scribo $2+z$, cujus terminum primum 2 duco in $3+z$, unde fit $6+2z$: Terminum secundum z duco in $2+\overline{z+1}$ $(=z+3)$ unde fit $2z+z\times\overline{z+1}$. Dein facta in unam summam colligendo, fit $\overline{z+2}\times\overline{z+3}=\begin{matrix}6+2z\\+2z\end{matrix}+z\times\overline{z+1}=6+4z+z\times\overline{z+1}$. Superest ut hoc ducatur in $z+7$. Itaque terminum primum 6 duco in $7+z$ $(=z+7)$ unde fit $42+6z$; terminum secundum $4z$ duco in $6+\overline{z+1}$ $(=z+7)$ unde fit $24z+4z\times\overline{z+1}$; terminum tertium $z\times\overline{z+1}$ duco in $5+\overline{z+2}$ $(=z+7,)$ unde fit $5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Factis itaque in unum collectis ut prius, fit $\overline{z+2}\times\overline{z+3}\times\overline{z+4}=42+30z+9z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Et ad eundem modum procedere licet in aliis casibus.

2. Sit

2. Sit autem exemplum Propositionis in fractione $\frac{1}{z\times\overline{z+2}\times\overline{z+5}}$. Restituendo factores $z+1$, $z+3$, $z+4$ in Denominatore deficientes, fractio fit $\frac{\overline{z+1}\times\overline{z+3}\times\overline{z+4}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$. Revocandus itaque est Numerator $\overline{z+1}\times\overline{z+3}\times\overline{z+4}$ ad formam requisitam. Itaque per methodum jam traditam fit primo $\overline{z+1}\times\overline{z+3}=1\times\overline{3+z}+z\times\overline{2+\overline{z+1}}$ $=3+\overline{z+2z}+z\times\overline{z+1}=3+3z+z\times\overline{z+1}$. Deinde $\overline{z+1}\times\overline{z+3}\times\overline{z+4}=3\times\overline{4+z}+3z\times\overline{3+\overline{z+1}}+z\times\overline{z+1}\times\overline{2+\overline{z+2}}=12+3z+9z$ $+3z\times\overline{z+1}+2z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$ $=12+12z+5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Applicando hoc factum ad Denominatorem $z\times\overline{z+1}\times$ &c. $\times\overline{z+5}$ fractio tandem revocatur ad hanc formam

$$\frac{12}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$$
$$+\frac{12}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$$
$$+\frac{5}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{1}{\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$$

Cujus denique Integrale est $\frac{-12}{5z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}$

$$+\frac{-12}{4\cdot\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-5}{3\cdot\overline{z+2}\times\overline{z+3}\times\overline{z+4}}$$
$$\frac{-1}{2\cdot\overline{z+3}\times\overline{z+4}}.$$

3. Quando duo tantùm sunt factores z & $z+a$, exhibebitur etiam Integrale per formulam $\frac{1}{2}-\frac{1-a}{2z\times\overline{z+1}}$

$$-\frac{\overline{1-a}\times\overline{2-a}}{3z\times\overline{z+1}\times\overline{z+2}}-\frac{\overline{1-a}\times\overline{2-a}\times\overline{3-a}}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}\ \text{\&c.}$$

Seriem nempe continuando donec abrumpatur per eva-

nescentiam terminorum. Si Factores duo sint z & $z-a$ exhibebitur Integrale per formulam $\frac{-1}{z-1}-\frac{-1+a}{2\cdot\overline{z-1}\cdot\overline{z-2}}$ $-\frac{\overline{-1+a}\times\overline{-2+a}}{3\cdot\overline{z-1}\cdot\overline{z-2}\cdot\overline{z-3}}-$ &c. Potest idem Integrale exprimi utroque modo, prout fractionis oblatæ factor vel minor vel major sumatur pro z.

4. Si primus valor z sit $a+1$, migrabit formula posterior in hanc $\frac{-1}{a}\times\overline{\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}+\&c.\text{ usque }\frac{1}{a}}$ inclusivè, quâ, cum signo contrario, exhibetur summa Seriei $\frac{1}{1\times\overline{1+a}}+\frac{1}{2\times\overline{2+a}}+\frac{1}{3\times\overline{3+a}}+$ &c. in infinitum continuatæ. Sit *e. gr.* $a=1$, atque Series erit $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\&c.=\frac{1}{1}\times\frac{1}{1}=1$. Si $a=2$, erit Series $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\&c.=\frac{1}{2}\times\overline{\frac{1}{1}+\frac{1}{2}}=\frac{3}{4}$; Si $a=3$, Series erit $\frac{1}{1\times4}+\frac{1}{2\times5}+\frac{1}{3\times6}+\frac{1}{4\times7}$ &c. $=\frac{1}{3}\times\overline{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}}=\frac{11}{18}$.

5. Ex eâdem Serie $\frac{1}{1\times\overline{1+a}}+\frac{1}{2\times\overline{2+a}}+\frac{1}{3\times\overline{3+a}}$ + &c. pro diverso valore a oriuntur Series plures formâ satis elegantes, quarum nonnullas Lectori ob oculos sistere, credo, ingratum non erit.

Si pro a sumantur successivè numeri pares, 2, 4, 6, 8, &c. Series erunt

Si $a=2$) $\frac{1}{1\times\overline{1+2}}+\frac{1}{2\times\overline{2+2}}+\frac{1}{3\times\overline{3+2}}+\frac{1}{4\times\overline{4+2}}+$ &c.

4) $\frac{1}{1\times\overline{1+4}}+\frac{1}{2\times\overline{2+4}}+\frac{1}{3\times\overline{3+4}}+\frac{1}{4\times\overline{4+4}}+$ &c.

6) $\frac{1}{1\times\overline{1+6}}+\frac{1}{2\times\overline{2+6}}+\frac{1}{3\times\overline{3+6}}+\frac{1}{4\times\overline{4+6}}+$ &c.

8) $\frac{1}{1\times\overline{1+8}}+\frac{1}{2\times\overline{2+8}}+\frac{1}{3\times\overline{3+8}}+\frac{1}{4\times\overline{4+8}}+$ &c.

Vel

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{9-4} + \frac{1}{16-1} + \frac{1}{25-4} + \frac{1}{36-4} + \&c.$$

$$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9} + \&c.$$

$$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16} + \&c.$$

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{4+1} + \frac{1}{9+3} + \frac{1}{16+5} + \frac{1}{25+7} + \&c.$$

$$\frac{1}{4+3} + \frac{1}{9+7} + \frac{1}{16+11} + \frac{1}{25+15} + \&c.$$

$$\frac{1}{4+5} + \frac{1}{9+11} + \frac{1}{16+17} + \frac{1}{25+23} + \&c.$$

Si pro a sumantur successive numeri impares 1, 3, 5, 7, &c. Series erunt

$$a=1)\ \frac{1}{1\times\overline{1+1}} + \frac{1}{2\times\overline{2+1}} + \frac{1}{3\times\overline{3+1}} + \frac{1}{4\times\overline{4+1}} + \&c.$$

$$3)\ \frac{1}{1\times\overline{1+3}} + \frac{1}{2\times\overline{2+3}} + \frac{1}{3\times\overline{3+3}} + \frac{1}{4\times\overline{4+3}} + \&c.$$

$$5)\ \frac{1}{1\times\overline{1+5}} + \frac{1}{2\times\overline{2+5}} + \frac{1}{3\times\overline{3+5}} + \frac{1}{4\times\overline{4+5}} + \&c.$$

$$7)\ \frac{1}{1\times\overline{1+7}} + \frac{1}{2\times\overline{2+7}} + \frac{1}{3\times\overline{3+7}} + \frac{1}{4\times\overline{4+7}} + \&c.$$

$$\text{Vel } \frac{1}{2}\times\ \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \&c.$$

$$\frac{1}{2}\times \frac{1}{3-1} + \frac{1}{6-1} + \frac{1}{10-1} + \frac{1}{15-1} + \&c.$$

$$\frac{1}{2}\times \frac{1}{6-3} + \frac{1}{10-3} + \frac{1}{15-3} + \frac{1}{21-3} + \&c.$$

$$\frac{1}{2}\times \frac{1}{10-6} + \frac{1}{15-6} + \frac{1}{21-6} + \frac{1}{28-6} + \&c.$$

$$\text{Vel } \frac{1}{2}\times \frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{6+0} + \frac{1}{10+0} + \&c.$$

$$\frac{1}{2}\times \frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4} + \&c.$$

$$\frac{1}{2}\times \frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8} + \&c.$$

$$\frac{1}{2}\times \frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12} + \&c.$$

6 Ante

6. Ante aliquot annos D. *Jac. Bernoulli* Geometra insignis invenit summam Seriei cujuslibet, cujus Numeratores constituunt Seriem æqualium, Denominatores verò constituunt, vel Seriem quadratorum dato aliquo quadrato Q minutorum, vel Seriem Triangulorum, dato aliquo Triangulo T minutorum. Hæc invenit ille observando quod hujusmodi Series oriantur ex ablatione Seriei Harmonicè proportionalium truncatæ ab eâdem Serie integrâ; nempe ita ut numerus terminorum deficientium in Serie truncata, sit, vel duplus lateris dati quadrati Q, vel duplus unitate auctus lateris dati Trianguli T. Idem etiam observavit frustrà quæri summam Seriei reciprocæ Quadratorum. Hoc idem etiam verum est de reciprocis Cuborum, vel aliarum quarumlibet dignitatum numerorum in progressione Arithmeticâ. Ratio est, quòd nulla intercedit differentia inter factores denominatorum, quod ad hujusmodi summationes semper requiri constat ex Methodo sumendi differentias in *Scholio Prop.* I. jam explicatâ. Nam si per formulam aliquam exhiberi posset summa quæsita, differentia istius formulæ exhiberet terminos Seriei propositæ: sed in tali differentiâ denominator semper afficitur per factores ab invicem diversos, quod quoniam in Seriebus prædictis non obtinet, summæ Serierum hujusmodi in terminis finitis haberi nequeunt. Ad eundem ferè modum, argumento petito à *Prop.* III. & IV. demonstrari potest summas Serierum exhiberi non posse in terminis numero finitis, quarum Numeratores constituunt Seriem æqualium. Denominatores vero constant ex certo numero terminorum in progressione Arithmeticâ, maximo factore cujusvis termini minore existente quàm factor minimus in termino proxime insequenti, cujusmodi est Series $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+$ &c.

7. Jam liceret regulas nonnullas tradere quas pro casibus quibusdam singularibus concinnavi; sed hæc

nos

nos longius abducerent. Sufficiat itaque quæ generaliora sunt explicasse, & simul monuisse, ad novæ hujusce Serierum infinitarum doctrinæ provectionem nihil magis facere, quam si excogitentur formulæ generaliores summarum, ex quarum differentiis, per regulas supra traditas computatis, deinde conficiantur Canones quantitatum summabilium; ita ferè ut jam factum est in Calculo Integrali, *h. e.* in Stylo *Newtoniano*, in Methodo Fluxionum.

8. Restituendo factores in Denominatore deficientes potuisset præsens Problema revocari ad *Propositionem* II. Sed & in terminis generalioribus proponi potest, nempe pro Numeratore sumptâ quâvis Formulâ, cujus differentia aliqua datur. Sub eâ tamen conditione ut dimensiones Denominatoris ad minimum binario superent Dimensiones Numeratoris; aliàs enim summa Seriei in terminis numero finitis haberi nequit. Sit hujus rei exemplum in Serie $\frac{1}{1.3.5.7} + \frac{4}{2.4.6.8}$ $\frac{9}{3.5.7.9} + \frac{16}{4.6.8.10} +$ *&c.* ubi Numeratores sunt numerorum naturalium quadrata. Applicando tum Numeratores tum Denominatores ad numeros naturales, Series revocatur ad formam simpliciorem $\frac{1}{3.5.7} + \frac{2}{4.6.8}$ $\frac{3}{5.7.9} + \frac{4}{6.8.10} +$ *&c.* Per p designatis numeris naturalibus 1, 2, 3, 4, *&c.* terminus Seriei designabitur per formulam $\frac{p}{p+2 \times p+4 \times p+6}$; vel per formulam $\frac{z-2}{z \times z+2 \times z+4}$, nempe pro $p+2$ scripto z. Quoniam progrediendo de termino in terminum augetur z per unitates, restituendi sunt factores in denominatore deficientes $z+1$, $z+3$, & hoc pacto revocatur terminus Seriei ad formulam $\frac{z-2 \times z+1 \times z+3}{z \times z+1 \times z+2 \times z+3 \times z+4}$. Per methodum in hâc Propositione jam explicatam re-

 vocatur

vocatur numerator ad formam $-6-6z-z\times\overline{z+1}$ $+z\times\overline{z+1}\times\overline{z+2}$. Unde habita ratione denominatoris Terminus revocatur ad formam $\frac{-6}{z\times\overline{z+1}\times\&c\times\overline{z+4}}$ $+\frac{-6}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-1}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}}$ $+\frac{1}{\overline{z+3}\times\overline{z+4}}$. Adeoque sumendo Integrale fit $\frac{6}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}+\frac{6}{3\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}$ $+\frac{1}{2\times\overline{z+2}\times\overline{z+3}}+\frac{-1}{z+3}$; quo, sub signo contrario, exhibetur summa Seriei in infinitum continuatæ, incipientis à termino $\frac{z-2}{z\times\overline{z+2}\times\overline{z+4}}$. Summa itaque Seriei integræ incipientis à termino $\frac{1}{3\cdot5\cdot7}$ est $\frac{31}{240}$.

Si per *Prop.* II. procedere esset animus, ex formulâ $\overline{z-2}\times\overline{z+1}\times\overline{z+3}$ collectis numeratoribus primis 24, 70, 144, 252, sumendo eorum differentias haberentur $46=b$, $28=c$, $6=d$, $e=0=\&c.$ existente $M=24$; unde per *Lem.* 2. prodiret formula $-6-6z$ $-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$, quâ designatur Terminus, eadem ac supra; atque pergendo per *Prop.* II. haberetur summa.

Prop. VI. *Prob.*

Invenire summam quotlibet terminorum Seriei Fractionum, quarum Numeratores & Denominatores constituunt lineas duas quasvis transversas in Triangulo Arithmetico *Paschalii*; nempe cujus generatores sunt unitates.

Solutio. Per n designetur Ordo Seriei Numeratorum in Triangulo Arithmetico, & sit p differentia inter ordinem Numeratorum & Denominatorum, & per q designetur numerus terminorum quorum summa requiritur.

quiritur. Tum ſi Denominatores ſint plurium dimenſionum quàm ſunt Numeratores, Summa exhibebitur per formulam primam ſequentem; ſi dimenſiones Numeratorum plures ſint quàm dimenſiones Denominatorum, Summa exhibebitur per formulam ſecundam.

Formula I.

$$\frac{n+p-1}{p-1} - \frac{n.\overline{n+1}.\overline{n+2}.\&c.\overline{n+p-1}}{p-1\times\overline{n+q}.\overline{n+q+1}.\&c.\overline{n+q+p-2}}$$

Formula II.

$$-\frac{n-p-1}{p+1} + \frac{\overline{q+n-1}.\overline{q+n-2}.\&c.\overline{q+n-p-1}}{\overline{p+1}\times\overline{n-1}.\overline{n-2}.\&c.\overline{n-p}}$$

Ex. 1. Inveniendum ſit aggregatum ſex primorum terminorum Seriei $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\frac{20}{84}+\frac{35}{210}+\frac{56}{462}+$ &c. ubi Numeratores conſtituunt lineam quartam, Denominatores conſtituunt lineam ſeptimam in Triangulo Arithmetico. Sunt itaque $n=4$, $p=3$, $q=6$; & quoniam dimenſiones Denominatorum ſuperant dimenſiones Numeratorum, dabitur ſumma per Formulam primam; nempe $\frac{4+3-1}{3-1}-\frac{4.5.6}{\overline{3-1}\times\overline{4+6}\times\overline{4+7}}$ ſive $3-\frac{6}{11}=2\frac{5}{11}$.

Ex. 2. Quæratur ſumma ſex primorum terminorum Seriei $\frac{1}{1}+\frac{7}{4}+\frac{28}{10}+\frac{84}{20}+\frac{210}{35}+\frac{462}{56}+$ &c. cujus termini ſunt terminorum Seriei prioris reciproci. Sunt itaque $n=7$, $p=3$, $q=6$, adeoque per formulam ſecundam ſumma fit $-\frac{3}{4}+\frac{12.11.10.9}{4\times6.5.4}=24$.

Scholium 1. Formulas in hac propoſitione exhibitas ante biennium communicavi cum Viris celeberrimis *Moivreo* & *Bernoulliis.* Facile autem derivari poſſunt ex præceptis in *Prop.* I. traditis. Sit exemplum in Serie priori $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+$ &c. Per p deſignato loco

Ter-

Termini in Serie hâc, exhibetur Terminus per formulam $\frac{4 \cdot 5 \cdot 6}{p+3 \cdot p+4 \cdot p+5}$; Unde regrediendo ad Integrale, summa Seriei incipientis à termino illo exhibetur per formulam $\frac{4 \cdot 5 \cdot 6}{2 \times p+3 \times p+4}$; adeoque pro p sumpto 1, Series integra fit $\frac{4 \cdot 5 \cdot 6}{2 \cdot 4 \cdot 5} = 3$, atque summa primorum sex terminorum fit $3 - \frac{4 \cdot 5 \cdot 6}{2 \cdot 10 \cdot 11}$, omninò ut per formulam jam exhibetur.

2. In formulâ primâ summa Seriei in infinitum continuatæ est $\frac{n+p-1}{p-1}$, evanescente jam parte alterâ formulæ. Sed in casu formulæ secundæ summa hæc est infinitum quid, cujus species, respectu numeri infiniti q, exhibetur per formulæ partem alteram, quæ in hoc casu fit $\frac{q^{p+1}}{p+1 \times n-1 \cdot n-2 \cdot \&c. n-p}$.

3. De hujusmodi Seriebus in epistolâ datâ mense *Maio* 1716, sic ad me scripsit Vir. Ill. *D. Leibnitius*, quem magno Scientiarum damno nobis nuper ereptum lugemus. " Il me semble qu'autrefois j'ay aussi sommé
" quelques Series ou suittes comme $\frac{1}{1} + \frac{2}{4} + \frac{3}{10} + \frac{4}{20}$
" $+ \frac{5}{35} + \frac{6}{56} +$ &c. Le terme de cette suitte exprimé
" Analytiquement est $\frac{x}{x \cdot x+1 \cdot x+2 \times \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3}}$
" $= \frac{1 \cdot 2 \cdot 3}{x+1 \cdot x+2} = \frac{6}{xx+3x+2}$. On demande donc
" la somme d'une suitte donneé, dont un terme soit
" $\frac{ll}{xx+3lx+2ll}$ ou x signifie les nombres naturales
" 1, 2, 3, 4, &c. & l signifie l'Unité, ou la difference
" des x. Supposons que le terme de la suitte som-
" matrice

" matrice demandée soit $\frac{fx}{mx+nl} = \frac{\odot}{\mathfrak{D}}$. Or Diff. $\frac{\odot}{\mathfrak{D}} =$
" $-\frac{\odot}{\mathfrak{D}} + \frac{\odot + d\odot}{\mathfrak{D} + d\mathfrak{D}} = \frac{\mathfrak{D}\,d\odot - \odot\,d\mathfrak{D}}{\mathfrak{D}\mathfrak{D} + \mathfrak{D}\,d\mathfrak{D}}$: sed $d\odot = fdx$,
" & $d\mathfrak{D} = mdx = ml$; donc la Difference de $\frac{\odot}{\mathfrak{D}}$ est $=$
" $\frac{nfll}{\begin{array}{l} mmxx + 2mnlx + nnll \\ \quad + mmlx + mnll \end{array}}$. Maintenant il faut faire
" $\frac{nfll}{\begin{array}{l} mmxx + 2mnlx + nnll \\ \quad + mmlx + mnll \end{array}} = \frac{mfll}{mmxx + 3mmlx + 2mmll}$
" c'est a dire, il faut identifier ces deux formules, ou la
" donnée est Multipliée per $\frac{nf}{mm}$: donc égalant les
" termes respectifs, puisque les xx conviennent, on
" aura par les x, $2n + m = 3m$, c'est adire il y aura
" $m = n$, & par les absolus on aura $nn + mn = 2mm$,
" ce qui donne encore $m = n$; donc l'identification
" reussit, & nous pouvons faire $n = m = l = 1$, &
" $f = 1$ (car f demeure arbitraire) & le terme de la
" suitte sommatrice sera $\frac{x}{x+1}$, car diff. $\frac{x}{x+1}$ donne
" $-\frac{x}{x+1} + \frac{x+1}{x+2} = \frac{1}{xx+3x+2}$, & par consequente
" $\frac{6x}{x+1}$ donne la somme des $\frac{x}{x.\overline{x+1}.\overline{x+2} \times \frac{1}{1}.\frac{1}{2}.\frac{1}{3}}$

" 3, 4, $\frac{9}{2}$, $\frac{24}{5}$, 5, $\frac{36}{7}$, *&c. Series summatrix, cujus terminus* $\frac{6x}{x+1}$.

" $\frac{1}{1} + \frac{2}{4} + \frac{3}{10} + \frac{4}{20} + \frac{5}{35} +$ *&c. Series summanda, cujus terminus* $\frac{x}{x.\overline{x+1}.\overline{x+2} \times \frac{1}{1} \times \frac{1}{2} . \frac{1}{3}}$ Et pour
" s'en servir aux sommations, les 5 termes, *par Ex.* de

" la suitte donnée seront $\frac{36}{7} - 3 = \frac{15}{7}$. Et generalle-
" ment la somme des termes jusqu'a quelque terme
" $\frac{x}{x \,.\, \overline{x+1} \,.\, \overline{x+2} \times \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3}}$ exclusivement, sera $\frac{6x}{x+1}$
" -3 : Et pour la somme de la suitte entiere a l'infi-
" nie, x devient infini, & $\frac{6x}{x+1} = 6$: donc la somme
" de toute la suitte est $6 - 3 = 3$, comme vous
" l'avez trouvé.
" Cette methode est le calcul des differences ap-
" pliqué aux Nombres; & il faut vous avouer qu'a-
" vant que de l'appliquer aux Figures, & même avant
" que d'avoir été Geometre, Je le prattiquai en quel-
" que façon dans les nombres; ayant trouvé encore
" jeune garçon que les suittes dont les Numerateurs
" fussent des Unites, & dont les Denominateurs fussent
" les Nombres figurés, comme Triangulaires Pyrami-
" daux &c. etoient les differences 1eres, 2es, 3emes, &c.
" multipliées par les constantes de la suitte $\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$
" $+ \frac{1}{4} +$ &c. & par consequent sommables. Mais
" quand je devins un peu Geometre & Analyste, Je
" vis qu'il y avoit moyen de venir a bout de telles
" sommations par une Methode generalle, autant qu'il
" etoit possible; & que le calcul des differences estoit
" encore plus commode dans la Geometrie que dans
" les Nombres, puis qu'il y a plus d'evanouissements,
" & que les differences repondent aux Tangentes, les
" sommes aux Quadratures. Cette methode generalle
" de chercher la suitte sommatrice de la suitte donnée,
" quand elle est possible, reussit toujours, quand le terme
" de la suitte donnée exprimé Analytiquement n'a
" point la quantité variable enveloppé dans une racine,
" ny entrant dans l'exposant; & alors, on peut tou-
" jours

" jours determiner la ſuitte ſommatrice, ou prouver
" qu'il eſt impoſſible d'en trouver. Et la choſe reuſſit
" méme bien ſouvent, lors méme que la variable en-
" tre dans l'Expoſant. Mais comme il y a quelque-
" fois des Quadratures particulieres de quelques por-
" tions d'une Figure, dont ou ne ſçauroit donner la
" Quadrature generalle ou la Figure quadratrice; de
" méme on peut trouver quelqueſois la ſomme de
" toute la ſuitte, ou d'un certaine partie, quoy qu'on
" ne puiſſe pas trouver la ſomme de chaque partie; &
" alors il faut avoir recours a des Methodes particulieres,
" dont on n'eſt pas toujours le maiſtre, noſtre Analyſe
" n'eſtant pas encore portée a ſa perfection.

Prop. VII. *Prob.*

Invenire ſummam Seriei cujus Numeratores conſtituunt lineam quamlibet erectam in Triangulo Arithmetico *Paſchalii*, Denominatores vero conſtituunt lineam quamlibet tranſverſam

Solutio. Deſignetur ordo lineæ erectæ per p, ordo lineæ tranſverſæ per q, & ſit m aggregatum tot terminorum primorum in lineâ erectâ ordinis $p+q-1$ quot ſunt unitates in $q-1$, atque ſumma quæſita erit $\overline{2^{p+q-2}-m} \times \frac{1 \cdot 2 \cdot 3 \,.\, \&c. \,\overline{q-1}}{p \cdot \overline{p+1} \,.\, \&c. \,\overline{p+q-2}}$.

Ex. 1. Proponatur Series $\frac{1}{1} + \frac{5}{4} + \frac{10}{10} + \frac{10}{20} + \frac{5}{35} + \frac{1}{56}$ Ubi Numeratores conſtituunt lineam ſextam erectam, Denominatores occupant lineam quartam tranſverſam. In hoc itaque caſu ſunt $p=6$, $q=4$, $p+q-1=9$, $q-1=3$, adeoque $m=1+8+28=37$ *i. e.* tribus terminis primis lineæ nonæ erectæ. Unde fit ſumma quæſita $\overline{2^8-37} \times \frac{1 \cdot 2 \cdot 3}{6 \cdot 7 \cdot 8} = \frac{219}{56}$.

Ex. 2. Conſtituant Numeratores lineam centeſimam erectam, & ſint Denominatores Numeri Trigonales, qui occupant lineam tertiam tranſverſam. Tum erunt

p

$p = 100, q = 3, m = 102$ atque adeo summa quæsita fit $\overline{2^{101} - 102} \times \frac{1 . 2}{100 . 101}$.

Cor. Si $q = 2$, formula fit $\frac{2^p - 1}{p}$, quâ exhibetur aggregatum primi termini, unâ cum semisse secundi, triente tertii, quadrante quarti, & sic porrò, lineæ cujusvis erectæ ordinis p Trianguli Arithmetici *Paschalii*. Sic *v. gr.* est $\frac{1}{1} + \frac{5}{2} + \frac{10}{3} + \frac{10}{4} + \frac{5}{5} + \frac{1}{6} = \frac{2^6 - 1}{6} = 10\frac{1}{2}$.

Prop. VIII. *Prob.*

Invenire summam ejusdem Seriei, quando terminorum signa sunt alternatim $+$ & $-$.

Solutio. Summa quæsita exhibetur per formulam simplicissimam $\frac{q - 1}{p + q - 2}$.

Ex. Invenienda sit summa Seriei $\frac{1}{1} - \frac{6}{9} + \frac{15}{45} - \frac{20}{165} + \frac{15}{495} - \frac{6}{1287} + \frac{1}{3003}$, ubi Numeratores constituunt lineam septimam erectam, Denominatores constituunt nonam transversam. In formulâ itaque pro p & q scriptis 7 & 9, fit summa $\frac{8}{14}$.

Manente eâdem Serie Numeratorum (nempe lineâ septimâ erectâ), si pro Serie Denominatorum sumantur successivè lineæ transversæ 2^{da}, 3^{tia}, 4^{ta}, *&c.* Summæ erunt $\frac{1}{7}$, $\frac{2}{8}$, $\frac{3}{9}$, $\frac{4}{10}$, $\frac{5}{11}$, *&c.* quæ sic possunt scribi, $\frac{1}{7}$, $\frac{7}{28}$, $\frac{28}{84}$, $\frac{84}{210}$, $\frac{210}{462}$, *&c.* ubi tam Numeratores, quàm Denominatores excerpuntur ex lineâ transversâ ordinis septimi. Idem eveniret si loco septimæ, Numeratores constituissent aliam quamlibet lineam erectam ordinis p; Summæ quippe orirentur ex applicatione terminorum

lineæ

lineæ transversæ ejusdem ordinis p ad terminos proximè sequentes in eadem lineâ.

Propositiones hæ duæ novissimæ potius elegantes sunt quàm utiles; quare Formularum nostrarum demonstrationem Lectoris solertia investigandam relinquimus, ad Propositionem ultimam jam properantes, quæ tertiam continet Serierum speciem, ob usum multiplicem satis insignem.

Lemma 5.

Sit Series quævis $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* cujus terminorum Denominatores constituunt progressionem quamlibet Geometricam h, h^2, h^3, h^4, *&c.* Sint etiam Numeratorum primus $A\ (=M)$, prima differentiarum primarum B, prima secundarum C, prima tertiarum D, quartarum E, & sic porrò; & sint $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, *&c.* respectivè, aggregata, Unius, Duorum, Trium, Quatuor, vel plurium terminorum Seriei $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}$, *&c.* atque sint Numeratorum primus $a\ (=\alpha)$ prima differentiarnm primarum b, prima secundarum c, prima tertiarum d, & sic porrò: & sit $h-1=q$. Tum ipsorum a, b, c, d, *&c.* valores erunt.

$$\begin{aligned}
a &= A = \alpha = M \\
b &= hA + B \\
c &= q\,hA + hB + C \\
d &= q^2 hA + q\,hB + hC + D
\end{aligned}$$

& sic porrò.

Demonstratio.

Satis constat esse $a=\alpha=A=M$.

Termini $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* Numeratoribus M, N, O, P,

&c. expressis per $A, B, C, D,$ &c. transformantur in terminos $\frac{A}{h}, \frac{A+B}{h^2}, \frac{A+2B+C}{h^3}, \frac{A+3B+3C+D}{h^4}$ &c. Unde colligendo summas terminorum, inveniuntur Numeratores $\alpha, \beta, \gamma, \delta,$ &c. nempe

$$\alpha = A$$
$$\beta = \overline{h+1}\,A + B$$
$$\gamma = \overline{h^2+h+1}\,A + \overline{h+2}\,B + C$$
$$\delta = \overline{h^3+h^2+h+1}\,A + \overline{h^2+2h+3}\,B + \overline{h+3}\,C + D$$

&c.

Unde sumendo differentias fiunt

$$b = hA + B$$
$$c = qhA + hB + C$$
$$d = qqhA + qhB + hC + D$$

& sic porrò, ut in Propositione exhibentur.

Cor. 1. Si Numeratorum $M, N, O, P,$ &c. differentia vel prima, vel secunda, vel alia quædam detur, terminis omnibus post primos aliquot in Serie $A, B, C, D,$ &c. evanescentibus, Differentiæ $b, c, d,$ &c. tandem incurrent in Progressionem Geometricam in ratione 1 ad q. Exempli gratiâ, si detur Numeratorum M, N, O, P &c differentia prima B, erunt $c, d,$ &c. in ratione continuâ Geometricâ 1 ad q; ut constat per ipsorum valores $qhA + hB$, $qqhA + qhB$, &c. existentibus $C = 0 = D =$ &c.

Cor. 2. Ordo autem primæ differentiarum $B, C, D,$ &c quæ hoc modo evanescunt, idem est ac ordo differentiæ vel b, vel c, &c. unde incipit Progressio illa Geometrica. Sic si $B = 0 = C =$ &c. erunt $b, c, d,$ &c. in Progressione Geometricâ; si $C = 0 = D =$ &c. erunt $c, d,$ &c. in Progressione Geometricâ. Et sic porrò.

Lemma 6.

Iisdem positis sit r terminus unde incipit Progressio Geometrica in Serie differentiarum $b, c, d,$ &c. & per

$p+1$

$p+1$ designetur ordo Termini in Serie $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, &c. Tum Terminus ille designabitur per fractionem cujus Denominatore existente h^{p+1} Numerator est

$$\frac{a+bp+cp\times\frac{p-1}{2}+dp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.+\frac{r}{q^n}}{\times\overline{h^p-1-qp-q^2p\times\frac{p-1}{2}-q^3p\times\frac{p-1}{2}\times\frac{p-2}{3}-\&c.}}$$

nempe per n designato ordine differentiæ evanescentis in Serie B, C, D, &c. ut & Numero terminorum $a+bp$, &c. item terminorum $-1-qp$, &c.

Demonstratio. Per *Lemma* 1. Termini istius Numerator exhibetur per formulam

$$a+bp+cp.\frac{p-1}{2}+dp\times\frac{p-1}{2}.\frac{p-2}{3}+\&c.$$ ($p+1$ subeunte vices x in Lemmate isto)

Ergò si sit, *ex. gr.* $n=2$, per *Lemm.* 5. *Cor.* 2. erunt c, d, &c. in ratione continuâ 1 ad q. Numerator itaque in hoc casu est

$$a+bp+cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+cq^2p\times\frac{p-1}{2}\times\frac{p-2}{3}\times\frac{p-3}{4}+\&c.$$ Sed si termini $cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.$ ducantur in $\frac{q^2}{c}$, & producti addantur termini $1+qp$, prodibit Series quâ exprimitur binomii $1+q$ dignitas $\overline{1+q}^p=h^p$. Ergo productum illud æquale est h^p-1-qp; adeoque termini $cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.=\frac{c}{q^2}\times\overline{h^p-1-qp}$. Quo pacto Numerator fit $a+bp+\frac{c}{q^2}\times\overline{h^p-1-qp}$, existentibus duobus terminis $a+bp$, ut & duobus $-1-qp$, juxta sensum Propositionis, quoniam $n=2$. Atque eadem est demonstratio in aliis casibus. De Denominatore verò per se satis constat.

Prop.

Prop. IX. *Prob.*

Invenire summam quotlibet terminorum Seriei cujusvis $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* cujus terminorum Denominatores constituunt progressionem quamlibet Geometricam h, h^2, h^3, h^4, *&c.* Numeratores autem sunt quantitates differentiâ aliquâ constanti gaudentes.

Solutio. Sunto Numeratorum M, N, O, P, *&c.* primus A, prima differentiarum primarum B, prima secundarum C, prima tertiarum D, & sic porrò; & sit ipsorum A, B, C, D, *&c.* numerus n, atque $h - 1 = q$, Tum fiat $a = A\,(= M)$ $b = hA + B$, $c = qhA + hB + C$, $d = q^2 hA + qhB + hC + D$, *&c.* ut sint tot termini a, b, c, d, *&c*, quot sunt unitates in $n + 1$. Terminorum istorum ultimus dicatur r, atque per $p + 1$ designetur numerus terminorum $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* quorum summa requiritur; Dico summam illam exhiberi per fractionem, cujus Denominatore existente h^{p+1}, Numerator est

$$\underline{a + bp + cp \times \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3} + \&c. + \frac{r}{q^n}}$$

$$\underline{\times h^p - 1 - qp - q^2 p \times \frac{p-1}{2} - q^3 p \times \frac{p-1}{2} \times \frac{p-2}{3} - \&c.}$$

$$- q^{n-1} p \times \frac{p-1}{2} \times \&c.$$

Demonstratio. Nam (per *Lem.* 6.) per hanc formulam repræsentatur terminus ordine $p + 1$ Seriei $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, *&c.* qui terminus (per constructionem *Lemmatis* 5.) æqualis est aggregato terminorum numero $p + 1$ Seriei propositæ $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$. *Q. E. D.*

Ex. 1.

Ex. 1. Invenienda ſit ſumma novem terminorum Seriei $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, $\frac{4}{16}$, &c. Sunt in hoc caſu $h=2$, q $(=h-1)=1$, $p+1=9$, $p=8$, $A=1$, $B=1$, $C=0,=D=$ &c. adeoque $n=2$, (quoniam ſunt duo A,B,) Hinc fit $a\,(=A)=1$, $b\,(=hA+B=2\times1+1)$ $=3$, $c\,(=qhA+hB+C=2\times1+2\times1+0)$ $=4=r$, Adeoque per formulam fit ſumma quæſita

$$\frac{1+3\times8+\frac{4}{1^2}\times\overline{2^8-1-1\times8}}{2^9}=\frac{1013}{512}.$$

Ex. 2. Quæratur ſumma ſex terminorum Seriei $1\times3+3\times3^2+6\times3^3+10\times3^4+15\times3^5+21\times3^6+$ &c. In hoc caſu ſunt $h=\frac{1}{3}$, $q=\frac{-2}{3}$, $p+1=6$, $p=5$, $A=1$, $B=2$, $C=1$, $D=0=E=$ &c. adeoque $n=3$, atque $a=1$, $b=\frac{1}{3}+2=\frac{7}{3}$, $c=\frac{-2}{9}+\frac{2}{3}+1=\frac{13}{9}$, $d=\frac{4}{27}-\frac{4}{9}+\frac{1}{3}=\frac{1}{27}=r$. Unde ſumma quæſita fit $=19956$. ſive

$$\frac{1+\frac{7}{3}\times5+\frac{13}{9}\times5\times\frac{4}{2}+\frac{-1}{8}\times\overline{\frac{1}{3^5}-1+\frac{2}{3}\times5-\frac{4}{9}\times5\times\frac{4}{2}}}{\overline{\frac{1}{3}}\big|^6}.$$

Cor. 1. Ejuſdem Seriei, à termino primo $\frac{M}{h}$ in infinitum continuatæ, ſumma exhibetur per formulam ſimpliciſſimam $\frac{A}{h-1}+\frac{B}{\overline{h-1}|^2}+\frac{C}{\overline{h-1}|^3}+\frac{D}{\overline{h-1}|^4}$ &c.

Cor. 2. Si $h=2$, Seriei totius in infinitum continuatæ ſumma habetur ſolâ additione terminorum A, B, C, D, &c. Et hæc ſumma eadem eſt ac ſumma lineæ erectæ reſpondentis termino primo A, in Triangulo Arithmetico, cujus lineam tranſverſam occupant Nume-

 ratores

ratores M, N, O, P, *&c.* Quod facile constat ex contemplatione Trianguli. Si itaque fuerint M, N, O, *&c.* Numeri figurati cujusvis ordinis n, summa Seriei $\frac{M}{2} + \frac{N}{4} + \frac{O}{8} + \frac{P}{16} +$ *&c.* æqualis erit Numeri binarii dignitati $\overline{2}|^{n-1}$. Sic Series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} +$ *&c.* $= 2^{1-1} = 1$, ut vulgò notum; Series $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} +$ *&c.* $= 2^{n-1} = 2$; Series $\frac{1}{2} + \frac{3}{4} + \frac{6}{8} + \frac{10}{16} +$ *&c.* $= 2^{3-1} = 2^2 = 4$, & sic porrò.

Scholium. Celeb. D. *Jac. Bernoulli*, in Tractatu suo de Seriebus infinitis, solvit illud Problema. " Invenire " summam Seriei infinitæ Fractionum quarum Denomi" natores crescunt in Progressione quacunque Geome" tricâ, Numeratores verò progrediuntur vel juxta Nu" meros naturales, 1, 2, 3, 4, *&c.* vel Trigonales 1, " 3, 6, 10, *&c.* vel Pyramidales 1, 4, 10, 20, *&c.* " aut juxta Quadratos 1, 4, 9, 16, *&c.* aut Cubos 1, " 8, 27, 64, *&c.* eorumve multiplices." Ipsius solutionem consulat Lector. Aliam verò, & quidem multo generaliorem invenit D. *Nic. Bernoulli* illius Nepos, eamque (postquam ei hæc miseram, sed sine demonstratione) mecum communicare dignatus est, in epistolâ datâ 18° *Septembris* 1715, miris quidem inventis refertissimâ, qualibus me crebro dignatur vir Doctissimus. De hoc vero Problemate sic scribit. " Pour la somme " d'un nombre determiné n de termes de la suitte de " vostre Theoreme 7. [*Corollarium* primum est hujus Propositionis] j'ay trouvé cette formule $\frac{1}{m^n} \times$

$$\times \frac{m-1}{m-1} a + \frac{A-n}{m-1} b + \frac{B-n.\frac{n-1}{2}}{m-1} c + \frac{C-n.\frac{n-1}{2}.\frac{n-2}{3}}{m-1} d$$

" + *&c.* ou les Lettres A, B, C, *&c.* marquent

" les

" les Coefficients des termes immediatement precedents. Et en mettant dans cette formule $p+1$ " pour n, h^m pour m, & en multipliant tout encore " par e^{m-1}, on a la solution de vostre *Prob.* " IX^{me}". Et me monuit Vir peritissimus hanc suam formulam generalem in nostram particularem (*Cor.* 1. hujus propositionis) migrare quando $n=\infty$; quippe tum evanescunt 1, n, $n \cdot \frac{n-1}{2}$, $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$, *&c* respectu ipsorum m^n, A, B, C, *&c.* adeo ut Series in eo casu sit $\frac{1}{m-1}a + \frac{A}{m-1}b + \frac{B}{m-1}c +$ *&c.* quæ omnino coincidit cum nostrâ $\frac{a}{m-1} + \frac{b}{\overline{m-1}|^2} + \frac{c}{\overline{m-1}|^3} +$ *&c.*

Adhuc aliam hujus Problematis solutionem, & quidem ab hisce admodum diversam, invenit D. *Taylor* ope Methodi suæ Incrementorum. Viri doctissimi rogatu, ad eum miseram formulam meam secundam pro solutione Problematis II^{di}, item formulas alias spectantes ad Propositiones tertiam, quartam & quintam, sed sine demonstrationibus: quippe non dubitabam quin Vir acutissimus, atque ipse Methodi istius Incrementorum Inventor, hisce, vel saltem paribus inveniendis par esset. Rescripsit se harum solutiones invenisse, & simul alia quædam communicavit ad hujus methodi profectum multum facientia, quæ jam nostro hortatu inductus hisce subjungere dignatur.

Appendix

APPENDIX

Quâ methodo diversâ eadem materia tractatur:
Auctore Brook Taylor, *LL. D. R. S. Secr.*

HOrtatu Viri Clariss. cui nos innumeris officiis devinctissimos esse libenter fatemur, sequentes jam Propositiones exhibemus, quas quidem in aliam occasionem reservandas esse decrevissemus, ni æquum visum fuisset parendum esse imperio amici qui, dum Propositiones quasdam præcedentes suas olim nobis investigandas proposuit, earum inveniendarum occasionem dedit.

Definitiones.

1. Quantitatis cujusvis variabilis valorem præsentem designo literâ simpliciter scriptâ, ut x; valores præcedentes distinguo lineolis eidem literæ ex parte superiori positis, sequentes lineolis ex parte inferiori scriptis. Ut vi hujus Definitionis sint $\overset{\prime\prime}{x}$, $\overset{\prime}{x}$, x, $\underset{\prime}{x}$, $\underset{\prime\prime}{x}$, ejusdem variabilis valores quinque continui, existente x valore præsenti, $\overset{\prime}{x}$ proximè præterito, $\overset{\prime\prime}{x}$ secundò præterito; $\underset{\prime}{x}$ proximè, atque $\underset{\prime\prime}{x}$ secundò futuro. Et sic de aliis. Ad eundem modum sunt interpretandæ lineolæ quæ incrementis apponuntur. Sic sunt $\overset{\prime\prime}{\underset{..}{x}}$, $\overset{\prime}{\underset{..}{x}}$, $\underset{..}{x}$, $\underset{\prime\,..}{x}$, $\underset{\prime\prime\,..}{x}$, ipsius $\underset{..}{x}$ valores quinque continui; ut sit $\overset{\prime\prime}{\underset{..}{x}}$ incrementum

secun-

secundum ipsius $\overset{\prime\prime}{x}$, sit $\underset{..}{\overset{\prime}{x}}$ incrementum secundum ipsius $\overset{\prime}{x}$. Et sic de aliis.

Cor. Vi hujus Definitionis, $x + \underset{.}{x} = x$, $x + \underset{.}{x} = \underset{\prime}{x}$, $\underset{.}{x} + \underset{..}{x} = \underset{\prime .}{x}$. Et sic de aliis hujusmodi.

Quando usu venit ut variabilis quantitas, puta x, spectanda sit tanquam Incrementum, ejus Integrale designo literâ inter uncos [ ] inclusâ. Istius etiam Integralis $[x]$ Integrale (vel ipsius x Integrale secundum,) designo numero binario uncorum priori superimposito, ut $\overset{2}{[}x]$. Istius etiam Integralis Integrale (vel ipsius x Integrale tertium,) ad eundem modum designo numero ternario, ut $\overset{3}{[}x]$. Et sic deinceps. Unde vi hujus Definitionis constituunt $\overset{3}{[}x]$, $\overset{2}{[}x]$, $[x]$, x Seriem terminorum, quorum quilibet est ipsum immediatè præcedentis incrementum primum, ut sit $\overset{2}{[}x] = \overset{3}{[}\underset{.}{x}]$, $[x] = \overset{2}{[}\underset{.}{x}]$, $x = [\underset{.}{x}]$.

Lemma.

Facti xv ex Multiplicatione duorum variabilium & v, incrementum est $\underset{.}{x}v + \underset{\prime}{x}\underset{.}{v}$.

Nam auctis variabilibus per propria incrementa, fit novum productum $\overline{x + \underset{.}{x}} \times \overline{v + \underset{.}{v}}$, sive $xv + \underset{.}{x}v + \overline{x + \underset{.}{x}} \times \underset{.}{v}$, hoc est $xv + \underset{.}{x}v + \underset{\prime}{x}\underset{.}{v}$ (pro $x + \underset{.}{x}$ scripto $\underset{\prime}{x}$ *per Def.* 1.)

Unde dempto priori producto xv, restat Incrementum $\underset{.}{x}v + \underset{\prime}{x}\underset{.}{v}$.

Prop. I. *Theor.*

Ejuſdem Facti $x\,v$ Incrementum, vel primum, vel ſecundum, vel tertium, vel aliud quodvis, cujus ordo deſignatur per ſymbolum n, exhibetur per formulam hanc generalem

$$\underset{n}{x}v + n\underset{\prime\,n-1}{x}\underset{\cdot}{v} + n\times\frac{n-1}{2}\underset{\prime\prime\,n-2}{x}\underset{\cdot\cdot}{v} + n\times\frac{n-1}{2}\times\frac{n-2}{3}\underset{\prime\prime\prime\,n-3}{x}\underset{\because}{v} + \&c.$$

In hâc formulâ hæc ſunt obſervanda, 1^mo^ Terminorum numeri coefficientes 1, n, $n\times\frac{n-1}{2}$, $n\times\frac{n-1}{2}\times\frac{n-2}{3}$ &c. iidem ſunt ac in binomii dignitate n. 2^do^ Numeri n, $n-1$, $n-2$, $n-3$, &c. ipſis x infraſcripti deſignant numeros punctorum quibus definiuntur Incrementa. 3^tio^ Lineolæ $_{\prime}$, $_{\prime\prime}$, $_{\prime\prime\prime}$, &c. ipſis x infraſcriptæ, interpretandæ ſunt per Def. 1. 4^to^. In quovis Termino numerus punctorum ipſis x & v ſimul infraſcriptorum, eſt n. Sit *v.g.* $n=4$: tum per formulam, ipſius $x\,v$ incrementum quartum prodit $\underset{::}{x}v + 4\underset{\prime\,\because}{x}\underset{\cdot}{v} + 6\underset{\prime\prime\,\cdot\cdot}{x}\underset{\cdot\cdot}{v} + 4\underset{\prime\prime\prime\,\cdot}{x}\underset{\because}{v} + \underset{\prime\prime\prime\prime}{x}\underset{::}{v}$.

Theorema hoc generale demonſtrari poteſt per Inductionem, incrementis continuò ſumptis juxta formam in Lemmate præcedenti traditam. Sed & collectâ formâ Seriei ex hujuſmodi calculo, Theorema etiam demonſtrari poteſt per Methodum Incrementorum, ad eum modum cujus ſpecimen mox dabimus in demonſtratione Propoſitionis tertiæ.

Prop. II. *Theor.*

Ipſius $x\,v$ Integrale primum $[x\,v]$ exhibetur per Seriem $[x]\,v - \underset{\prime}{\overset{2}{[x]}}\,\underset{\cdot}{v} + \underset{\prime\prime}{\overset{3}{[x]}}\,\underset{\cdot\cdot}{v} - \underset{\prime\prime\prime}{\overset{4}{[x]}}\,\underset{\because}{v}$ &c.

Series autem ita terminatur, ut ſit $[x\,v] = [x]\,v$ —

$$-\left[[\underset{\prime}{x}]\,\underset{\cdot}{v}\right] = [x]\,v - \overset{2}{[\underset{\prime}{x}]}\,\underset{\cdot}{v} + \left[\overset{2}{[\underset{\prime\prime}{x}]}\,\underset{\cdot\cdot}{v}\right] = \&c.$$

Nam sumendo incrementa restituitur propositum xv.

Cor. 1. Datis duobus ex istis $[x]$, $[xv]$, $\left[[\underset{\prime}{x}]\,\underset{\cdot}{v}\right]$, datur tertium. Item datis tribus ex istis $[x]$, $\overset{2}{[\underset{\prime}{x}]}$, $[xv]$, $\left[\overset{2}{[\underset{\prime\prime}{x}]}\,\underset{\cdot}{v}\right]$, datur quartum, Et sic porrò.

Cor. 2. Si $\underset{\cdot}{v}=0$, datur $[xv]$ ex dato $[x]$. Si $\underset{\cdot\cdot}{v}=0$ datur $[xv]$ ex datis duobus $[x]$, & $\overset{2}{[x]}$, Si $\underset{\cdot\cdot\cdot}{v}=0$, datur $[xv]$, ex datis tribus $[x]$, $\overset{2}{[x]}$, $\overset{3}{[x]}$. Et sic porrò.

Ex. 1. Sit exemplum hujus formulæ in inventione Integralis ipsius $\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$, dato nempe $\underset{\cdot}{z}$, atque existente $\underset{\cdot\cdot\cdot}{v}=0$, qui casus est specialis Propositionis secundæ Tractatus præcedentis D^{ni} *Monmort*. Facto itaque $x = \frac{1}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$, sunt $[x] = \frac{-1}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}}$, $\overset{2}{[\underset{\prime}{x}]} = \frac{1}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}}$, atque $\overset{3}{[\underset{\prime\prime}{x}]} = \frac{-1}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime\prime}{z}}$. Unde per formulam fit $[xv]$, hoc est $\left[\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}\right] = -\frac{v}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}}$ $-\frac{\underset{\cdot}{v}}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}} - \frac{\underset{\cdot\cdot}{v}}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime\prime}{z}}$.

Ex.

Ex. 2. Sit aliud exemplum in inventione Integralis ipsius na^z, ubi est $\dot{z} = 1$, atque datur a, Tum pro x sumpto a^z, & pro v sumpto n, fit $\underset{\prime}{x} = a^{\overset{z}{\prime}}$ hoc est $\underset{\prime}{x} = ax$, seu $x + \dot{x} = ax$, adeoque $\dot{x} = \overline{a-1}\, x$,

atque $x = \dfrac{\dot{x}}{a-1}$. Regrediendo itaque ad Integralia fit

$[x] = \dfrac{x}{a-1}$; item $\overset{2}{[x]} = \dfrac{[x]}{a-1} = \dfrac{x}{\overline{a-1}|^2}$, item $\overset{3}{[x]} = \dfrac{x}{\overline{a-1}|^3}$; & sic porrò. Adeoque (quoniam $\underset{\prime}{x} = ax$,) sunt

$[x] = \dfrac{x}{a-1}$, $[\underset{\prime}{x}] = \dfrac{ax}{\overline{a-1}|^2}$, $[\underset{\prime\prime}{x}] = \dfrac{a^2 x}{\overline{a-1}|^3}$, &c. Unde

per formulam prodit $[na^z] = \dfrac{a^z n}{a-1} - \dfrac{a^{z+1} \dot{n}}{\overline{a-1}|^2} + \dfrac{a^{z+2} \ddot{n}}{\overline{a-1}|^3}$ &c.

In hoc exemplo continetur Solutio Problematis, de quo agit D^nus de *Monmort* in Propositione nona. Coincidit autem formula cum ea quam exhibet ille in Corollario primo ejusdem Propositionis.

Scholium. Possunt etiam ex hâc formulâ alii derivari valores Integralis quæsiti, pro vario modo quo interpretantur Incrementi propositi factores. Sic in exemplo secundo integrale ipsius na^z exhiberi potest per

formulam $a^z [n] - \overline{a-1}\, a^z \overset{2}{[\underset{\prime}{n}]} + \overline{a-1}|^2\, a^z \overset{3}{[\underset{\prime\prime}{n}]}$

$-$ &c. pro x nempe sumpto n, & pro v sumpto a^z. Sed de his fortasse aliâ occasione fusius dicemus.

Prop. III. *Theor.*

Ejusdem xv Integrale, vel primum, vel secundum, vel tertium, vel aliud quodvis cujus ordo designatur symbolo n, exhibetur per Seriem in hâc formâ generali prodeuntem $\overset{n}{[xv]} = \overset{n}{[x]}\, v - n \overset{n+1}{[\underset{\prime}{x}]}\, \dot{v} +$

$+ nx$

$$+ n \times \frac{n+1}{2} \overset{n+2}{[\underset{''}{x}]} \underset{..}{v} - n \times \frac{n+1}{2} \times \frac{n+2}{3} \overset{n+3}{[\underset{'''}{x}]} \underset{\because}{v} + \&c.$$

Collectâ formâ Seriei ex Propositione præcedenti, Coefficientes $1, -n, n \times \frac{n+1}{2}, -n \times \frac{n+1}{2} \times \frac{n+2}{3}$, &c. sic inveniuntur per Methodum Incrementorum. Pone

$$\overset{n}{[\dot{x} v]} = \dot{A} \overset{n}{[x]} v + B \overset{n+1}{[\underset{'}{x}]} \underset{.}{v} + C \overset{n+2}{[\underset{''}{x}]} \underset{..}{v} + D \overset{n+3}{[\underset{'''}{x}]} \underset{\because}{v} + \&c.$$

Tum aucto n incremento suo $\dot{n} = 1$, atque ipsis A, B, C, D, &c. incrementis suis contemporaneis $\dot{A}, \dot{B}, \dot{C}, \dot{D}$, &c. ut jam evadant $\underset{'}{n}, \underset{'}{A}, \underset{'}{B}, \underset{'}{C}, \underset{'}{D}$, &c. fiet novum Integrale (quod Integrale est ipsius $\overset{n}{[x v]}$,) $\overset{n+1}{[x v]} = \underset{'}{A} \overset{n+1}{[x]} v + \underset{'}{B} \overset{n+2}{[\underset{'}{x}]} \underset{.}{v} + \underset{'}{C} \overset{n+3}{[\underset{''}{x}]} \underset{..}{v} + \underset{'}{D} \overset{n+4}{[\underset{'''}{x}]} \underset{\because}{v} + \&c.$ Hujus itaque Incrementum primum coincidere debet cum Integrali prius posito. Sumptis ergo incrementis, fit

$$\overset{n}{[x v]} = \underset{'}{A} \overset{n}{[x]} v \begin{matrix} + \underset{'}{A} \\ + \underset{'}{B} \end{matrix} \overset{n+1}{[\underset{'}{x}]} \underset{.}{v} \begin{matrix} + \underset{'}{B} \\ + \underset{'}{C} \end{matrix} \overset{n+2}{[\underset{''}{x}]} \underset{..}{v} \begin{matrix} + \underset{'}{C} \\ + \underset{'}{D} \end{matrix} \overset{n+3}{[\underset{'''}{x}]} \underset{\because}{v} +$$

idem ac Integrale prius positum. Itaque terminos homologos inter se comparando fit 1^{mo} $\underset{'}{A} = A$. Unde est A datum quid. Sed ubi $n = 0$, est $A = 1$, ergo $A = 1$. 2^{do}. $B = \underset{'}{B} + \underset{'}{A}$, hoc est $B = B + \dot{B} + 1$, seu $\dot{B} = -1 = -\dot{n}$. Ergo regrediendo ad Integralia, fit $B = -n + a$. Sed ubi $n = 0$, est $B = 0$. Ergo $a = 0$, atque $B = -n$. 3^{tio}. $C = \underset{'}{C} + \underset{'}{B}$, hoc est $\dot{C} = \underset{'}{n}$. Regre-

diendo itaque ad Integralia fit $C = \frac{n\overset{n}{,}}{2} + b$. Sed ubi $n = 0$, est $C = 0$. Ergo $b = 0$, atque $C = \frac{n\overset{n}{,}}{2}$, hoc est, $n \times \frac{n+1}{2}$. 4to. Ad eundem modum invenitur $\dot{D} = -\dot{n} \times \frac{n+1}{2} \times \frac{n+2}{3}$. Et sic pergendo inveniuntur cæteri Coefficientes.

Scholium. 1. In hâc Propositione comparatâ cum Propositione primâ, cernitur singularis quædam relatio Incrementa inter & Integralia. Ut enim in Arithmeticâ vulgari, Multiplicatio & Divisio sunt invicem ita contrariæ, ut si Multiplicatio designetur per Indicem affirmativum, Divisio designabitur per Indicem cum signo negativo; sic etiam in Methodo Incrementorum, si Incrementum designetur per Indicem affirmativum, Index negativus Integrale sistet. Sic in Propositione primâ, si pro n sumatur Numerus binarius 2, per formulam exhibebitur ipsius $x v$ incrementum secundum, nempe $\ddot{x} v + 2 \dot{x}_{,} \dot{v} + x_{,,} \ddot{v}$; Sed si pro n sumatur numerus negativus -2, ut jam quæratur ipsius $x v$ incrementum (ita loqui liceat) negativè secundum, (quod idem est ac Integrale secundum) prodeunt coefficientes iidem ac si sumatur n affirmativè in Propositione præsenti: atque interpretatis insuper ipsis $\overset{-2}{x}$, $\overset{-3}{x_{,}}$, $\overset{-4}{x_{,,}}$, &c. per $[\overset{2}{x}]$, $[\overset{3}{x_{,}}]$, $[\overset{4}{x_{,,}}]$, &c. Series fit omnino eadem ac per Propositionem præsentem prodit, ubi quæritur Integrale secundum.

2. Ex his autem formulis quasi suâ sponte procedunt formulæ Propositionum undecimæ atque duodecimæ Libri de Methodo Incrementorum. Nam pro incre-

incrementis scribe Fluxiones, atque evanescentibus incrementis fiant jam omnes $x, x_{'}, x_{''}, x_{'''}$, *&c.* inter se æquales, atque migrabit statim hæc Propositio secunda in illam undecimam, atque præsens tertia in illam duodecimam. Quod quidem exemplum satis insigne est Methodi *Newtonianæ*, quâ colligit ille rationes Fluxionum ex rationibus ultimis Incrementorum evanescentium, vel ex primis nascentium.

Additamentum.

PRæcedentium impressioni intentus dum Typothetarum erroribus corrigendis do operam, atque eâ occasione in animo illa sœpius revolvo, subiit Artificium illud quo jam olim usus est D. *Jac. Bernoulli* in inventione quarundam Serierum, ope Progressionis Harmonicæ, cujus meminit D. de *Monmort* in *Scholio* 6. *Prop.* V. præcedente commodè etiam applicari posse ad inventionem ipsius *Monmortii* Propositionum 2^{dam}, 3^{tiam}, 4^{tam}, 5^{tam}, atque id genus aliarum aliquanto fortasse generaliorum. Hoc in sequentibus paucis ostendisse, credebam Lectori non fore ingratum.

Theorema.

Sit Progressio Arithmetica $p,\ p+n,\ p+2n,$ *&c.* cujus termini singuli successivè designentur per x, & sunto $b, c, d,$ *&c.* quivis multiplices differentiæ datæ n terminorum Progressionis istius Arithmeticæ. Sint $A, B, C, D,$ *&c.* Numeri quilibet dati, & constituantur fractiones quotvis $\frac{A}{x}, \frac{B}{x+b}, \frac{C}{x+c}, \frac{D}{x+d},$ *&c.* Pro x successive scriptis valoribus suis $p, p+n, p+2n,$ *&c.*

ex

ex harum fractionum quâlibet, oritur Series Harmonicè proportionalium Sic *v g.* ex fractione primâ $\frac{A}{p}$, oritur Series $\frac{A}{p}$, $\frac{A}{p+n}$, $\frac{A}{p+2n}$, &c. Dico quod aggregatum quotlibet hujusmodi Serierum in infinitum continuatarum in terminis numero finitis exhiberi potest, si modo fuerit numeratorum A, B, C, D, &c. aggregatum æquale nihilo. Duobus exemplis hoc fiet manifestum.

Ex. Sint duæ tantùm fractiones $\frac{A}{x}$, atque $\frac{-A}{x+3n}$, existente $b = 3n$. Scribantur Series harmonicæ ex his formulis ortæ, eo ordine, ut termini, in quibus sunt denominatores æquales, sibi invicem respondeant, & collectis summis terminorum homologorum, prodibit aggregatum Serierum in terminis numero finitis, ut in calculo apposito videre est.

$$\frac{A}{p} + \frac{A}{p+n} + \frac{A}{p+2n} + \frac{A}{p+3n} + \frac{A}{p+4n} + \&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+ \frac{-A}{p+3n} + \frac{-A}{p+4n} + \&c. = \text{Seriei ex } \frac{-A}{x+3n}$$

$$\frac{A}{p} + \frac{A}{p+n} + \frac{A}{p+2n} + 0 + 0 + \&c. = \text{Aggreg. Serierũ.}$$

Ex. 2. Sint tres fractiones $\frac{A}{x}$, $\frac{B}{x+2n}$, $\frac{C}{x+3a}$, existentibus $b = 2n$, $c = 3n$, atque $A + B + C = 0$. In hoc casu Calculus sic se habet.

$$\frac{A}{p} + \frac{A}{p+n} + \frac{A}{p+2n} + \frac{A}{p+3n} + \ldots + \&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+ \frac{B}{p+2n} + \frac{B}{p+3n} + \ldots + \&c. = \text{Seriei ex } \frac{B}{x+2n}$$

$$+ \frac{C}{p+3n} + \ldots + \&c. = \text{Seriei ex } \frac{C}{x+3n}$$

$$\frac{A}{p} + \frac{A}{p+n} + \frac{A+B}{p+2n} + \frac{A+B+C=0}{p+3n} + \&c. = \text{Aggregato Serierum.}$$

Ubi

Ubi etiam prodit aggregatum Serierum in terminis numero finitis, nempe $\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}$, ob Numeratorum A, B, C, aggregatum æquale nihilo. Et ad eundem modum demonstratur Theorema in aliis casibus quibusvis.

Cor. 1. Ex his principiis derivari possunt innumeræ Series in infinitum continuatæ, in terminis tamen numero finitis summabiles.

Cas. 1. Sint $\frac{A}{x}$ & $\frac{-A}{x+b}$ formulæ duarum Serierum harmonicarum quarum aggregatum prodit in terminis numero finitis per superius demonstrata, Tum, formulis istis in unam summam collectis, fit $\frac{Ab}{x\times\overline{x+b}}$ formula Seriei summabilis. Sint *v.gr.* $A=\frac{1}{6}$, $p=1$, $n=2$, atque $b=3n=6$. Tum formulæ Serierum harmonicarum erunt $\frac{1}{6x}$, & $\frac{-1}{6\times\overline{x+6}}$, formula Seriei compositæ summabilis erit $\frac{1}{x\times\overline{x+6}}$, Serie illa existente $\frac{1}{1\times7}+\frac{1}{3\times9}+\frac{1}{5\times11}+\frac{1}{7\times13}+$ &c. atque summa Seriei, per calculum in præmissis demonstratum, erit $\frac{1}{6\times1}+\frac{1}{6\times3}+\frac{1}{6\times5}$. Sint tres formulæ Serierum harmonicarum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, (existente $A+B+C=0$, ut sit Serierum aggregatum finitum per præmissa.) Tum formulis in unam summam collectis fit $\frac{A\times\overline{x+b}\times\overline{x+c}+B\times x\times\overline{x+c}+C\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}}$, seu (terminis revocatis ad formam factorum x, $x\times\overline{x+b}$, $x\times\overline{x+b}\times\overline{x+c}$,) $\frac{Acb+\overline{Ac+c-b\,B}\times x+\overline{A+B+C}\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}}$, hoc est

 (o

(ob $A + B + C = 0$) $\frac{Acb + \overline{Ac + B\times\overline{c-b}}\times x}{x\times\overline{x+b}\times\overline{x+c}}$, formula Seriei ſummabilis. Si quatuor ſint Fractiones $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, (exiſtente $A + B + C + D = 0$) ad eundem modum invenietur formula Seriei ſummabilis

$$\frac{Abcd + \overline{Acd + B\times\overline{c-b}\times\overline{d-b}}\times x + \overline{Ad + B\times\overline{d-b} + C\times\overline{d-c}}\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$$

Et ſic pergere licet ad formulas adhuc magis compoſitas.

Caſ. 2. Et ſi plures ſint formulæ Serierum hujuſmodi ſummabilium, quarum denominatorum factores excerpantur ex diverſis progreſſionibus Arithmeticis, ex iſtarum formularum quotvis in unam ſummam additione, conficietur formula nova Seriei ſummabilis. Sint *e. gr.* formulæ duæ Serierum ſummabilium $\frac{1}{x\times\overline{x+3}}$ & $\frac{1}{z\times\overline{z+2}}$, excerptis x ex Progreſſione Arithmeticâ 1, 2, 3, 4, *&c.* z ex Progreſſione Arithmeticâ 1, 3, 5, *&c.* Tum ex his formulis in unam ſummam collectis fiet formula nova $\frac{z\times\overline{z+2} + x\times\overline{x+3}}{x\times\overline{x+3}\times z\times\overline{z+2}}$, vel, (expoſito z per x & numeros datos) $\frac{\overline{2x-1}\times\overline{2x+1} + x\times\overline{x+3}}{x\times\overline{x+3}\times\overline{2x-1}\times\overline{2x+1}}$

Cor. 2. Hinc omnis Series in infinitum continuata ſummabilis eſt, cujus termini deſignantur per Fractionem, cujus denominatoris factores excerpuntur ex datâ quâlibet Progreſſione Arithmeticâ, numerator autem eſt multinomium, cujus dimenſiones ſunt ad minimum binario pauciores, quam ſunt dimenſiones Denominatoris. Nam omnis hujuſmodi fractio reſolvi poteſt in tot fractiones ſimplices, quot ſunt dimenſiones (hoc eſt, quot ſunt factores) Denominatoris, quarum numeratorum aggregatum eſt nihil. Sit exempli gratiâ, formula

formula oblata $\frac{\alpha + \beta x + \gamma x x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$. Pone hanc formulam æquari aggregato fractionum $\frac{A}{x} + \frac{B}{x+b} + \frac{C}{x+c} + \frac{D}{x+d}$. Tum fractionibus istis in unam summam collectis fiet $\overline{Abcd + \overline{Acd + B\overline{c-b} \times \overline{d-b}} \times x}$
$+ \overline{Ad + B \times \overline{d-b} + C \times \overline{d-c}} \times xx \times \overline{x+b}$
$+ \overline{A+B+C+D} \times xxx \times \overline{x+b} \times \overline{x+c}$ applicatum ad
$x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d} = \frac{\alpha + \beta x + \gamma xx \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$.

Unde per comparationem terminorum homologorum fit $Abcd = \alpha$, $Acd + B \times \overline{c-b} \times \overline{d-b} = \beta$, $Ad + B \times \overline{d-b} + C \times \overline{d-c} = \gamma$, $A + B + C + D = 0$. adeoque $A = \frac{\alpha}{bcd}$, $B = \frac{\beta - Acd}{\overline{c-b} \times \overline{d-b}}$,

$C = \frac{\gamma - Ad - B \times \overline{d-b}}{d-c}$, $D = -A - B - C$, Quo pacto formula oblata resolvitur in fractiones simplices $\frac{\alpha}{bcdx}$
$+ \frac{\beta - Acd}{\overline{c-b} \times \overline{d-b} \times \overline{x+b}} + \frac{\gamma - Ad - B \times \overline{d-b}}{\overline{d-c} \times \overline{x+c}}$
$+ \frac{-A-B-C}{x+d}$, ex quibus ortarum Serierum aggregatum, hoc est, summa Seriei ortæ ex formulâ oblatâ $\frac{\alpha + \beta x + \gamma x x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$, per jam dicta prodit in terminis numero finitis. Quod verò dimensiones numeratoris in formulâ oblatâ, debeant esse binario ad minimum pauciores, quam sunt dimensiones Denominatoris, hinc constat, quòd in reductione fractionum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, quilibet numerator A, B, C, D,

ducitur

ducitur in omnes denominatores excepto uno, nempe ſuo; unde prodeunt Numeratoris Dimenſiones unitate pauciores quam ſunt dimenſiones Denominatoris. Sed per æquationem $A + B + C + D = 0$ perit altiſſima dimenſio in numeratore; Unde ſuperſunt Numeratoris Dimenſiones ad minimum binario pauciores quam ſunt dimenſiones Denominatoris. Ad hoc verò Corollarium revocari poſſunt D. de *Monmort* Propoſitiones 2[da] & 5[ta].

Cor. 3. Item oblatâ formulâ juxta *Caſ.* 2. *Cor.* 1. adhuc magis compoſitâ, ex iidem principiis perſpici poteſt an ſit Series ſummabilis. Sint progreſſiones duæ Arithmeticæ 1, 3, 5, &c. 2, 4, 6, &c. quarum termini homologi deſignentur per x & z, & ſit formula Seriei oblata $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+2} \times z \times \overline{z+2}}$, vel (pro z ſcripto $x + 1$, & factoribus Denominatoris in ordinem coactis) $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$. Pone formulam hanc æquari aggregato formularum $\frac{P}{x \times \overline{x+2}}$, $\frac{Q}{\overline{x+1} \times \overline{x+3}}$, Serierum per ſuperius dicta ſummabilium, ut (formulis his noviſſimis in unam ſummam collectis) ſit $\frac{P \times \overline{x+1} \times \overline{x+3} + Q \times x \times \overline{x+2}}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$ ſeu

$$\frac{3P + \overline{4P + 2Q}\,x + \overline{P + Q}\,x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}} = \frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}.$$

Hinc comparando terminos homologos oriuntur æquationes $3P = \alpha$, $4P + 2Q = \beta$, $P + Q = \gamma$. Unde eliminatis P & Q per debitas operationes Analyticas, prodit æquatio $2\alpha - 3\beta + \gamma = 0$, qua definitur relatio quæ inter coefficientes α, β, γ intercedere debet, ut Series orta ex formulâ oblatâ $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$ ſit

sit summabilis. Ad eundem modum si formulæ oblatæ Denominatoris factores excerpantur ex tribus Progressionibus Arithmeticis, invenientur duæ æquationes quibus definiantur relationes coefficientium Numeratoris, ut sit Series summabilis. Si quatuor sint Progressiones Arithmeticæ, Coefficientium relatio definietur per tres æquationes. Et sic porrò. Et in hujusmodi formulis ut sint Series summabiles, hæc insuper observanda sunt, Primò ut Numeratorum dimensiones sint ad minimum binario pauciores quam sunt dimensiones Denominatorum, Deinde ut ex singulis Progressionibus Arithmeticis excerpantur ad minimum duo factores Denominatoris. Denique, quod si sint duo vel plures factores Denominatoris inter se æquales, ponendum sit tot etiam Progressiones Arithmeticas, ex quibus excerpuntur, esse inter se æquales. Præmissis attentius perpensis, hæc obvia erunt. Ad hoc vero Corollarium facile revocantur D. de *Monmort* Propositiones 3tia & 4ta.

F I N I S.

ERRATUM in N°. 352.

PAge 586, after the end of line 15, add *black Cloud, from behind which there issued a.*

LONDON:

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *Octob. Nov.* and *Decemb.* 1717.

The CONTENTS.

I. *An Advertisement to* Astronomers, *of the Advantages that may accrue from the Observation of the* Moon's *frequent Appulses to the* Hyades, *during the Three next ensuing Years.*

OF all the Methods hitherto proposed for finding the *Longitudes* of Places for Geographical Uses, none seems more adapted to the purpose, than that by the *Occultations* of the fixt Stars by the *Moon* observed in distant Parts: For those Immersions of the Stars which happen on the dark Semicircle of the Moon, and their Emersions from the same, are perfectly momentaneous, without that Ambiguity, to which the Observations of the Eclipses of the *Moon* and those of *Jupiter's Satellites* are subject. Besides, whilst the Moon is horned, and her weaker Light less dazling, an ordinary short Telescope, such as by Experience is found to be manageable on Ship board, suffices to observe those Moments, even in the Occultations of very minute Stars: On which account, this way seems to bid fairest for the desired Solution of the grand Problem of finding the Longitude at Sea. But since it would be needless to enquire exactly what Longitude a Ship is in, when that of the Port to which she is bound is still unknown it were to be wisht that the Princes of the Earth would cause such Observations to be made, in the Ports and on the principal Head-Lands of their Dominions, each for his own, as might

might once for all settle truly the Limits of the Land and Sea. This Work however being likely to be left to the Care and Curiosity of private Persons, it may not be amiss hereby to give notice of the present Opportunity of performing it, in this our Northern Hemisphere, by help of the frequent Appulses of the Moon to the more Southerly of the *Hyades*, many of which she eclipses in each monthly Revolution, and will continue so to do, during the Years 1718, 1719, and 1720.

These Stars are but Three or Four in all former Catalogues, but the *British* of Mr *Flamsteed* encreases them to Sixteen; to them we have added Three others somewhat smaller, *viz.* *c i*, and *n* in the Figure of the *Hyades* hereto annext. In it the principal Stars are markt with *Bayer's* Marks, and the rest with the Letters of the *Italick* Alphabet; their Longitudes are fitted to the beginning of the Year 1718, and being truly laid down, may serve to instruct the curious Observer, when and where to look for them, when the Moon is among them.

It appears by this Scheme that the Distance between *a* and *a* or *Palilicium*, is about Nine Hours Motion of the Moon, in which time supposing her to pass from one to the other, she must eclipse *γ* and *ε*, and Four or Five of those about *θ*, and must apply very close, with her Southern Limb, to all those which have about Six Degrees South-Latitude; which would be a very entertaining Sight for the Lovers of these Arts. But if the Times of the Occultations of any One of these Stars, or even of any Two of them in the same Night, be accurately observed under distant Meridians, the difference of those Meridians may be truly obtain'd thereby; especially since the *Moon's Parallax*, and all other parts of her Theory thereto required, are at present sufficiently stated and known.

For the sake of such as are willing to make use of this Method, we have added the Places of all the *Hyades* fitted to the present Time, and chiefly taken from the *British* Catalogue, which being faulty in the Stars we call *k* and *l*, we have here rectified them.

Catalogus

Catalogus Hyadum, *ineunte Anno* 1718.

Stellarum NOMINA.		Long. II			Lat. Aust.			Ma.
Quæ præcedit γ Tauri ———	a	0	51	3	5	50	14	7
In naribus Tauri, Bayero	γ	1	50	54	5	46	22	3
Quæ sub γ ——— —	b	1	56	31	6	19	57	7
In Origine Nasi Tauri ———	c	2	54	25	4	47	5	7
Inter nares & oculum Tauri Boreum —— ——	δ	2	54	47	4	0	34	3/4
Huic contigua ad Austrum.----	d	3	10	33	4	9	4	6
Præcedentium θ Borealis ———	e	3	17	21	5	41	50	8
Earum Australis clarior. ———	f	3	25	32	6	2	44	6
Quæ sequitur δ —— ———	g	3	35	2	3	43	27	5
Contiguarum inter nares & Palilicium Borea ——	θ	3	59	45	5	47	16	4
Earundem Australior ———	θ	4	0	11	5	52	55	4
Duarum supra θ Borea ———	h	4	2	32	5	23	43	7
Earundem Australior ———	i	4	7	44	5	36	40	8
Sub θ trium in recta præcedens	k	4	19	27	6	9	45	[illegible]
Earum media —— —	l	4	26	55	6	7	35	[illegible]
2[um] *θ sequentium Borea* ———	m	4	30	26	5	37	49	7
Oculus Boreus Tauri ———	ε	4	30	31	2	35	58	3
Sequentium θ Australis ———	n	4	32	35	5	41	00	8
Trium sub θ sequens ———	o	4	45	55	6	0	35	7
Palilicium Bayero	α	5	50	20	5	29	50	1
Quæ hanc sequitur proxime —	p	6	17	35	6	3	20	7
Contig. sequentium Australis—	q	6	30	34	6	19	19	6
Borea & clarior —— ———	σ	6	33	12	6	12	35	7

II. *Solutio*

II. *Solutio Problematis à Dom*^no G. G. Leibnitio, *Geometris* Anglis *nuper propositi. Per* Brook Taylor, *LL. D. &* R. S. *Secr.*

CUM Dom. *G. G. Leibnitius* nuper defunctus, in controversiâ jampridem ortâ circa Inventionem Methodi Fluxionum, (quam is Differentialem vocare maluit, sibique pertinaciter appropriari nisus est,) nihil omnino responsi dederit argumentis, quibus inclyti istius Inventi gloria Dom^no *Neutono* vendicatur ; en tandem, hortante Dom^no *Joh. Bernoulli*, Problema Geometris *Anglis* solvendum proposuit ; quo scilicet vires eorum in Methodo istâ experiretur ; quasi Problematis istius Solutioni si cæteri istius Nationis deprehendantur impares, rectè concludatur, nec ipsum *Neutonum*, qui, fatente etiam *Leibnitio*, ab hujusmodi contemplationibus jam jure immunis esse debet, olim fuisse parem inventioni istius Methodi. Sive Problema solvatur, sive insolutum maneat, nihil exinde consequetur quod *Neutonum* afficiat ; nec istis certè *Leibnitii* Fautoribus, qui Problematis solutionem etiamnum continenter efflagitant, jus ullum est nos ad certamen ingeniorum tantâ cum licentiâ provocandi ; adeoque Problema eorum jure merito negligi posset. Verùm ne aliquando exinde occasionem triumphandi arripiant, si hoc Problema maneat ab *Anglis* omninò intactum, ipse, Geometra longè non summi inter nostrates subsellii, inducor, ut solutionem edam qualem qualem Problematis, nec usu, nec difficultate adeò insignis.

Problema à *Leibnitio* primò propositum, ita fuit intellectum quasi nihil aliud requisitum fuisset, quam ut secarentur ad angulos rectos Hyperbolæ Conicæ iisdem Centro & Verticibus descriptæ. Verùm cum illi nunciatum

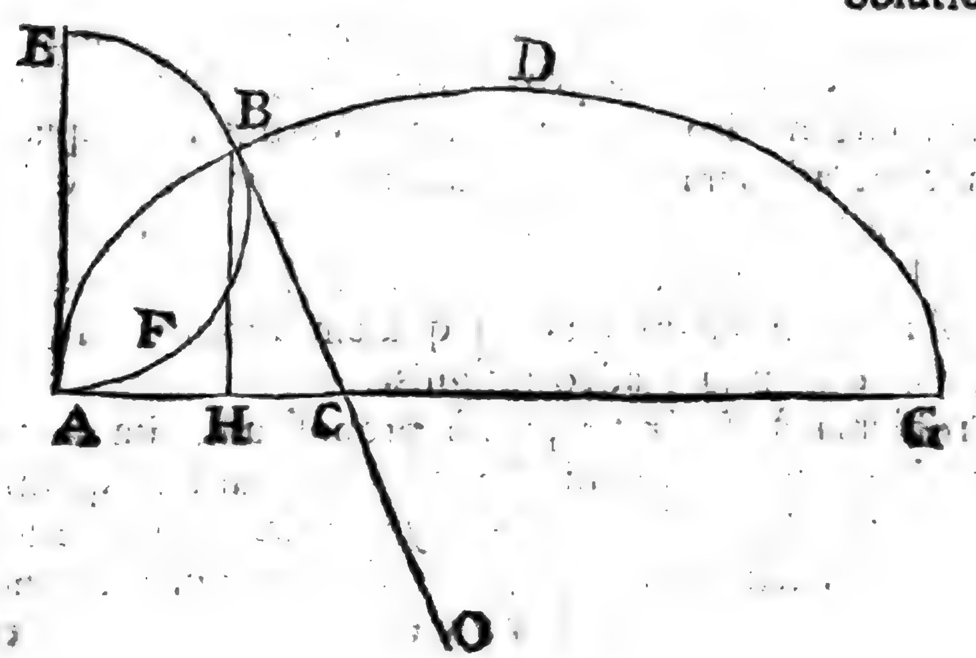
tum fuerat hunc casum à quibusdam *Anglis* fuisse illicò solutum, rescripsit, non solutionem casûs particularis, sed generalem requiri. Quo factum est ut solutiones istæ particulares non editæ fuerint; verùm in Transactione Philosophicâ N° 347. subinde prodiit Solutio maximè generalis. Sed nec illâ contenti fuerunt *Leibnitius* & Fautores ejus, quin illam derisui habuêre, quasi qui illam excogitaverat non potuisset eam ad casum specialem applicare. Si nondum viderint quomodo ex illâ æquationes sint deducendæ, id profectò illorum imperitiæ tribuendum erit. Paulò ante *Leibnitii* obitum prodiit tandem Problema sequens; quod quidem diversimodè solvi potest, premendo vestigia Solutionis generalis modò citatæ, sed quod in præsentia solvimus ut sequitur.

Problema.

Super rectâ AG *tanquam axe, ex puncto* A *educere infinitas Curvas, qualis est* ABD, *ejus naturæ, ut radii Osculi, in singulis punctis* B *& ubique ducti,* BO *secentur ab axe* AG *in* C, *in datâ ratione, ut nempe sit* BO *ad* BC *ut* 1 *ad* n.

Deinde construendæ sunt Trajectoriæ EBF *primas Curvas* ABD *normaliter secantes.*

Solutionis



Solutionis Pars prima;

Nempe Inventio Curvarum secundarum ABD.

1. DUctâ ordinatâ BH ad axem AG normali, sint, Abscissa $AH = z$, Ordinata $HB = x$, Curva $AB = v$. Tum per Methodum Fluxionum directam erit $BC = \frac{\dot{v}}{\dot{z}}x$, & fluente uniformiter v, $BO = \frac{\dot{v}\dot{x}}{\ddot{z}}$. Unde per conditionem Problematis fit $BO\ \left(\frac{\dot{v}\dot{x}}{\ddot{z}}\right)$: BC $\left(\frac{\dot{v}}{\dot{z}}x\right)$ :: 1 : n; adeoque $\ddot{z}x - n\dot{z}\dot{x} = 0$.

2. Collatâ hâc æquatione cum formulâ Fluxionum secundâ, in calce *Prop.* 6. Methodi Incrementorum, invenitur $\dot{z}x^{-n} = \dot{v}\alpha^{-n}$; existente α linea data, per cujus valorem potest Curva ABD accommodari conditioni alicui Problemati annexæ.

3. Pro v scripto ipsius valore $\sqrt{\dot{x}^2 + \dot{z}^2}$, migrat æquatio $\dot{z}x^{-n} = \dot{v}\alpha^{-n}$ in hanc $\dot{z} = \frac{\dot{x}x^n}{\sqrt{\alpha^{2n} - x^{2n}}}$. Unde datur z ex datâ x, per quadraturam Curvæ cujus abscissâ existente x est ordinata $\frac{x^n}{\sqrt{\alpha^{2n} - x^{2n}}}$.

4. Sint σ & τ numeri integri, vel affirmativi vel negativi, tales ut sit Curvarum isto modo provenientium simplicissima, ea cujus est Abscissa y, & Ordinata $y^{\frac{1-n+2\sigma n}{2n}} \times \overline{z-y}|^{\tau-\frac{1}{2}}$; tum erit ea omnium Curvarum simplicissima, per quarum Quadraturam datur Abscissa z ex datâ Ordinatâ x.

5. Est Curva ABD Geometrica, quoties pro n sumitur reciprocum numeri cujusvis imparis.

 6. In

6. In prædictis Curvam ABD consideravimus ut versus axem AG concavam, quo in casu maxima ordinata x æqualis est lineæ datæ α, quam Parametrum Curvæ commodè vocare licet. Et in hoc casu Curva actu occurret Axi. Unde fluente ipsius $\frac{\dot{x}x^n}{\sqrt{\alpha^{2n}-x^{2n}}}$ debitè sumptâ, hoc est, ita ut simul evanescant z & x, transibit Curva per punctum datum A, sicut postulat Problema.

7. Sed si quæratur Curva ABD, quæ sit versùs axem convexa, ad eundem modum pervenietur ad æquationem $\dot{z}=\frac{\alpha^n\dot{x}}{\sqrt{x^{2n}-\alpha^{2n}}}$; quæ etiam ex æquatione priori derivari potest mutando signum ipsius n. Et in hoc casu est curva ABD Geometrica, quoties pro n sumitur reciprocum cujusvis numeri paris. In hoc verò casu Ordinata omnium minima x æqualis est Parametro α; adeoque Curva nusquam occurrit Axi. Quare limitatur Problema ad casum priorem.

8. Ex præmissis facilè colligitur Curvas omnes ABD esse inter se similes, & circa punctum datum A similiter positas, lateribus earum homologis existentibus proportionalibus Parametris α.

Solutionis Pars altera;

Nempe Inventio Curvæ secantis.

9. Ex § 2. fit $\dot{v}:\dot{z}::\alpha^n:x^n$. Sed est $BC:BH::\dot{v}:\dot{z}$, Unde fit $BC:BH::\alpha^n:x^n$. Ex conditione verò Problematis est BC tangens Curvæ quæsitæ EBF. Quare si jam sumantur AH (z) & BH (x) pro coordinatis Curvæ EBF, Curvâ ipsâ EB existente r, erit, per Meth. Flux. direct. $\dot{r}:-\dot{x}::(BC:BH::)\,\alpha^n:x^n$. Unde fit $\frac{x^n}{\alpha^n}=\frac{-\dot{x}}{\dot{r}}$.

10. In

10. In Curva ABD finge æquationem $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$ transformari in æquationem signis radicalibus non affectam $\dot{z} = A\dot{x}\frac{x^n}{a^n} + B\dot{x}\frac{x^{3n}}{a^{3n}} +,$ *&c.* Tum regrediendo ad Fluentes fiet $z = \frac{1}{n+1}A\frac{x^{n+1}}{a^n} + \frac{1}{3n+1}B\frac{x^{3n+1}}{a^{3n}} +,$ *&c.* coefficiente novâ introductâ nullâ, quoniam per conditionem Problematis debent simul nasci z & x. Hinc vice $\frac{x^n}{a^n}$ substituto ipsius valore $\frac{-\dot{x}}{\dot{r}}$ in § 9 invento, fit $z = \frac{1}{n+1}Ax\frac{-\dot{x}}{\dot{r}} + \frac{1}{3n+1}Bx\frac{-\dot{x}^3}{\dot{r}^3} +,$ *&c.* quæ æquatio fluxionalis est primi gradûs ad Curvam quæsitam EBF. Revocatur autem ad formulam simpliciorem in terminis numero finitis, modo sequenti.

11. Fluat uniformiter r, & existente a quantitate non fluente, fit $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$. Substituto hoc valore ipsius $\frac{-\dot{x}}{\dot{r}}$ in æquatione novissimè inventâ, atque ductâ æquatione in $\frac{s}{x}$, transformatur ea in hanc $\frac{zs}{x} = \frac{1}{n+1}A\frac{s^{n+1}}{a^n} + \frac{1}{3n+1} \times B\frac{s^{3n+1}}{a^{3n}} +$ *&c.* Unde capiendo Fluxiones fit $\frac{s\dot{z}x + \dot{s}zx - sz\dot{x}}{x^2} = A\dot{s}\frac{s^n}{a^n} + B\dot{s}\frac{s^{3n}}{a^{3n}} = \frac{\dot{s}s^n}{\sqrt{a^{2n} - s^{2n}}}$. Quod ultimum constat ex Analogia Serierum $A\dot{x}\frac{x^n}{a^n} +$ *&c.* & $A\dot{s}\frac{s^n}{a^n} +$ *&c.* Hinc pro s & $\dot{s}$ substitutis eorum valoribus ex æquatione $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$ collectis, elicitur æquatio $n\dot{x}^2\dot{z}z - \ddot{x}x\dot{z}z - n\dot{x}x\dot{z}^2 - \ddot{x}\dot{x}x^2 = 0$. Quæ ad Fluxiones primas revocatur modo sequenti.

fit

12. In termino ultimo $-\ddot{x}\dot{x}x^{2}$ vice $\ddot{x}\dot{x}$ scripto ipsius valore $-\ddot{z}\dot{z}$, & æquatione deinde applicata ad z, fit $n\dot{x}^{2}z - \ddot{x}xz - n\dot{x}x\dot{z} + x\dot{x}\ddot{z} = 0$. Quæ æquatio in x^{-n-1} ducta est Fluxio æquationis $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$; existentibus a & r non fluentibus. Est ergo $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$, seu $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^{n}$, æquatio fluxionalis primi gradûs ad Curvam quæsitam EBF.

13. In istâ autem æquatione est a valor Ordinatæ BH, quando incidit punctum H in punctum A.

14. Haud proclive est æquationem $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^{n}$, manente n in terminis generalibus, revocare ad æquationem Fluentes tantùm involventem, vel ad quadraturam Curvarum. Sed puncta curvæ EBF possunt commodè inveniri per descriptionem Curvæ ABD, & Curvæ cujusdam Geometricæ. Per Geometricam hic intelligo Curvam, cujus æquationem non ingrediuntur Fluxiones, nec fluentes in Indicibus dignitatum. Secetur enim Curva ABD, cujus Parameter sit α, in B, à Curvâ geometricâ cujus æquatio est $a\alpha^{n}x^{n} - za^{n}x^{n} = xa^{n}\sqrt{a^{2n} - x^{2n}}$; atque erit punctum illud intersectionis B ad unam ex Trajectoriis quæsitis, nempe quæ transit per punctum E, existente $AE = a$ & normali ipsi AG.

15. Hinc si ABD sit Curva Geometrica, erit etiam EBF geometrica.

Scholium. Potest & alio modo inveniri æquatio $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^{n}$. Nam certâ quâdam Analysi quam nunc celare statuo, inveni æquationem $\frac{\dot{\alpha}}{\alpha} = \frac{r\dot{r}}{zz + xx}$. Quâ comparatâ cum æquatione $\frac{x^{n}}{\alpha^{n}} = \frac{-\dot{x}}{\dot{r}}$ (§ 9) eliminando α & $\dot{\alpha}$, tandem pervenitur ad prædictam æquationem $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^{n}$.

Exemplum.

Exemplum. Ad demonstrationem Solutionis nostræ suffecerit exemplum simplicissimum. Sit itaque $n=1$; quo in Casu est ABD semicirculus diametro AG descriptus, atque est EBF item semicirculus descriptus diametro AE. Est autem in hoc Casu $\frac{\dot{x}x^n}{\sqrt{a^{2n}-x^{2n}}}=\frac{\dot{x}x}{\sqrt{a^2-x^2}}$. Unde in § 3. fit $\dot{z}=\frac{\dot{x}x}{\sqrt{a^2-x^2}}$; adeoque $z=a-\sqrt{a^2-x^2}$, quæ æquatio est ad Circulum diametro $AG=a$ descriptum, ut fieri debuit. Item pro n scripto 1, æquatio $\dot{z}x-z\dot{x}\times x^{n-1}=\dot{r}x^n$ (§ 12.) migrat in hanc $\dot{z}x-z\dot{x}=\dot{r}x$. Unde exterminando $\dot{r}$ ope æquationis $r\dot{r}=x\dot{x}+z\dot{z}$, fit $\frac{2z\dot{z}x-\dot{x}z^2}{x^2}=-\dot{x}$; adeoque regrediendo ad Fluentes $\frac{zz}{x}=-x+a$, quæ æquatio est ad Circulum diametro $AE=a$ descriptum, ut etiam fieri debuit.

III. *Extract of a Letter of Dr.* Chr. Hunter, *M.D. to Dr.* J. Woodward, *R. S. S. from* Durham, *giving an Account of a* Roman *Inscription, lately dug up in the North of* England; *with some Historical and Chronological Remarks thereon.*

THE Inscription which comes herewith, (*Fig.* II.) was dug up, two Years ago, in the *Roman CASTRUM*, near *Lanchester:* The Inscription is very legible, and gives me reason to hope, a Search after the first Fortifying this Place will not be unnecessary; especially, being able to fix the Time of *Gordian*'s Repair-

Repairing this Fortress, to the 243d Year of Christ. We may reasonably ascribe the Foundation to the prudent Administration of *Julius Agricola*, in the Reign of *Fl. Vespasian*, about 169 Years before. In Confirmation of this, I find the following Particulars very material, and think it not unbecoming to begin my Enquiry with *Vespasian*'s first Appearance upon the Theatre of Fame in *Britain*.

In the Second Year of the Emperor *CLAUDIUS*, *Ann. Dom.* 44. the *Romans* invaded *Britain*, under the Command of *Aulus Plautius*, in which Expedition *Vespasian* *, then Legate of the Second Legion, made a glorious Figure; having been engaged in no less than thirty Battels, and reduced two powerful Provinces, above twenty Towns, and the Isle of *Wight*. All these Successes, tho' continued with good Improvements in some of the following Years and Governments, could not frighten the Natives into an entire Submission; especially, no Advance being made into the Country of the *Brigantes*, till the Advancement of *Vespasian* to the Imperial Throne, about 26 Years after, *Ann. Dom.* 70. Then the whole Empire was deliver'd from the Miseries of *Nero*'s, and the short but lamentable Devastations of the three succeeding Reigns: *Vespasian* then resolv'd to push on his begun Conquests in *Britain*; choice Armies, commanded by experienced Generals, are sent over; and the XX^th Legion, having in the preceding Troubles acted seditiously, (not without Difficulty) was reduced to submit to *Vespasian* (most of the Officers as well as Soldiers having been advanced by *Vitellius*). *Julius Agricola* is constituted Legate, who, under the Governour *Petilius Cerealis*, bore a considerable Share

* *Suetonius*, *Vespasian*, Cap. 4.

in

in the Successes against the *Brigantes*; * " Sed primò
" *Cerealis* modò labores & discrimina, mox & gloriam
" communicabat: Sæpe parti Exercitus in experimen-
" tum, aliquando majoribus copiis ex eventu præfecit.
Tacitus afterwards in a few Words sums up the Whole
of *Cerealis* his Acquisitions, † " Terrorem statim intu-
" lit *Petilius Cerealis*, *Brigantum* Civitatem, quæ nu-
" merosissima Provinciæ totius perhibetur, aggressus;
" multa prælia, & aliquando non incruenta; magnam-
" que *Brigantum* partem aut victoria amplexus, aut
" bello. Notwithstanding these Advantages, I dare not
suppose the *Romans* to have then penetrated so far into
this Province as our *Longovicum*, which is situate so
near the Northern Bounds of the *Brigantes*, that at pre-
sent it's not distant above twelve Miles from *Corbridge*,
the Roman *Curia*, the chief Town of the adjoining Peo-
ple the *Otadini*. I now advance to my principal Mo-
tive, (I hope its Length may deserve Pardon, being un-
der no Obligation to account for the Government of
Jul. Frontinus Successor to *Cerealis*) to fix upon the se-
cond Year of *Julius Agricola*'s Government for this Work,
which *Tacitus* thus describes, ‖ " Sed ubi Æstas adve-
" nit contracto Exercitu — loca Castris ipse capere,
" æstuaria ac sylvas ipse prætentare: & nihil interim
" apud Hostes quietum pati, quo minus subitis Excur-
" sibus popularetur; atque ubi satis terruerat, parcen-
" do rursus irritamenta Pacis ostentare. Quibus rebus
" multæ Civitates quæ in illum diem ex æquo egerant,
" datis Obsidibus iram posuere, & Præsidiis Castellis-
" que circumdatæ, tantâ ratione curâque, ut nulla an-
" tè Britanniæ nova pars illacessita transierit. This ex-
cellent Conduct *Tacitus* further confirms from the Ob-

* *Tacit. Vit. Agric.* 8. † Cap. 17. ‖ Cap. 20.

servation of Others. "Adnotabant periti, non alium "Ducem Opportunitates locorum sapientius legisse, "nullum ab *Agricola* positum Castellum aut vi Ho-"stium expugnatum, aut pactione aut fugâ desertum.

Agricola, this Summer, having quieted so large a Tract, and finished so many Fortresses, it cannot be expected all should be built with the most exquisite Art, sufficient to perpetuate them. I proceed to *Gordian*'s Repairs; whose Historian *Julius Capitolinus* having never once named *Britain*, yet giving so many Hints of the excellent Oeconomy of his Government, under the prudent Administration of his Father in-Law *Misitheus*, I dare not fix this Work till the Third Year of his Reign, He having before been under the Direction of the Eunuchs and Officers of the Court, whom *Capitolinus* represents, in *Misitheus* his Letter to *Gordian*, to have prostituted all Employments to their own Covetousness and mercenary Creatures.

Durham, July 5.
1717.

IV. *A New*

IV. *A new* Genus *of* PLANTS, *call'd* Araliastrum, *of which the famous* Nin-zin *or* Ginseng *of the* Chineses, *is a* Species. *Communicated by Mr.* Vaillant *Prædemonstrator at the Royal Garden at* Paris, *to the Learned Dr.* Will. Sherrard, *LL. D. late Consul at* Smyrna, *and by him to the* Royal Society.

ARaliastrum is a *Genus* of Plants, whose *Flower* A * is complete †, regular, polypetalous, and hermaphrodite, standing on the *Ovary* B. The *Ovary*, which is crown'd by a *Calyx* cut into several Parts, becomes a Berry D, in which are, for the most part, two flat Seeds, like a Semicircle, which both together represent a sort of a Heart. Add to this the Stalk, which is single, ending in an Umbel of which each Ray bears but one Flower. Above the Middle of the Stalk come out several *Pedicles*, (as on that of the *Anemone*) on the Extremities of which grow several Leaves like Rays, or like an open Hand.

The Species *of this* Genus *are*,

1. *Araliastrum Quinquefolii folio, majus, Nin-zin vocatum D.* Sarrazin. *Gin-seng.* Des lettres edifiantes & curieuses, *Tom.* X. *pag:* 172:
2. *Araliastrum Quinquefolii folio, minus.* D. Sarrazin. *Plantula Marilandica, foliis in summo caule ternis, quorum unumquodque quinquefariam dividitur, circa margines serratis.* N° 36. Raii Hist. III. 658.

* *vid.* ARALIA *Inst. rei herb.* Tab. 154.
† Complete, *that is to say, that has a* Calyx.

3. *Araliastrum Fragariæ folio, minus.* D. Vaillant. *Nasturtium Marianum Anemones sylvaticæ foliis, enneaphyllon, floribus exiguis.* Pluk. Mantiss. 135. Tab. 435. Fig. 7.

To shew wherein *Araliastrum* differs from *Aralia*, (from whence it takes its Name) 'tis convenient to give allso the Character of this last *Genus*, such as Mr. *Vaillant* establish'd it, in his Demonstrations of the Year 1717.

Aralia * is altogether like the *Araliastrum*, as to the Structure and Situation of its Flower; but its Berry consists of Five Seeds plac'd round an *Axis*. Moreover, its Leaves are branched, almost like those of *Angelica*; and its Stalks (which in some *Species* are naked, and in others have Leaves set alternately) bear each several Umbels at their top, in the Form of a Bunch of Grapes.

The Species *of* Aralia, *are*

1. *Aralia caule aphyllo, radice repente.* D. Sarrazin. *Christophoriana Virginiana Zarza radicibus surculosis & fungosis, Sarsaparilla nostratibus dicta.* Pluk. Almag. 98. Tab. 238. Fig. 5. *Zarsaparilla Virginiensibus nostratibus dicta, lobatis umbellifera foliis, Americana.* Ejusd. Almag 396.
2. *Aralia caule folioso lævi,* D. Sarazzin. *Aralia Canadensis.* Inst. rei Herb 300.
3. *Aralia caule folioso & hispido.* D. Sarazzin.
4. *Aralia arborescens spinosa,* D. Vaillant. *Angelica arborescens, spinosa, seu Arbor Indica, Fraxini folio, cortice spinoso* Raii Hist. II. 1798. *Christophoriana arbor aculeata Virginiensis* Pluk. Almag. 98. Tab 20.

* vid. *Inst. rei Herb.* 300. Tab. 154.

All

All the *Species* of these two *Genera*, except the last of each of them, are common in *Canada*, whence Mr. *Sarrazin*, Counsellor in the upper Council, Physician to his Majesty, and Correspondent of the Royal Academy of Sciences, sent them to the Royal Garden first in 1700.

The Inhabitants of that Colony, and those of *Virginia*, call the first *Species* of *Aralia* by the Name of *Sarsaparilla*, because its Roots have almost the same Figure and Vertues.

Mr. *Sarrazin* writes, that he had a Patient who had been cured of an *Anasarca*, about two Years before, by the use of a Drink made of these Roots. That able Physician assures us also, that the Roots of the second *Species*, well boyl'd and apply'd by way of *Cataplasme*, are very excellent for the curing of old Ulcers; as also the Decoction of them, with which they bath and syringe the Wounds. He does not at all doubt, but the Virtues of the third *Species* (which I shall briefly describe) are the same with those of the second.

Its Roots creep, and send forth Stalks, which rise commonly to the Height of a Foot and half, and sometimes to two Foot; the bottom part of them is rough, with reddish, stiff and prickling Hairs. These Stalks are set from the Bottom to almost the Top (which are divided successively into several naked Branches charg'd with Umbels) with branch'd alternate Leaves, almost like those of *Podagraria hirsuta Angelicæ folio & odore* D. Vaillant; which Plant is grav'd in the second Tome of *Boccone's Musæum*, by the Name of *Cerefolium rugoso Angelicæ folio, Aromaticum.* Tab. 19. and in *Rivini* by that of *Myrrhis folio Podagrariæ*.

See the Account of the Chinese *Gin-seng*, in Phil. Transact. *Anni* 1713. p. 237.

V. *Extract*

V. *Extract of a Letter of Mr.* Edw. Berkeley *from* Naples, *giving several curious Observations and Remarks on the Eruptions of Fire and Smoak from Mount* VESUVIO. *Communicated by Dr.* John Arbuthnot, *M. D. and* R. S. S.

A*Pril* 17. 1717. with much Difficulty I reach'd the top of Mount *Vesuvius* in which I saw a vast Aperture full of Smoak, which hinder'd the seeing its Depth and Figure. I heard within that horrid Gulf certain odd Sounds, which seem'd to proceed from the Belly of the Mountain; a sort of Murmuring, Sighing, Throbbing, Churning, Dashing (as it were) of Waves, and between whiles a Noise like that of Thunder or Cannon, which was constantly attended with a Clattering, like that of Tiles falling from the Tops of Houses on the Streets. Sometimes, as the Wind changed, the Smoak grew thinner, discovering a very ruddy Flame, and the Jaws of the Pan or *Crater*, streak'd with Red, and several Shades of Yellow. After an Hour's stay, the Smoak, being moved by the Wind, gave us short and partial Prospects of the great Hollow, in the flat Bottom of which I could discern two Furnaces almost contiguous; that on the Left, seeming about three Yards in Diameter, glow'd with red Flame, and threw up red-hot Stones with a hideous Noise, which, as they fell back, caus'd the fore-mentioned Clattering. *May* 8. in the Morning, I ascended to the Top of *Vesuvius* a second time, and found a different face of things. The Smoak ascending upright, gave

gave a full Profpect of the *Crater*, which, as I could judge, is about a Mile in Circumference, and an Hundred Yards deep. A conical Mount had been formed fince my laft Vifit, in the middle of the Bottom. This Mount I could fee was made of the Stones thrown up and fallen back again into the *Crater*. In this new Hill remained the two Mouths or Furnaces already mention'd; that on our Left Hand was in the *Vertex* of the Hill which it had formed round it, and raged more violently than before, throwing up every three or four Minutes, with a dreadful Bellowing, a vaft Number of red-hot Stones, fometimes in appearance above a Thoufand, and at leaft 300 Foot higher than my Head as I ftood upon the Brink. But there being little or no Wind, they fell back perpendicularly into the *Crater*, increafing the conical Hill. The other Mouth to the Right was lower in the fide of the fame new formed Hill. I cou'd difcern it to be fill'd with red-hot liquid Matter, like that in the Furnace of a Glafs-Houfe, which raged and wrought as the Waves of the Sea, caufing a fhort abrupt Noife like what may be imagin'd to proceed from a Sea of Quickfilver dafhing among uneven Rocks. This Stuff wou'd fometimes fpew over and run down the convex fide of the conical Hill, and appearing at firft red-hot, it changed Colour, and harden'd as it cool'd, fhewing the firft Rudiments of an Eruption, or, if I may fo fay, an Eruption in Miniature. Had the Wind driven in our Faces, we had been in no fmall Danger of ftifling by the fulphurous Smoak, or being knock'd on the Head by Lumps of molten Minerals, which we faw had fometimes fallen on the Brink of the *Crater*, upon thofe Shots from the Gulf at Bottom. But as the Wind was favourable, I had an opportunity to furvey this odd Scene for above an Hour and a half

together;

together; during which it was very observable, that all the Volleys of Smoak, Flame, and burning Stones, came only out of the Hole to our Left, while the liquid Stuff in the other Mouth wrought and overflow'd as hath been already described. *June* 5. After a horrid Noise, the Mountain was seen at *Naples* to spew a little out of the *Crater*. The same continu'd the 6th. The 7th, nothing was observ'd till within two Hours of Night, when it began a hideous bellowing, which continued all that Night and the next Day till Noon, causing the Windows, and as some affirm, the very Houses in *Naples* to shake. From that time it spew'd vast Quantities of molten Stuff to the South, which stream'd down the side of the Mountain, like a great Pot boyling over. This Evening I return'd from a Voyage thro' *Apulia*, and was surprised, passing by the North side of the Mountain, to see a great Quantity of ruddy Smoak lie along a huge Tract of Sky over the River of molten Stuff, which was it self out of sight. The 9th, *Vesuvius* raged less violently: that Night we saw from *Naples*, a Column of Fire shoot between whiles out of its Summit. The 10th, when we thought all wou'd have been over, the Mountain grew very outragious again, roaring and groaning most dreadfully. You cannot form a juster Idea of this Noise in the most violent Fits of it, than by imagining a mix'd Sound made up of the raging of a Tempest, the Murmur of a troubled Sea, and the Roaring of Thunder and Artillery, confused all together. It was very terrible as we heard it in the further End of *Naples*, at the Distance of above twelve Miles. This moved my Curiosity to approach the Mountain. Three or four of us got into a Boat, and were set ashoar at *Torre del Greco*, a Town situate at the Foot of *Vesuvius* to the South-

South-West, whence we rode four or five Miles before we came to the burning River, which was about Mid-Night. The Roaring of the *Volcano* grew exceeding loud and horrible as we approach'd. I observed a Mixture of Colours in the Cloud over the *Crater*, green, yellow, red and blue; there was likewise a ruddy dismal Light in the Air over that Tract of Land where the burning River flowed; Ashes continually shower'd on us all the way from the Sea-Coast. All which Circumstances, set off and augmented by the Horror and Silence of the Night, made a Scene the most uncommon and astonishing I ever saw; which grew still more extraordinary as we came nearer the Stream. Imagine a vast Torrent of liquid Fire rolling from the Top down the Side of the Mountain, and with irresistible Fury bearing down and consuming Vines, Olives, Fig-trees, Houses, in a word, every thing that stood in its way. This mighty Flood divided into different Channels, according to the Inequalities of the Mountain. The largest Stream seem'd half a Mile broad at least, and five Miles long. The Nature and Consistence of these burning Torrents hath been described, with so much Exactness and Truth, by *Borellus*, in his Latin Treatise of *Mount Ætna*, that I need say nothing of it. I walked so far before my Companions, up the Mountain along the side of the River of Fire, that I was oblig'd to retire in great haste, the sulphureous Steam having surpriz'd me, and almost taken away my Breath. During our Return, which was about Three-a-Clock in the Morning, we constantly heard the Murmur and Groaning of the Mountain, which between whiles would burst out into louder Peals, throwing up huge Spouts of Fire and burning Stones, which falling down again resembled the Stars in our Rockets. Sometimes I ob-

serv'd two, at others three distinct Columns of Flame, and sometimes one vast one that seem'd to fill the whole *Crater*. These burning Columns, and the fiery Stones seem'd to be shot a 1000 Foot perpendicular above the Summit of the *Volcano*. The 11th at Night, I observ'd it, from a Terrass in *Naples*, to throw up incessantly a vast Body of Fire and great Stones to a surprising Height. The 12th in the Morning, it darken'd the Sun with Ashes and Smoak, causing a sort of Eclipse. Horrid Bellowings this and the foregoing Day were heard at *Naples*, whither part of the Ashes also reached. At Night I observed it throw up Flame, as on the 11th. On the 13th, the Wind changing, we saw a Pillar of black Smoak shot upright to a prodigious Height. At Night I observed the Mount cast up Fire as before, tho' not so distinctly because of the Smoak. The 14th, A thick black Cloud hid the Mountain from *Naples*. The 15th, in the Morning, the Court and Walls of our House in *Naples* were cover'd with Ashes. In the Evening, Flame appear'd on the Mountain thro' the Cloud. The 16th, the Smoak was driven by a Westerly Wind from the Town to the opposite side of the Mountain. The 17th, the Smoak appear'd much diminish'd, fat and greasy. The 18th, the whole Appearance ended, the Mountain remaining perfectly quiet without any visible Smoak or Flame. A Gentleman of my Acquaintance, whose Window look'd toward *Vesuvius*, assur'd me, that he observ'd this Night several Flashes, as it were of Lightening, issue out of the Mouth of the *Volcano*. It is not worth while to trouble you with the Conjectures I have formed concerning the Cause of these Phænomena, from what I observed in the *Lacus Amsancti*, the *Solfatara*, &c. as well as in Mount *Vesuvius*. One thing I may venture to say, that I saw the fluid Matter rise out of the Centre

of

of the Bottom of the *Crater*, out of the very middle of the Mountain, contrary to what *Borellus* imagines, whose Method of explaining the Eruption of a *Volcano* by an inflexed Syphon, and the Rules of *Hydrostaticks*, is likewise inconsistent with the Torrent's flowing down from the very *Vertex* of the Mountain. I have not seen the *Crater* since the Eruption, but design to visit it again before I leave *Naples*. I doubt there is nothing in this worth shewing the Society; as to that, you will use your Discretion.

E. Berkeley.

VI. *An Account of an extraordinary* TUMOUR *or* WEN *lately cut off the Cheek of a Person in* Scotland. *Communicated to the Royal Society by Dr.* Thomas Bower, *M. D.* and *F. R. S.*

IT is generally acknowledg'd, that the exact Observation of internal Diseases, and the faithful Accounts of external Tumours, and extraordinary Cases in Chirurgery, have contributed very much to the Advancement of Medicine. *Hippocrates* and *Galen*, and other ancient Fathers of Medicine, have set us fair Copies of this; and the Moderns, happily following their Footsteps, have illustrated this Matter by many curious Observations and Reflections. The Royal Societies and Colleges of *Virtuosi*, that are now over all *Europe*, have taken much pains in this Affair, and have given us many Instances and Examples of Extraordinary Cases in Medicine, which are of great use to all the Practisers of Physick and Chirurgery. According to these laudable Examples I shall, for the Satisfaction

of the Curious and Ingenious, give a true and faithful Account of an extraordinary Excreſcence cut off from the Cheek of a Man, which weighed Nineteen Pounds, and the Patient entirely recover'd in few Weeks time. I never before ſaw the like, nor never read of it in any Author, tho' I have conſulted many on the Head. This Excreſcence is preſerved among the Rarities of the College of Phyſicians at *Edinburgh.* The Phyſician concern'd in this Affair, was Dr. *Alexander Ruſſel* of *Elgine*, a learned and ingenious Man, and the Operator was the ingenious Mr. *George Gordon* of *Keith*, from whom I had the following Information.

Alexander Palmer, of the Pariſh of *Keith*, in the County of *Bamff*, in the North of *Scotland*, now about Fifty Four Years of Age, obſerved, when about Twenty Seven, a little hard Swelling in the Muſcle of the lower Jaw on the Left Side, without any Hurt or manifeſt Occaſion, which at firſt went on ſlowly, but afterwards it proceeded more quickly, and the older it grew, it ſtill came on the faſter; until it increaſed to a prodigious Bulk and Weight: From the firſt Appearance of this Tumour to the total Exciſion of it, there were about Twenty Seven Years. He had exceſſive Pains and Uneaſineſs in it, and at laſt it mightily extenuated and emaciated him, who was otherwiſe a ſtrong and robuſt Man.

This Excreſcence was of the natural Colour of the Skin, and ſeem'd to be an *Atheroma*, being a glandulous Subſtance with ſeveral big Blood-Veſſels in it, and had Hair growing on it, as on the other Parts of the Body, as may yet be ſeen. It was almoſt round and very hard, and was as ſenſible as any other Part of the Body; for, when the poor Man was working in the Fields,

Fields, ſome ſix or ſeven Years ago, he accidentally made a great Gaſh or Wound in it with a ſharp Iron, which was very painful, but was cured by a Surgeon, after the manner of an ordinary Wound; the Cicatrice is ſtill to be ſeen in it.

This Excreſcence having grown ſo big, was attach'd to the Muſcle under the Left Eye, call'd *Obliquus minor* or *inferior*, to the Ear and its Muſcles, and to the Muſcle of the lower Jaw, named *Deprimens*. By reaſon of its great Bulk and Weight, it could not hang down freely without ſome Support, therefore it reſted on the top of the Shoulder, which made a conſiderable Dimple in it, that is yet very obſervable; beſides, it was holden up by the Man's Hand in the Day-time, and laid on a Pillow in the Night-ſeaſon.

Some three or four Days before the total Exciſion was made, the Patient obſerved this Tumour begin to mortify at the lower end, which made him ſo uneaſy, that he took a Knife and cut off a good part of it. This occaſion'd a great Hæmorrhage; ſo that he reckon'd there was loſt a *Scots* Pint or four Pounds of Blood, before it could be ſtopt. The Patient, after ſo great Trouble and Pain, at laſt applied himſelf to Mr *Gordon*, Surgeon of the Place, who made a total Extirpation of it, on the *19th* of *January*, 1717.

He made a cloſe Ligature, taking in the Baſis of the Excreſcence, and all the looſe Skin, and contracting it as much as poſſible, he cut it entirely off with a ſharp Raſour. There guſh'd out of the Excreſcence, after it was cut off, and was lying on the Ground, as near as could be gueſs'd, two Pounds of Blood; for it was nouriſh'd by ſeveral large Blood-Veſſels, perhaps by ſome Branches of the Carotide Artery much inlarged, and other Blood-Veſſels coming from

from the Ear, and the Muscles of the Eye and lower Jaw abovementioned. When Mr. *Gordon* brought it to us, which was full three Months after it was cut off, we cut off with a Knife about a quarter of an Inch broad of the Basis of it; and in that small Space we observed four big Blood Vessels. The Basis, as it now appears, is five Inches Diameter, which seems too large for the whole side of the Face: So that after the Exsection, I think the loose Skin has turn'd backwards, which may make the Basis now appear so big.

After all this Blood was lost, the Excrescence was weighed, and was full Nineteen Pound Weight; so that before his own Incision and this Operation, it behoved to be several Pounds heavier, which is a most prodigious Weight to be depending on such a place. This Tumour was of a Spheroidical Figure, and when measured, was Thirty four Inches about by the longest way, and Twenty eight by the broadest.

The Hæmorrhage, which was great, was stopped by the Vitriolic Powders and other Astringents, and the ordinary Dressing was used. So this great Cure was completed in six Weeks time, and the Patient entirely recover'd, and goes about his Business, to the great Admiration and Astonishment of every body. The Lid of his Left Eye continues still downwards a little, as does that same side of the Mouth, which was occasion'd by the great Weight depending on that side of the Face; but it may be expected they may come again to their right Posture; for the Head, at first after cutting, enclined much to the Right side, by reason of the great Weight on the Left Cheek having been removed; but it now begins to stand upright, since he is perfectly recovored. Tho' the Skin, and even a deal of the musculous part of the Cheek and lower

lower Jaw was cut away, yet, according to the Information I have from Mr. *Gordon* the Operator, it is grown up again, and is of the ordinary Colour of the Skin, and like the other ſide of the Face; ſo that there grows Hair on that ſide of the Face as well as on the other, which he ordinarily ſhaves; and this is as ſurprizing as any thing in the whole Affair.

I have given a true and plain Account of this extraordinary Caſe from certain Information; I have contented my ſelf to relate only Matters of Fact, without making any Obſervations or Reflections on it; for I leave it to the Philoſophers and *Virtuoſi* to make their own Reaſonings and Refinements as ſeems beſt to themſelves.

VII. *An Account of an Experiment to prove an interſpers'd* Vacuum; *or to ſhew that all Places are not equaly full.*

THIS Experiment was *made before the* KING, and HER Royal Highneſs the Princeſs of *Wales*, at *Hampton-Court, in the Month of* September 1717. *afterwards before the* ROYAL-SOCIETY, *on* Thurſday, December 5. 1717. *and ſince that, in* Channel-Rowe, Weſtminſter, *before ſome Members of the* Royal-Society, *by* J. T. Deſaguliers, *M. A. F. R. S.* as follows:

Having had the Honour to make ſome Experiments laſt Year before his Majeſty and their *Royal Highneſſes* the *Prince and Princeſs of* Wales; among others, I ſhew'd that of a Guinea and a Piece of fine Paper; then of a Guinea and a Feather dropt together from the top of an exhauſted Glaſs Receiver about 20 Inches high; both which

which fell to the Bottom at the same Instant of Time: Now since the chief *Resistance* of a *Medium* (and indeed almost all of it) depends upon the † Quantity of its Matter; therefore this Diminution of Resistance, whereby the Feather fell as soon as the Guinea, shew'd a Diminution of the Quantity of Matter, and consequently prov'd an *interspers'd Vacuum*. Some time after this, I was inform'd that some *Plenists* here in *England* objected against the Shortness of the Glass-Receiver; as if the Difference of Time in the Fall of the two Bodies, which they affirm'd to be real, could not be perceiv'd in such a Glass; and that some Philosophers from abroad affirm'd that in a Glass Receiver 7 or 8 Foot long, there would be such a manifest Difference in the Time of the Fall of the said Bodies, as to shew this Experiment no Proof of a *Vacuum*; though at the same time, some of the Objectors well knew that there could be no Receivers of half that Length made at the Glass House, and therefore thought the Experiment impracticable. To obviate this, I contriv'd a Machine for the purpose, which consisted of a strong wooden Frame 15 Foot high, that held the Air-Pump and four Cylindric Glass-Receivers of about two Foot long each, and six Inches Diameter: Of these, having set the first upon the Air-Pump Plate, I laid on the Top of it a Brass-Plate of seven Inches Diameter, that had an oil'd Leather fix'd to it above and below, with an Hole through the middle, of between four and five Inches Diameter; then on that Plate I set the next Receiver, with a like Plate at top; and after the same manner fix'd the other two with Plates between them: The upper Receiver being a little narrower at the Neck,

† See Sir *If. Newton's Principia*, Book II. Prop. 40.

went

went into the Hole of a Board, whereby it was screw'd down pretty hard on the other Glasses, and fix'd to the whole Machine. On the top of this upper Receiver I laid the Brass Plate, wet Leather, and Brass Springs which contain'd the Bodies to be dropt.

Having acquainted His Majesty with what I had prepar'd, he order'd me to shew him the Experiment with this long compounded Receiver, at *Hampton-Court*; and when I made it before him and her Royal Highness, he was pleas'd (by pulling down a String fix'd to a Leaver at the top of the Machine) to let loose the Bodies himself, to see that the Experiment was fair.

When the Receiver was full of common Air before Pumping, the Guinea came to the Bottom, just as the Paper was about the Middle of the second Glass; but when the Receiver was exhausted, the Guinea and Paper came to the Bottom precisely in the same Instant of Time.

Upon my giving an account of the Success of this Experiment to the *Royal Society*, they order'd me to repeat it before them on the 5th Day of *December* 1717, being the *Thursday* next after the Yearly Meeting for choosing Officers on St. *Andrew*'s Day; on which Day an annual Experiment is appointed to be made, in Conformity to the Will of their late worthy Member and Benefactor Sir *Godfrey Copley*.

I made the Experiment first with two of the Receivers; then with all the four; dropping a Guinea and a small Piece of Paper together; and the Success answer'd Expectation: But not being willing to try with a Down-Feather, because I fear'd the Air might Insinuate between some of the Glasses, by reason the Number of Persons present shak'd the Room, the Society order'd me to make the Experiment at home before one or more of their Members.

Martin Foulkes, Esq; a very ingenious Member of the Society, did me the favour to be present when I made the Experiment at my House; where we made four Tryals in the following manner.

The whole Machine being fix'd, as above mention'd, we first let fall a Guinea and two Papers; the one placed over, and the other under it, (before any Air was pump'd out) and the Guinea came to the Bottom when the Papers were only in the Middle of the second Glass from the Top. Then having laid a Feather on the Brass-Springs close by the Guinea, we let them loose both together; and the Feather was fallen only down to the 4th part of the Length of the first Glass or $\frac{1}{16}$ of the whole Distance, when the Guinea was got down to the Bottom of the Receiver. We then laid two Papers and two Feathers, one of each under, and the other over the Guinea between the Springs; and having drawn out so much of the Air as to bring up the Mercury in the Gage-Tube within a quarter of an Inch of the greatest Height to which it could be then rais'd by the Pressure of the external Air, we caus'd the Bodies to fall all at once: And tho' the Papers came down to the Bottom at the same time as the Guinea, yet the Feathers, being much lighter, wanted about three Inches. But at last, having laid the Papers, Feathers, and Guinea, as before, we pump'd out all the Air, and then the Feathers, as well as the Papers, came to the Bottom of the Receiver at the same instant of time as the Guinea.

VIII. *An*

VIII. *An Account of a small* Telescopical Comet *seen at* London *on the* 10th *of* June 1717. *by* Edm. Halley, *LL.D.* R. Soc. Secr.

THAT the Number of Comets traversing our Solar System is much greater than some, on account of the late rareness of their Appearance, have supposed it, may be collected from several small ones which have within few Years been described in the Memoirs of the *French* Royal Academy of Sciences; those diligent Observers assuring us that they discover'd one in *Sept.* 1698. another in *Febr.* 1699. a third in *April* 1702: and again a fourth in *Novemb.* 1707. none of which, as far as I can learn, were ever seen in *England*; all of them having been very obscure and without Tails, by means whereof Comets usually first shew themselves. And besides these, two other Comets with remarkably long Tails, the one in *Novemb.* 1689. the other in *Febr.* 1702. past by unobservable in these our Northern Climats, they having great South Latitude, and their Motions directed toward that Pole. Hence we may justly conclude that the Returns of Comets are much more frequent than is vulgarly reckoned, and that it is only contingent that for these 35 Years no one of them has been seen and observed by our Astronomers.

But there may be still a much greater Number of these Bodies, which by reason of their Smallness and Distance are wholly invisible to the naked Eye; so that unless Chance do direct the Telescope of a proper Observer, almost to the very Points where they are (against which there are immense Odds) it will not be possible

 for

for them to be discovered: And that this is not barely a Conjecture, take the following Instance.

On *Monday*, *June* 10. in the Evening, the Sky being very serene and calm, I was desirous to take a View of the Disk of *Mars* (then very near the *Earth*, and appearing very glorious) to see if I could distinguish, in my 24 Foot Telescope, the Spots said to be seen on him. Directing my Tube for that purpose, I accidentally fell upon a small whitish Appearance near the Planet, resembling in all respects such a *Nebula* as I lately described in *Philos. Transact.* N° 347. but smaller. It seemed to emit from its upper part a very short kind of Radiation directed towards the East, but Northerly withal; which, considering its Situation, was nearly towards the Point opposite to the Sun. The great Light of the Moon, then very near it and also near the Full, hinder'd this *Phænomenon* from being more distinctly seen; but its Place in the Heavens was sufficiently ascertained from the Neighbourhood of *Mars*, from whom it was but about half a Degree distant towards the Southwest, the difference of Latitude being somewhat more than that of Longitude; and *Mars* being at that time in ♐ 17°. 30′ with 3°. 48′ South Latitude, I concluded the place thereof in ♐ 17°. 12′ with 4°. 12′. Lat. South, or thereabouts; the which may yet be more securely determined by help of two small fixt Stars I found near it, the more northerly of which I judged to have the same Latitude with it, and to follow it at about the Distance of six Minutes; the other Star was about four Minutes more southerly than the former, and about one Minute in consequence thereof; the Angle at the Northern Star was a little obtuse, as of about 100 Degrees, and the Distance of our *Nebula* from it *sesquialter* to the Distance of the two Stars, or rather a little more. The Reverend Mr. *Moses Williams*, Mr. *Alban Thomas*, and my self, contem-

contemplated this Appearance for above an Hour, *viz.* from $10\frac{1}{2}^h$ to near twelve, and we could not be deceiv'd as to its Reality; but the Slowneſs of its Motion made us at that time conclude that it had none, and that it was rather a *Nebula* than a *Comet.*

However, ſuſpecting that it might have ſome Motion, I attended the next Night, *June* 11th, at the ſame Hours and in the ſame Company, when with ſome Difficulty by reaſon of the Thickneſs of the Air, we found the two little Stars, but the *Nebula* could not at that time be ſeen, which we then imputed to the want of a clearer Sky. But on *Saturday, June* 15. the Moon being abſent, and the Air perfectly clear, we had again a diſtinct View of the two Stars, with an entire Evidence that there remained no Footſtep or Sign of it, in the place where we had firſt ſeen this *Phænomenon*, which we therefore now found to be a Comet, and that being far without the Orb of the Earth, and in it ſelf a very ſmall Body, it appeared only like a little Speck of a Cloud, ſuch as would ſcarce have been diſcerned in an ordinary Teleſcope, much leſs by the naked Eye.

IX. *An Account of Books*, I. Joannis Poleni in Gymnaſio Patavino Phil. Ord. Prof. & Scient. Societatum Regalium, quæ Londini & Berolini ſunt, Sodalis, *De Motu Aquæ mixto*, Libri duo, &c. 4to. Patavii 1717.

THE Subject here treated of not having hitherto fallen under the Conſideration of Mathematical Writers, the Learned Author is obliged to make uſe of ſeveral Terms, which are either wholly new, or at leaſt

least are apply'd in a sense somewhat different from their common Acceptation; for which reason he begins his Work with a Sett of Definitions.

Aqua mortua, or a dead Water. is that whose Surface being every where equally distant from the *Centrum gravium*, no part of it can descend any lower, without forcing some other upward, and consequently the Whole is without Motion.

Aqua viva, or a running Water, is that which is put into motion by the Pressure of the Incumbent Water, and whose Motion is opposed by no other Water lying in its way.

The motion of a running Water is call'd *Motus simplex*, or the simple Motion.

If a running Water moving over the Surface of a dead Water, do, by its Pressure communicate part of its Motion to the dead Water; the compound Motion with which the whole Body of the Water flows, is called *Motus mixtus*, or the mixt Motion.

If a Water at different Depths from the Surface run with different Velocities, the *mean Velocity* is that, which being the same at all Depths, will discharge the same Quantity of Water.

Next follows a short History of the Original, and Progress of the Doctrine of running Waters, the Invention of which our Author justly asserts to the Learned *Castellus*, and defends him against *Fabretti*, who has maintain'd that *Castellus*'s fundamental Proposition of the Quantity discharged being *cæteris paribus* in proportion to the Velocity, was known, and publickly taken notice of before him by *Frontinus*.

The Author allows *Castellus* to have been mistaken in determining the Velocity of Water running out at the bottom of a Vessel, he having asserted that Velocity to be as the Depth of the Water.

Three

Three Years after *Castellus's* Book came out, this Mistake was corrected by the famous *Torricellius*, who was the first that maintain'd, that the Velocity of the Water running out was in a subduplicate *Ratio* of the Depth, but gave no Demonstration of it.

This Proposition, says our Author, was confirm'd by the Experiments of *Maggiotti*, *Mariotte*, and *Guglielmini*, and has since been demonstrated by Mr. *Varignon*, by *Herman* in his *Phoronomia*, and *John Bernoulli*, as reported by *Herman* in the *Acta Lipsiensia*.

Here it may not be improper to take notice, that the Demonstrations of those three Learned Persons are all grounded upon this Supposition, that the Water running out from the Hole is prest upon by the Column of Water incumbent upon it, which may easily be demonstrated to be a Mistake. Likewise, if their Demonstrations be just, it will follow, that the first Drops of Water, which issue out from the Hole, must run with the same Velocity, as after the Water has been running some time; the Contrary of which appears to be true in Fact by the Experiments of the famous Mr. *Mariotte*.

The Author might have found a juster Account of this matter in the Writings of a Great Man, whom he cites on another Occasion; the second Edition of whose Book was come out some time before *Herman* publish'd either of those Demonstrations, and had been seen by him, as appears by his quoting it frequently, and mentioning the Difference in this very Particular between the first and second Edition.

Our Author goes on to consider the simple Motion of Water running out by a Section perpendicular to the Horizon, in the side of a Receptacle, which is always entertain'd at the same Height. He shews, that the Velocities, with which the Water issues out at different

Depths,

Depths, being as the Roots of thoſe reſpective Depths, may be repreſented by the Ordinates of a *Parabola*, whoſe Axis repreſents the entire Depth of the Water. Conſequently, ſince the Quantities of Water, running out at different Depths, are as thoſe Velocities, they likewiſe may be repreſented by the ſame Ordinates, and the Quantity of Water diſcharg'd from the whole Section, will be repreſented by the Parabolick Space; and the mean Velocity by that ſame Space divided by the Abſciſſe.

The Times, being as the Quantities of Water diſcharged, may be repreſented in the ſame manner as thoſe Quantities.

Hence he derives his general Theorem, That the Quantities of Water diſcharged, are in a *ratio* compounded of the ſeſquiplicate *ratio* of the Depths of the Water, the *ratio* of the Breadths of the Section, and of that of the times of the Efflux.

The Author proceeds now to the mixt Motion of Water; in order to diſcover the Nature of which he has made ſome curious Experiments, after the following manner:

A large cylindrical Veſſel, with a perpendicular Section through the ſide of it, was placed upright in a dead Water; ſo that the bottom of the Veſſel was a conſiderable Depth below the Surface of the Water; and the Veſſel was kept immovable in this ſituation.

Above this was fixt another Veſſel, full of Water, whoſe Bottom was pierced with 16 Holes, exactly round, and of the ſame Bore, and ſo order'd, as to be open'd, or ſtopt at pleaſure. The Water in this Veſſel was always kept at the ſame Height, by means of a third Veſſel, which ſupply'd the Water, as faſt as it ran out at the round Holes in the Bottom; and a large Aperture, in the ſide of the ſecond Veſſel near the Top, prevented

prevented the Water in it from exceeding the due Height. To break the Force of the Water running into the two lowermost Vessels, they were each of them divided by a Board, placed perpendicular, but not reaching the Bottom, which separated the Part where the Water came in, from that where it went out.

The *Apparatus* being thus fixt, three of the round Holes in the Bottom of the second Vessel were unstopt, to let the Water run into the lower Vessel. Where not running out at the Section in the side, so fast as it came in from above, it rose to a considerable Height above the Surface of the dead Water; after which, the Efflux of the Water becoming equal to the Influx, it rose no higher.

In other Tryals the Water being suffer'd to run from 6, from 9, 12, and 15 of the round Holes, the Water rose successively to greater Heights, before the Section discharged it as fast, as it came in.

The Experiment being repeated with opening other Numbers of the round Holes, with Sections of different Breadths, and at different Depths of the dead Water, the several Heights, to which the Water rose in the Vessel, were carefully observ'd and set down.

Other Experiments were made by placing the lower Vessel on dry Ground, and the several Heights to which the Water rose in the Vessel, according as different Quantities were suffer'd to run in, were likewise observ'd, and found agreeable to the Heights deduced by Calculation from the general Theorem above-mention'd, concerning the simple Motion of Water.

The Learned Author comes now to apply these Experiments, in order to discover the Theory of mixt motion, to which end he lays down these two Hypotheses.

First, he supposes, that the Velocity of the running Water is every where in a subduplicate *ratio* of the Depth, and consequently the Quantities discharged may be represented by the Parabolick Spaces, just as in the case of the simple Motion of Water.

Secondly, that the Velocity of the dead Water, is the same at all Depths, and equal to the greatest Velocity of the running Water. Wherefore the Quantity of dead Water discharged may be represented by a Rectangle, whose Height represents the Depth of the stagnant Water, and whose Base is the greatest Ordinate of the Parabolick Space abovementioned.

Having thus contrived a way of representing the Quantities of Water discharged by the mixt Motion, as had been done before for the simple Motion of Water, our Author observes that the Velocities of the Water issuing out at different Depths, and consequently the Parabolick Spaces representing the Quantities of Water expended, must be less in the mixt, than in the simple Motion.

In order therefore to find a general Rule for determining the Proportion between the Parabolick Spaces, which represent the Quantities discharged by the mixt and simple Motion, or between the Parameters of those *Parabolas*, he draws some Observations from the foregoing Experiments, by the help of which he hopes such a Rule may be found out.

First, he observes that, if the Depth of the running Water continue unchanged, a greater Depth of dead Water requires a less Parameter.

Secondly, That this Parameter does not decrease in so great a Proportion, as the Depth of the Water increases

Thirdly, That, if the Depth of the dead Water decrease, or the Depth of the running Water increase in such

such manner, that the latter becomes infinitely great in proportion to the former, then the Parameter of the mixt Motion must become equal to that of the simple Motion.

Fourthly, That, if the Depth of the dead Water become infinitely great in comparison of the Depth of the running Water, the Parameter of the mixt Motion vanishes, or becomes equal to nothing.

The Rule, therefore, which is to be found, ought to agree with all these Observations, and besides must produce the same Quantities of Water by Calculation, as were found by Experiment to answer to the several Depths of running and dead Water, in the above mention'd Tryals.

Upon this Foundation the Learned Author proceeds, in a tentative Method, to find his Rule, and having discover'd it, he shews by Calculation, that it answers all the Conditions before requir'd.

This Rule is exprest in a pretty high Equation, which, besides other Operations, requires the extracting the Root of the sixth Power.

From this Equation is derived another, serving to find either the Quantity of Water discharg'd, the Depth of the running, or that of the dead Water, the other two of them being given; as likewise a third Equation, to find the mean Velocity.

Our Author goes on to shew the Usefulness and Necessity of considering the Doctrine of mixt Motion, in all Questions relating to the Course of Rivers, the Quantities of Water which they discharge, the enlarging or narrowing their Outlets, the scouring and deepening their Channels, and the Motion of the Tides in Harbours. These he illustrates by several Deductions from the Equations above mention'd; to render which of greater Evidence, it were to be wisht, that those Equations

were built upon a more solid Foundation than a tentative *Calculus*; and that Allowance had been made for the Velocity imprest upon the preceding Water in Rivers, by the *impetus* of that which follows, which is omitted by the Author in his Theory, both of mixt and simple Motion.

In the Second Book, this Learned Writer proposes the State of the *Laguna* of *Venice*, as a proper Example, to demonstrate the Usefulness of his new Theory. He considers very minutely the several Causes of choaking up the *Laguna*, examins the Methods proposed by various Authors for scouring and keeping it clear, some of which he rejects as impracticable on account of the Expence, others as useless, or prejudicial; and lastly delivers his own Opinion.

The principal Causes, which he assigns, of filling up the *Laguna*, are the Rivers running into it, and the Sea.

The Rivers, which enter it, arising out of the *Alps*, and running down with great Rapidity, carry with them, especially after Rains, great Quantities of Soil, which is easily suspended in the Water, so long as that Swiftness of Motion continues. But when they come into the *Laguna*, the Water, upon extending it self over that vast Surface, looses almost all its Velocity, and consequently the Soil and Filth which before it carry'd with it, subsides in great Quantities to the Bottom.

The Remedy our Author proposes for this Inconvenience, is either wholly to divert the Course of the Rivers and carry them, by another Way, directly into the Sea; or at least, to secure their Outlets with Sluices, so as to suffer them to pass into the *Laguna*, when their Waters are clear; but after great Rains, when they run foul and turbid, to stop their Passage that way, and let them out by the other Channel into the Sea.

The

The second, principal Cause of choaking up the *Laguna*, is the Sea. Concerning which our Author observes, that the Tide of Flood sets into the *Laguna* from along the Coast of *Istria* and *Friuli*, where it is perpetually washing away the Land in great Quantities, with which, and the Sand which it raises upon high Winds in the Shallows near the Shore, it enters the *Laguna* exceedingly turbid and foul; especially, when the Wind blows hard at South-East, at which times the Tide of Flood is several Hours longer than the Ebb. This occasions very high Tides in the *Laguna*, and a great part of the Water, which enters by the Flood, not being carry'd out by the subsequent Ebb, has the more time to discharge its Soil and Sand in the *Laguna*.

This is an Enemy very hard to deal with, however our Author proposes some Works of strong Piles, and large Stones thrown in between them, to be carried directly forward into the Sea, in order to break the Violence of the Waves, and prevent their washing and carrying away the Land.

He seems likewise to favour a Proposal made by the late famous *Guglielmini*, and some others, to let the Tide enter the *Laguna* by more Passages than it is to go out at, in order to make it run out with a greater Velocity, and thereby scour and deepen the Channels. But he thinks this Contrivance will scarcely perform all that is expected from it; besides that, it will be attended with great Difficulties in making Works, and Flood-gates of a sufficient Strength, to resist the Violence of the Waters.

He occasionally combats the Opinion of *Guglielmini*, and most other Mathematicians who have thought upon the Subject, that in order to give a greater Velocity to the Water of a River, thereby to scour and cleanse the Channel, it is proper to make the Outlet narrower.

narrower. This our Author maintains to be oftner false, than true, and endeavours to shew from his Theorem above-mention'd, that making the Outlet narrower, will frequently cause the *mean Velocity* of the Waters to become less than it was before. But whether a Proposition of such Consequence, and seemingly so well supported by Reason and Experience, ought to be condemn'd upon the Authority of a Theorem founded only upon a tentative Calculation, must be left to the Judgment of the Learned.

II. Apollonii Pergæi *Conicorum Libri Octo*, & Sereni Antissensis *de Sectione Cylindri & Coni Libri duo.* *Fol. Reg.* *E Theatro* Oxon. 1710.

THE worthy Curators of the *Oxford* Press having obliged the Publick with a very elegant Edition of the Works of *Euclid*, *Græco-Latinè*, were pleas'd further to proceed in the laudable Intention of giving the rest of the ancient *Greek Mathematicians* in the same beautiful Form: In this Design they were chiefly animated by the late learned and beneficent *Dean* of *Christ Church*, Dr. *Henry Aldridge*, who pitching upon *Apollonius*, as most proper to succeed *Euclid*, engaged the two *Savilian* Professors to take upon them the Care and Pains of the Edition: Dr. *David Gregory* promising his Assistance as to the first Four Books, which are still extant in *Greek*; and Dr. *Edm. Halley* undertaking to translate the Fifth, Sixth, and Seventh Books out of *Arabick* (in which Language they were only to be found) and to endeavour to restore the Eighth, long since wholly lost. But Dr. *Gregory* soon after dying, the Care of the Whole devolved on Dr. *Halley*, who hath spared no Pains to render the Work complete.

He in his Preface tells us what Helps he had to perfect the Text, That he had the use of two *Greek MSS.* of the first Four Books, one of which was Sir *Henry Savil's*, and is in the *Savilian Study* at *Oxford*, the other is now in the

Royal

Royal Society's *Museum*, having been lately presented them by that skilful Mathematician Mr. *William Jones*, F. R. S. That he had only one Manuscript of *Eutocius*'s Commentary, out of the *Bodley* Library; and two *Greek* Copies, from the *Savilian Study*, of *Pappus*'s Collections, out of whose 7th Book he took the *Lemmata*, which serve as a Comment on the more difficult Places of his Author; and that he was forced to revise and correct the Mistakes and Improprieties of the Latin Translation of *Commandine*.

As to the latter Books, which were only in *Arabick*, he informs us, that he made use of the *Bodley* Transcript of a *Manuscript* that is at *Leyden*, which it self is a late Copy of that ancient *Arabick* Book of the *Conicks*, heretofore *Golius*'s, but since purchased by that great Patron of Universal Learning, *Narcissus* late Primate of *Ireland*, who was pleased to favour him so far as to send over into *England* this Original Book whereby he not only amended several Faults committed by the Copyists in a double Transcription, but was also assured that this *Arabick* Book was a verbal Translation from the Greek; the same Schemes markt with the same Letters, and the whole Context being the same in the first four Books of it, as in the Greek *Apollonius*. This valuable Manuscript, with about 800 others, *Oriental* and *Greek*, has since, by the Donation of that most venerable Prelate, made a noble Accession to the *Bodley* Library, wherein it is now deposited. It appears by an *Epigraphe* at the end, to have been written in the Year of Christ 1303. and to have been a Copy of a Translation of the Conicks, made some Ages before by *Thebit Ben Corah*, but then newly revised by that famous *Persian* Mathematician *Nasir-eddin*, who flourish'd about the middle of our thirteenth Century.

Besides this, the Editor tells us, that on occasion he consulted another *Arabick* Manuscript (heretofore *Ravius*'s) of great Antiquity, being an *Epitome* of the same Books by *Abdolmelec* of *Schiraz*, every where agreeing in the Order and Argument with the former, but abridg'd. So that having had these Helps, he is in hopes that he has so far retrieved those Three Books of *Apollonius*, that the Loss of the *Greek* Text may henceforth be less lamented.

The

The Eighth Book of these Conicks, was wanting in the *Greek* Copies even before the Traduction of them into *Arabick* by *Thebit*: But it having been observ'd that there was a very near relation between the Arguments of the VIIth and VIIIth Books, for that the same *Lemmata* of *Pappus* were common to them both, which are different to all the rest, it seemed that the *Theoremata Dioristica* of the VIIth Book were designed to determine the Limits of the *Problemata Dioristica* of the VIIIth; and therefore supposing what those Problems might have been, and their Order from that of the said Theorems, Dr. *Halley* has in XXXIII Propositions given the *Analyses* and *Syntheses* of them, after the Method of the Ancients, every where following the Steps of *Apollonius* to be found in his VIIth Book. This he calls *Conicorum Liber Octavus restitutus*, and may serve the turn, till such time as the Original Eighth Book come to light; if that be not now to be despair'd of.

Because of the Affinity of the Subject he hath subjoin'd the two Books of *Serenus Antissensis*, the Greek Text of which was never before in print. This was procured by the abovesaid Reverend *Dean* of *Christchurch*, Dr. *Aldrich*, in a collated Copy of three Manuscripts extant in the King's Library at *Paris*, and by him, according to his wonted Goodness and Generosity, freely communicated for the use of the Publick. To this also is added the Latin Translation of *Commandine*, which in many Cases needed Castigation.

As to the Authors themselves little needs be said, they having stood the Test of so many Ages, and been highly valued by the Learned in all Times, especially the Conicks, justly esteemed a Masterpiece in the Geometry of the Ancients: So that it may seem strange, that a Book so excelling in its kind, should not till now have been printed in its native *Greek*, a Tongue so peculiarly adapted to Mathematical Purposes. But this present Edition may make ample Amends, the Paper and the Elegance and Correctness of the Print being remarkable. The Book is now to be had of Mr. *Christopher Bateman* in *Pater noster Row*, *London*.

Printed for *W.* and *J. Innys*, at the *Prince's Arms* in St. *Paul's* Church-yard. 1718.

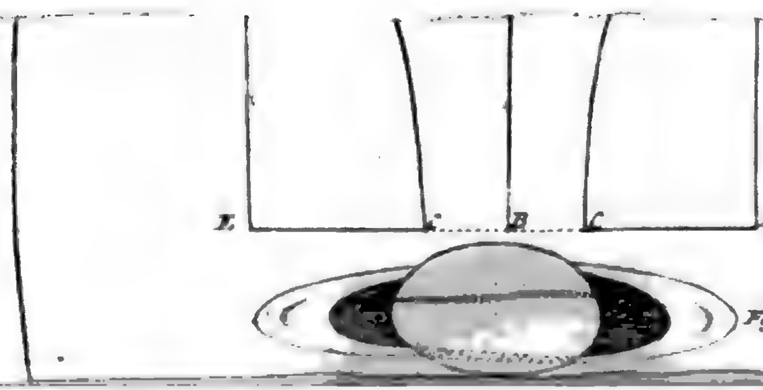


Fig. 12

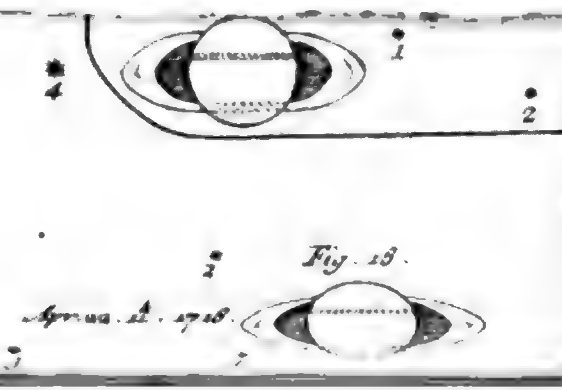


Fig. 13

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Jan. Feb. March* and *Apr.* 1718.

The CONTENTS.

I. *Considerations on the Change of the Latitudes of some of the principal fixt Stars.* By Edmund Halley, R. S. Sec.

HAving of late had occasion to examine the quantity of the Precession of the Equinoctial Points, I took the pains to compare the Declinations of the fixt Stars delivered by *Ptolomy*, in the 3*d* Chapter of the 7*th* Book of his *Almag.* as observed by *Timocharis* and *Aristyllus* near 300 Years before *Christ*, and by *Hipparchus* about 170 Years after them, that is about 130 Years before *Christ*, with what we now find: and by the result of very many Calculations, I concluded that the fixt Stars in 1800 Years were advanced somewhat more than 25 degrees in Longitude, or that the Precession is somewhat more than 50″ *per ann.* But that with so much uncertainty, by reason of the imperfect Observations of the Ancients, that I have chosen in my Tables to adhere to the even proportion of five Minutes in six Years, which from other Principles we are assured is very near the Truth. But while I was upon this Enquiry, I was surprized to find the Latitudes of three of the principal Stars in Heaven directly to contradict the supposed greater *Obliquity* of the *Ecliptick*, which seems confirmed by the Latitudes of most of the rest; they being set down in the old Catalogue, as if the Plain of the Earths Orb had changed its Situation, among the fixt Stars, about 20′ since the time of *Hipparchus*. Particularly all the Stars in *Gemini* are put down, those to the *Northward* of the Ecliptick, with so much less Latitude than we find, and those to the *Southward* with so much more *Southerly* Lati-

Latitude. Yet the three Stars *Palilicium* or the *Bulls* Eye, *Sirius* and *Arcturus* do contradict this Rule directly: for by it, *Palilicium* being in the days of *Hipparchus* in about 10 *gr.* of *Taurus* ought to be about 15 Min. more *Southerly* than at present and *Sirius* being then in about 15 of *Gemini* ought to be 20 Min. more *Southerly* than now; yet *è contra Ptolomy* places the first 20 Min. and the other 22 more *Northerly* in Latitude than we now find them. Nor are these errors of Transcription, but are proved to be right by the declinations of them set down by *Ptolomy*, as observed by *Timocharis*, *Hipparchus* and himself, which shew that those Latitudes are the same as those Authors intended. As to *Arcturus*, he is too near the Equinoctial Colure, to argue from him concerning the change of the Obliquity of the Ecliptick, but *Ptolomy* gives him 33′ more *North* Latitude than he now has; and that greater Latitude is likewise confirmed by the Declinations delivered by the abovesaid Observers. So then all these three Stars are found to be above half a degree more *Southerly* at this time than the Antients reckoned them. When on the contrary at the same time the bright Shoulder of *Orion* has in *Ptolomy* almost a degree more *Southerly* Latitude than at present. What shall we say then? It is scarce credible that the Antients could be deceived in so plain a matter, three Observers confirming each other. Again these Stars being the most conspicuous in Heaven, are in all probability the nearest to the Earth, and if they have any particular Motion of their own, it is most likely to be perceived in them, which in so long a time as 1800 Years may shew it self by the alteration of their places; though it be utterly imperceptible in the space of a single Century of Years. Yet as to *Sirius* it may be observed that *Tycho Brahe* makes him 2 Min. more *Northerly* than we now find him, whereas he ought to be above as

much

much more *Southerly* from his Ecliptick, (whose Obliquity he makes 2 ½ greater than we esteem it at present) differing in the whole 4 ½ Min. One half of this difference may perhaps be excused, if refraction were not allowed in this Case by *Tycho*; yet two Minutes, in such a Star as *Sirius*, is somewhat too much for him to be mistaken.

But a further and more evident proof of this change is drawn from the Observation of the application of the Moon to *Palilicium Anno Christi* 509 *Mart.* 11°. when in the beginning of the Night the Moon was seen to follow that Star very near, and seemed to have Eclipsed it. ἐπέβαλλε γὰρ ὁ ἀστὴρ τῷ παρὰ τὴν διχοτομίαν μέρει τῆς κυρτῆς περιφερείας τοῦ πεφωτισμένου μέρους. *i. e. Stella apposita erat parti per quam bissecabatur limbus Lunæ illuminatus*, as *Bullialdus*, to whom we are beholden for this Antient Observation has translated it. Now from the undoubted principles of Astronomy, it was impossible for this to be true at *Athens*, or near it, unless the Latitude of *Palilicium* were much less than we at this time find it. *Vide Bullialdi Astr. Philolaica, pag.* 172.

This Argument seems not unworthy of the *Royal Society*'s Consideration, to whom I humbly offer the plain Fact as I find it, and would be glad to have their Opinion.

But whether it were really true that the Obliquity of the Ecliptick was, in the time of *Hipparchus* and *Ptolemy*, really 22 Min. greater than now, may well be questioned; since *Pappus Alexandrinus*, who lived but about 200 Years after *Ptolemy*, makes it the very same that we do. *Vide Pappi Collect. Lib.* VI. *Prop.* 35.

II. *An*

II. *An Account of ſome Experiments ſhown before the* Royal Society; *with an enquiry into the cauſe of the Aſcent and Suſpenſion of Water in Capillary Tubes.* *By* James Jurin, *M.D. and* R. Soc. S.

SOme Days ago a Method was propoſed to me by an ingenious Friend, for making a perpetual Motion, which ſeem'd ſo plauſible, and indeed ſo eaſily demonſtrable from an Obſervation of the late Mr. *Hawksbee*, ſaid to be grounded upon Experiment, that, tho' I am far from having any Opinion of attempts of this Nature, yet, I confeſs, I could not ſee why it ſhould not ſucceed. Upon tryal indeed I found my ſelf diſappointed. But as ſearches after things impoſſible in themſelves are frequently obſerv'd to produce other diſcoveries unexpected by the Inventer; ſo this Propoſal has given occaſion not only to rectify ſome miſtakes into which we had been led, by that ingenious and uſeful Member of the *Royal Society* above named, but likewiſe to detect the real Principle, by which Water is rais'd and ſuſpended in Capillary Tubes, above the Level.

My Friend's Propoſal was as follows.

Fig 1. Let *ABC* be a capillary Siphon, compos'd of two Legs *AB*, *BC*, unequal both in length and Diameter; whoſe longer and narrower Leg *AB* having its orifice *A* immerſt in Water, the Water will riſe above the Level, till it fills the whole Tube *AB*, and will then continue ſuſpended. If the wider and ſhorter Leg *BC*, be in like manner immerſt, the Water

 will

will only riſe to ſome height as *FC*, leſs than the entire height of the Tube *B C*.

This Siphon being fill'd with Water, and the Orifice *A* ſunk below the Surface of the Water *D E*, my Friend reaſons thus.

Since the two Columns of Water *AB* and *FC*, by the Suppoſition, will be ſuſpended by ſome Power acting within the Tubes they are contain'd in, they cannot determine the Water to move one way, or the other. But the Column *B F*, having nothing to ſupport it, muſt deſcend, and cauſe the Water to run out at *C*. Then the preſſure of the Atmoſphere driving the Water upward through the Orifice *A*, to ſupply the Vacuity, which would otherwiſe be left in the upper part of the Tube *B C*, this muſt neceſſarily produce a perpetual Motion, ſince the Water runs into the ſame Veſſel, out of which it riſes. But the Fallacy of this reaſoning appears upon making the Experiment.

Exp. 1. For the Water, inſtead of running out at the Orifice *C*, riſes upward towards *F*, and running all out of the Leg *B C*, remains ſuſpended in the other Leg to the height *A B*.

Exp. 2. The ſame thing ſucceeds upon taking the Siphon out of the Water, into which its lower Orifice *A* had been immerſt, the Water then falling in drops out of the Orifice *A*, and ſtanding at laſt at the height *A B*. But in making theſe two Experiments it is neceſſary that *A G* the difference of the Legs exceed *F C*, otherwiſe the Water will not run either way.

Exp. 3. Upon inverting the Siphon full of Water, it continues without Motion either way.

The reaſon of all which will plainly appear, when we come to diſcover the Principle, by which the Water is ſuſpended in Capillary Tubes.

Mr.

Mr. *Hawkshee*'s Obſervation is as follows.

Fig. 2. Let *ABFC* be a capillary Siphon, into the which the Water will riſe above the Level to the height *CF*, and let *BA* be the depth of the Orifice of its longer Leg below the Surface of the Water *DE*. Then the Siphon being fill'd with Water, if *BA* be not greater than *CF*, the Water will not run out at *A*, but will remain ſuſpended.

This ſeems indeed very plauſible at firſt ſight. For ſince the Column of Water *FC* will be ſuſpended by ſome power within the Tube, why ſhould not the Column *BA*, being equal to, or leſs than the former, continue ſuſpended by the ſame Power?

Exp. 4. In fact, if the orifice *C* be lifted up out of the Water *DE*, the Water in the Tube will continue ſuſpended, unleſs *BA* exceed *FC*.

Exp. 5. But when *C* is never ſo little immerſt in the Water, immediately the Water in the Tube runs out in drops at the Orifice *A*, tho' the length *AB* be conſiderably leſs than the height *CF*.

Mr. *Hawkshee* in his Book of Experiments has advanced another Obſervation, namely, that the ſhorter Leg of a Capillary Siphon, as *ABFC*, muſt be immerſt in the Water to the depth *FC*, which is equal to the height of the Column, that would be ſuſpended in it, before the Water will run out at the longer Leg.

Exp. 6. From what miſtake this has proceeded, I cannot imagine; for the Water runs out at the longer Leg, as ſoon as the Orifice of the ſhorter leg comes to touch the Surface of the ſtagnant Water, without being at all immerſt therein.

Having proceeded thus far in obedience to the commands of this Illuſtrious Society, I beg leave to go a little farther, and to enquire into the cauſe of the aſcent and ſuſpenſion of Water in capillary Tubes.

 That

That this Phænomenon is no way owing to the pressure of the Atmosphere, has been, I think sufficiently prov'd by Mr. *Hawksbee*'s Experiments.

And that the cause assign'd by the same ingenious and inquisitive Person, namely the attraction of the concave Surface, in which the suspended Liquor is contain'd, is likewise insufficient for producing this effect, I thus demonstrate.

Since in every capillary Tube the height, to which the Water will spontaneously ascend, is reciprocally as the Diameter of the Tube, it follows, that the Surface containing the suspended Water in every Tube is always a given Quantity: but the Column of Water suspended is, as the Diameter of the Tube. Therefore, if the attraction of the containing Surface be the cause of the Waters suspension; it will follow, that equal causes produce unequal effects, which is absurd.

To this it may perhaps be objected, that, in two Tubes of unequal Diameters, the circumstances are different, and therefore the two Causes, tho' they be equal in themselves, may produce effects that are unequal. For the lesser Tube has not only a greater Curvature, but those parts of the Water, which lie in the middle of the Tube, are nearer to the attracting Surface, than in the wider. But from this if any thing follows, it must be, that the narrower Tube will suspend the greater quantity of Water, which is contrary to Experiment. For the Columns suspended are as the Diameters of the Tubes.

But as Experiments are generally more satisfactory in things of this nature, than Mathematical reasonings, it may not be amiss to make use of the following, which appear to me to contain an *Experimentum Crucis.*

Fig.

Fig. 3. The Tube *C D* is composed of two Parts, in the wider of which the Water will rise spontaneously to the height *B F*, but the narrower Part, if it were of a sufficient length, would raise the Water to a height equal to *C D*.

Exp. 7. This Tube being fill'd with Water, and the wider end *C* immerst in the stagnant Water *A B*, the whole continues suspended.

Exp. 8. *Fig.* 4. The narrower end being immerst, the Water immediately subsides, and stands at last at the height *D G* equal to *B F*.

From which it is manifest, that the suspension of the Water in the former of these Experiments is not owing to the attraction of the containing Surface: since, if that were true, this Surface being the same, when the Tube is inverted, would suspend the Water at the same height.

Having shown the insufficiency of this Hypothesis, I come now to the real cause of that Phænomenon, which is the attraction of the Periphery, or Section of the Surface of the Tube, to which the upper Surface of the Water is contiguous and coheres.

For this is the only part of the Tube, from which the Water must recede upon its subsiding, and consequently the only one, which by the force of its cohesion, or attraction, opposes the descent of the Water.

This likewise is a cause proportional to the effect, which it produces; since that Periphery, and the Column suspended, are both in the same proportion as the Diameter of the Tube.

Tho' from either of these particulars it were easy to draw a just Demonstration, yet to put the matter out of all doubt, it may be proper to confirm this assertion, as we have done the former, by actual Experiment.

Fig.

Fig. 5. Let therefore *EDC* be a Tube, like that made use of in the 7th and 8th Experiments, except that the narrower Part is of a greater length; and let *AF* and *BG* be the heights, to which the Water would spontaneously rise in the two Tubes *ED* and *DC*.

Exp. 9. If this Tube have its wider Orifice *C* immerst into the Water *AB*, and be fill'd to any height less than the length of the wider Part, the Water will immediately subside to a Level with the point *G*; but if the Surface of the contain'd Water enter never so little within the smaller Tube *ED*, the whole Column *DC* will be suspended, provided the length of that Column do not exceed the height *AF*.

In this Experiment it is plain, that there is nothing to sustain the Water at so great a height, except the contact of the Periphery of the lesser Tube, to which the upper Surface of the Water is contiguous. For the Tube *DC*, by the Supposition, is not able to support the Water at a greater height than *BG*.

Exp. 10 *Fig.* 6. When the same Tube is inverted, and the Water is rais'd into the lower extremity of the wider Tube *CD*, it immediately sinks, if the length of the suspended Column *DH* be greater than *GB*; whereas in the Tube *DE* it would be suspended to the height *AF*. From which it manifestly appears, that the suspension of the Column *DH* does not depend upon the attraction of the Tube *DE*, but upon the Periphery of the wider Tube, with which its upper Surface is in contact.

For the sake of those, who are pleas'd with seeing the same thing succeed in different manners, we subjoin the two following Experiments, which are in substance the same with the 9th and 10th.

Fig. 7. *ABC* is a Siphon, in whose narrower, and shorter Leg *AB*, if it were of a sufficient length, might

be

be ſuſpended a Column of Water of the height *E F*; but the longer and wider Leg *B C* will ſuſpend no more than a Column of the length *G H*.

Exp. 11. This Siphon being fill'd with Water, and held in the ſame Poſition as in the Figure, the Water will not run out at *C* the Orifice of the longer Leg, unleſs *D C*, the difference of the Legs *A B* and *B C*, exceed the length *E F*.

Fig. 8. *Exp.* 12. If the narrower Leg *B C* be longer than *A B*, the Water will run out at *C*, if *D C* the difference of the Legs exceed *E F*; otherwiſe it will remain ſuſpended.

In theſe two Experiments it is plain, that the Columns *D C* are ſuſpended by the attraction of the Peripheries at *A*, ſince their lengths are equal to *E F*, or to the length of the Column, which by the ſuppoſition thoſe Peripheries are able to ſupport; whereas the Tubes *B C* will ſuſtain Columns, whoſe lengths are equal to *G H*.

Tho' theſe Experiments ſeem to be concluſive, yet it may not be improper to prevent an Objection, which naturally preſents it ſelf, and which at firſt view may be thought ſufficient to overturn our Theory.

Fig. 5. For ſince a Periphery of the Tube *E D* is able to ſuſtain no more than a Column of the length *A F*, contain'd in the ſame Tube; how comes it to ſuſtain a Column of the ſame length in the wider Tube *D C*, which is as much greater than the former, as the Section of the wider Tube exceeds that of the narrower?

Fig. 6. Again, if a Periphery of the wider Tube *D C* be able to ſuſtain a Column of Water in the ſame Tube, of the length *B G*; why will it ſupport no more than a Column of the ſame length in the narrower Tube *E D*?

Which

Which Queries may likewise be made with regard to the 11th and 12th Experiments.

The answer is easy, for the Moments of those two Columns of Water are precisely the same, as if the sustaining Tubes *E D* and *C D*, were continued down to the Surface of the stagnant Water *A B* ; since the velocities of the Water, where those Columns grow wider, or narrower, are to the velocities at the attracting Peripheries, reciprocally as the different Sections of the Columns.

Fig. 9. *Exp.* 13. From which consideration arises this remarkable Paradox. That a Vessel being given of whatsoever Form, as *A B C*, and containing any assignable quantity of Water, how great soever; that whole quantity of Water may be suspended above the Level, if the upper part of the Vessel *C* be drawn out into a capillary Tube of a sufficient fineness.

But whether this Experiment will succeed, when the height of the Vessel is greater than that, to which Water will be rais'd by the pressure of the Atmosphere, and how far it will be alter'd by a *Vacuum*, I may perhaps have the honour of giving an account to the *Society* some other time, not being perfectly satisfy'd with those Tryals which I have hitherto had the opportunity of making.

Having discover'd the cause of the suspension of Water in capillary Tubes, it will not be difficult to account for the seemingly spontaneous ascent of it. For, since the Water, that enters a capillary Tube as soon as it's Orifice is dipt therein, has it's gravity taken off by the attraction of the Periphery with which it's upper Surface is in contact, it must necessarily rise higher, partly by the pressure of the stagnant Water, and partly by the attraction of the Periphery immediately above that, which is already contiguous to it.

It

It might now be ſhown, how naturally the various, and ſeemingly contrary appearances of the above mention'd Experiments are deducible from this Theory; but this is ſo eaſy, that it is needleſs to inſiſt upon it; and our diſcourſe upon this minute Subject has been already ſo tedious, that we could ſcarce hope for Pardon, unleſs it were directed to thoſe, who are ſenſible to how many of the greater, and more conſiderable, Phænomena of Nature this Doctrine is applicable.

P S. When this Paper was reading before the Society, I found that our incomparable Preſident was already acquainted with the above-mentioned Principle, and I have ſince met with ſeveral Paſſages in the 31ſt Query ſubjoin'd to the late Edition of his *Opticks* which plainly ſhew, that he was Maſter of it, when they were written.

I muſt do the ſame Juſtice to that excellent Mathematician Mr. *John Machin*, Profeſſor of Aſtronomy in *Greſham* College.

To theſe two worthy Perſons I am obliged for the following Obſervation, That, what I call a Periphery, or Section of the concave ſurface of the Tube, is really a ſmall Surface, whoſe Baſe is that Periphery, and whoſe height is the diſtance, to which the attractive power of the Glaſs is extended.

III. *De Motu Aquarum fluentium. Authore eodem D.* Jacobo Jurin, *M. D.*

AQuæ Motum ex imi vasis foramine defluentis sæpe videmus, tum in ipsa re Hydraulica, tum in ejus Principiis ad Oeconomiam Animalem applicandis, aliis cum Potentiis comparari. Cujus Motus quantitatem cum hactenus nemo, quod sciam, recte determinaverit, usurpare solent ejus loco scriptores Hydraulici Columnæ aqueæ pondus foramini incumbentis. Quod qui faciunt, id sane neutiquam animum advertunt fieri omnino non posse, ut Motus aliquis cum pondere quiescente conferatur. Poterit autem Aquæ defluentis Motus facili opera definiri hunc in modum.

Fig. 10. Sit SHAHS Aquæ superficies infinita, *CC* foramen circulare in fundo factum, *AB* recta perpendicularis per foraminis centrum ducta, *SGCCGS* Columna sive Cataracta Aquæ per foramen *CC* decurrentis, *SGC* Curva, cujus rotatione circa Axem *AB* generatur Solidum, sive Cataracta, *SGCCGS*. Aqua enim cum libere, & motu accelerato descendat ad normam corporum omnium gravium, necessariò in minorem amplitudinem contrahitur, prout majorem velocitatem acquirit inter cadendum, & profluit ex foramine *CC* ea cum velocitate, quæ cadendo ab altitudine *AB* comparatur.

Velocitas autem corporis gravis cadendo genita, ex *Galilæi* demonstratis, rationem obtinet subduplicatam altitudinis unde cecidit. Quare, si ducatur ad Curvam *SGC* Ordinata quævis *DE*, atque ipsa *DE* vocetur *y*, & *AD x*, exponetur velocitas Aquæ in sectione *EE*

per

per $\sqrt{x}$, & Factum ex ea velocitate ducta in ipsam sectionem per $\sqrt{x} \times y^2$.

Quod Factum est ut moles Aquæ dato temporis spatio per eam sectionem transeuntis; cumque eadem Aquæ moles dato tempore per singulas Cataractæ sectiones transeat, proinde Factum istud perpetuo sibi constabit, eritque $\sqrt{x} \times y^2 = 1$, & $x y^4 = 1$.

Quæ est Æquatio Curvæ *S G C*, cujus partem, intra datum vas comprehensam, delineavit, ejusdemque Æquationem non obscure indicavit Magnus *Newtonus*, *Prop.* 36. *Libr.* 2. *Princip.* qui primus omnium veram Aquæ effluentis velocitatem, ex genuinis Principiis deductam, Orbi Literato exposuit.

Est autem ipsa Curva Hyperboloeides quarti Ordinis, cujus altera Asymptotos est recta *A S* ad Horizontem parallela, altera *A B* eidem perpendicularis.

Hujus Potestas est Quadrato-Cubus Ordinatæ *F G*, ductæ ad punctum *G*, ubi recta *A G*, bisecans angulum ab Asymptotis comprehensum, Curvæ occurrit.

Spatium *S A D E S*, inter Curvam *S G E*, Ordinatam *D E* & Asymptotos *A D*, *A S* inclusum, æquale est quatuor partibus tertiis Rectanguli *H D*, sub Abscissa *A D* & Ordinata *D E* contenti. Estque proinde Spatium *S H E* pars tertia ejusdem Rectanguli.

Solidum *S G E E G S*, convolutione spatii *S A D E S*, circa Axem *A D*, generatum, duplum est Cylindri incumbentis sectioni *E E*. Unde Solidum cavum, quod gignit conversio spatii *S H E G S*, circa eundem Axem, Cylindro incumbenti æquale est. Quæ omnia facili calculo inveniuntur per Methodum Fluxionum inversam.

Theorema I.

Aquâ ex vase amplitudinis infinitæ, per foramen circulare in fundo factum, decurrente, Motus totius Cataractæ aqueæ Horizontem versus æqualis est Motui Cy-

lindri aquei, sub ipso foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis; vel æqualis est Motui molis Aquæ, quæ dato quovis tempore effluit, cujus ea sit velocitas, qua percurratur eodem dato tempore spatium æquale altitudini Aquæ.

Demonstratio primæ partis.

Ducatur ad Curvam SGC alia Ordinata de, priori DE quam proxima.

Curvâ circa Axem AB conversâ, generabunt Ordinatæ DE, de, Circulos duos, quibus intercipitur Solidum nascens $EEee$. Id solidum æquale est Facto ex altitudine Dd ducta in sectionem EE, & Motus ejus æquatur Facto ex ipso solido ducto in velocitatem ejusdem, sive Facto ex altitudine Dd, sectione EE, & velocitate Aquæ in ea Sectione. Cumque supra ostensum sit, Factum ex quavis Sectione Cataractæ & velocitate Aquæ in ea Sectione, quantitatem esse constantem, erit proinde Motus totius Cataractæ æqualis Facto ex quantitate illa constante ducta in Summam omnium altitudinum Dd, sive in ipsam AB, hoc est, Motui Cylindri sub ipso foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis. *Q. E. D.*

Corol. 1. Data altitudine Aquæ, erit Motus Cataractæ in ratione foraminis.

2. Dato foramine, erit Motus Cataractæ in ratione sescuplicata altitudinis, sive in ratione triplicata velocitatis, qua Aqua per foramen exit.

3. Dato Motu Cataractæ, erit foramen reciprocè in ratione sescuplicata altitudinis, vel reciprocè in ratione velocitatis triplicata.

Demonstratio secundæ partis.

Moles Aquæ dato tempore effluentis est ad Cylindrum sub ipso foramine & altitudine Aquæ, ut longitudo quam Aqua effluens æquabili velocitate dato isto

tempore

tempore percursura sit, ad altitudinem Aquæ. Cumque velocitas, quæ tribuitur moli Aquæ effluentis, sit ad velocitatem Cylindri reciproce in eadem ratione, erunt Motuum quantitates utrinque æquales. *Q. E. D.*

Corol. 1. Data altitudine Aquæ & mole effluente; Motus Cataractæ est in ratione inversa temporis quo ista moles effluit.

2. Data altitudine & tempore, Motus Cataractæ est ut moles Aquæ tempore isto effluentis.

3. Dato tempore & mole Aquæ effluentis, erit Motus Cataractæ in ratione altitudinis.

4. Dato Motu Cataractæ & altitudine, moles effluens est in ratione temporis.

5. Dato Cataractæ Motu & mole Aquæ effluentis, altitudo est ut tempus.

6. Dato tempore & Motu Cataractæ; erit Aquæ effluentis moles reciprocè ut altitudo.

Theorema II.

Fig. 11. Si capiatur BA, quæ sit ad BD, ut DG^4 ad $DG^4 - BC^4$; Aqua decurrente ex dato vase Cylindrico semper pleno $GGEE$, per foramen circulare CC in fundo medio factum, Motus Cataractæ aqueæ Horizontem versus æqualis erit Motui Cylindri sub foramine & altitudine AB, cujus velocitas æquet velocitatem Aquæ per foramen exeuntis; vel erit æqualis Motui molis Aquæ quæ dato quovis tempore effluit, cujusque ea sit velocitas, qua percurratur eodem dato tempore spatium æquale altitudini AB.

Demonstratio primæ partis.

Ducatur AS ipsi DG parallela, & Asymptotis AS, AB, per puncta G, C descripta concipiatur Curva *Newtoniana* SGC.

Ut constet Aquæ altitudo, supplendus est exeuntis locus Cylindro aqueo $ggGG$, descendente cum ea velocitate

locitate uniformi, quæ acquiritur cadendo ab *A* ad *D*, quemadmodum docet Vir incomparabilis Propositione prædicta.

Motui hujus Cylindri æquatur, per Theorema superius, Motus Cataractæ *SSGG*. Ergo Motus Aquæ descendentis, cum sit compositus ex Motu Cylindri aquei *ggGG*, & Motu Cataractæ *GGCC*, æqualis est Motui Cataractæ integræ *SGCCGS*, *h. e.* per Theorema primum, Motui Cylindri aquei sub foramine & altitudine *AB*, cujus velocitas æqualis sit velocitati Aquæ per foramen decurrentis. *Q. E. D.*

Pars secunda sequitur ex priore.

Corol. 1. Oriuntur hinc omnia Propositionis præcedentis Corollaria, substituendo altitudinem *AB*, pro Aquæ altitudine.

2. Si vas alia figura fuerit, atque Cylindrica; aut foraminis figura pro circulari fuerit quadrata, triangularis, vel qualiscunque; aut ipsum foramen non sit in medio fundo situm, vel etiam in latere vasis factum; idem erit Motus Cataractæ, scilicet æqualis Motui Prismatis aquei sub foramine & altitudine *AB*, cujus velocitas par sit velocitati Aquæ effluentis. Nam eadem Aquæ moles, cum eadem velocitate atque in priori Hypothesi, tum per ipsum foramen, tum per singulas Cataractæ sectiones transibit.

3. Si vasis Diameter permagnam rationem obtineat ad Diametrum foraminis, negligi poterit altitudo *AD*, & vasis ipsius altitudo pro altitudine Cylindri, vel Prismatis aquei, usurpari.

Hactenus casum illum particularem, quo Aqua, Gravitatis vi, ex vase defluit, seorsim consideravimus. Id eo fecimus lubentius, tum quod illum fere solum adhibere soleant Mathematici, quoties agitur de Fluidorum impetu, tum quod Curvæ Hyperbolicæ supra expositam proprietatem, qua Cataractam Aquæ descendentis format,

mat, non indignam censeamus contemplatione Geometrarum. Alioqui potuisset iste casus nullo negotio deduci ex Theoremate generali, quod proximo loco proponemus.

Theorema III.

Fig. 12. Aqua fluente per Canalem plenum quemcunque *ABCD* secundum lineam *EF*, cui sit perpendiculare utrumque Canalis orificium *AB* & *CD*, Motus Aquæ versus Orificium *CD*, sive Motus impedimenti, quod in ipso orificio oppositum sistat Motum totius Aquæ, æqualis est Motui Prismatis aquei sub qualibet Sectione Canalis *CH* & linea directionis, sive longitudine Canalis *EF*, quod moveatur eadem cum velocitate, qua Aqua fluit per istam Sectionem: sive æqualis Motui molis Aquæ, quæ dato quovis tempore effluit ex Canali, cujusque ea sit velocitas, qua percurratur eodem dato tempore spatium æquale longitudini Canalis.

Cas. 1. Sit linea directionis recta quævis *EF*.

Facile demonstratur pars prima eodem modo, quo Theorema primum. Est enim Factum ex quavis sectione Canalis *CH*, & Aquæ velocitate in ea Sectione, quantitas constans.

Pars secunda sequitur ex prima.

Cas. 2. *Fig.* 13. Si linea directionis *ABCDE*, ex pluribus rectis *AB*, *BC*, *CD*, *DE*, ad sese invicem inclinatis sit composita, idem erit Aquæ Motus. Nam Motus Aquæ in toto Canali composito *ABCDE*, conficitur ex Motibus Aquæ in partibus Canalis *AB*, *BC*, *CD*, *DE*, additis sibi invicem. Statuimus autem Aquam fluentem secundum rectam *AB*, mutata ista directione in aliam, qua feratur secundum rectam *BC*, nihil ex Motu deperdere. Leges enim illas, quæ in motu corporum solidorum observantur, quoties eorundem directio

directio mutatur, fluida non sequuntur. Alioqui fluidum mutata directione in aliam priori perpendicularem, penitus sisteretur, quod Experimentis neutiquam deprehenditur. Aqua porro ex Vasis foramine exiliens, sive deorsum, sive secundum Horizontis planum, sive rectâ sursum feratur, eandem obtinet velocitatem. Quod si aliquando vel ratiocinio subtiliori, vel Experimentis innotescet, aliquam Motus imminutionem ex mutata directione proficisci, erit ejusdem ratio habenda.

Si Curva fuerit linea directionis *A B*, referetur ad hunc Casum, quippe quæ ex pluribus rectulis confecta concipi queat. *Fig.* 14.

Cas. 3. *Fig.* 15. Si divisus fuerit Canalis *A B* in plures ramos *B C*, *B D*, *B E*, longitudine æquales, eadem ratione invenietur Aquæ Motus, usurpando pro linea directionis longitudinem *A B D*, compositam ex longitudine Canalis principis *A B*, & longitudine cujusvis rami *B D*. Perinde autem est, sive Aqua à Canali principe versus ramos, sive à ramis fluxerit versus principem Canalem. Quod si rami fuerint inæquales, inveniendus est Motus Aquæ in singulis ramis, adhibendo pro linea directionis longitudinem confectam ex longitudine cujusque rami, & longitudine principis Canalis.

Nullo negotio deducitur ex Casu secundo.

Cas 4. *Fig.* 16. Si rami æquales, in quos distributus est Canalis *A B*, iterum in Canalem unicum *F G* colligantur, ad Motum Aquæ inveniendum adhibenda est pro linea directionis longitudo integra *A B D F G*, confecta ex longitudine principis Canalis *A B*, rami cujusvis *B D F*, & Canalis recompositi *F G*. Si Rami sint inæquales, inveniendus est in singulis Aquæ Motus, & eorum Motuum Summa Motui Aquæ in Canali recomposito addendus. Sequitur ex Casu 2, & 3.

Corol. 1. Data longitudine Canalis, & qualibet Sectione ejusdem, erit Motus Aquæ in ratione velocitatis, qua

qua Aqua fluit per istam Sectionem.

2. Data quavis Sectione, & velocitate Aquæ Sectionem istam præterfluentis, erit Motus Aquæ ut longitudo Canalis.

3. Data Canalis longitudine, & velocitate Aquæ in quavis Sectione, erit Aquæ Motus in ratione illius Sectionis.

4. Dato Motu Aquæ, & aliqua Sectione, erit longitudo Canalis in ratione inversa velocitatis.

5. Dato Aquæ Motu, & longitudine Canalis, erit Sectio quævis reciprocè ut velocitas.

6. Data velocitate in qualibet Sectione, & Motu Aquæ, erit ista Sectio in ratione reciproca longitudinis.

7. Data longitudine Canalis, & mole Aquæ certo quovis tempore effluentis, erit Aquæ Motus reciprocè ut istud tempus.

8. Data Canalis longitudine, & tempore, erit Aquæ Motus ut moles effluens.

9. Dato tempore, & mole Aquæ effluentis, erit Aquæ Motus ut longitudo Canalis.

10 Dato Motu Aquæ, & longitudine Canalis, moles effluens est in ratione temporis.

11. Dato Aquæ Motu, & mole effluente, erit tempus ut longitudo Canalis.

12. Dato tempore, & Motu Aquæ, erit moles effluens reciproce ut longitudo Canalis.

13. Si binæ moles Aquæ motu contrario in directum occurrant, & pares sint utrinque tum superficies quibus in se invicem impingant, tum velocitates quibus istæ superficies in adversum moveantur, fuerit autem altera moles Aquæ guttulæ uni æqualis, altera Aqua omnis Oceano contenta, vel etiam quantitas Aquæ infinita; fieri potest ut una ista guttula Aquam omnem Oceani, vel quantitatem Aquæ infinitam, non solum sustineat, sed

 post

post occursum, eadem ac prius velocitate, ipsa in plagam eandem moveri pergat, eadem illam in partes contrarias repellat. Quod est mirabile Paradoxon in re Hydraulica.

14. Si certa moles Aquæ, per canalem ex tubis duobus cylindricis, Diametro inæqualibus, compositum, à tubo ampliore versus angustiorem fluat, & motus Aquæ neque minuatur inter fluendum neque augeatur, simul ac prima pars Aquæ tubi minoris initium ingressa fuerit, statim tardius fluere incipiet, & continuato effluxu ex tubo latiore in angustiorem, gradatim magis retardabitur Aqua in tubo angustiore, usque dum tota in eum tubum pervenerit. Contrario modo res eveniet, fluente Aqua à tubo minore versus ampliorem. Quod est alterum Paradoxon in re Hydraulica. Ponitur autem Aqua ubique sibi cohærere.

Oriuntur bina ista Corollaria ex Casu 1.

15 Ex Casu secundo datur Methodus æstimandi Motum Sanguinis in qualibet Arteria.

16. Datis quibuscunque Arteriis binis, æqualem Sanguinis molem transmittentibus, major est impetus Sanguinis in Arteria à Corde remotiore quam in propiore. Quod est Paradoxon notatu dignum in Oeconomia Animali.

17. Ex Casu tertio oritur alterum Paradoxon in Oeconomia Animali, nempe majorem esse Sanguinis motum sive impetum, in Arteriis omnibus Capillaribus simul sumptis, quam in ipsa Aorta. Item, major est in Capillaribus Venis, quam Arteriis.

18. Ex Casu quarto deducitur Methodus definiendi motum Sanguinis in quavis Vena.

19. Ex eodem deducitur tertium in Oeconomia Animali Paradoxon, nempe majorem esse Sanguinis impetum in Vena quavis, quam in Arteria ei Venæ respondente,

dente, & proinde majorem esse in Vena Cava, quam in Aorta.

Problema I.

Invenire motum Aeris ex Pulmone effluentis.

Sit $l =$ Longitudo totius ductus aerei, ab Ore & Naribus ad extremos ramos Trachææ.

$q =$ Quantitas Aeris mediocri exspiratione ex Pulmone emissa.

$Q =$ Aeris copia validissima exspiratione expulsi.

$t =$ Tempus mediocris exspirationis.

$T =$ Tempus exspirationis fortissimæ.

Inde, per Theorema 3, *Cas.* 3, Motus Aeris ex Pulmone effluentis, in exspiratione mediocri $= \frac{ql}{t}$.

fortissima $= \frac{Ql}{T}$.

Hoc est, Motus Aeris ex Pulmone exeuntis æqualis est motui molis Aeris, quæ unica exspiratione emittitur, cujus ea sit velocitas, qua percurratur tempore exspirationis longitudo totius Canalis Aerei. *Q. E. I.*

Aeris quantitatem exspiratione mediocri emissam Vir Clarissimus, *Alphonsus Borellus*, facto Experimento 18 circiter, vel 20 unciis cubicis definit. Est autem diversa, non solum in diversis Hominibus, sed etiam temporibus diversis, in Homine eodem. Ipse Experimentum in hunc modum institui.

Vesicæ madefactæ à parte inferiore pondus appendebam, & aptato eidem superius tubo vitreo Diametro circiter unciali, naribus obturatis Aerem vesicæ leniter inspirabam, per spatium trium minutorum secundorum, pondere interim in mensa quiescente. Postea Vesicam cum Aere incluso & pondere appenso, sub Aquam in vase Cylindrico contentam, demergebam, notata diligenter altitudine, ad quam Aqua attollebatur.

 Deinde,

Deinde, Aere ex vesica expresso, iterum eandem cum pondere in Aquam immittebam. Quod cum esset factum, facile inveniebatur Aquæ moles, quæ vasi infusa altitudinem prius notatam conficeret. Experimento decies repetito, & additis sibi invicem quantitatibus singulis inventis, earum decima, sive media moles Aquæ vasi infusa, reperiebatur 35 unciis cubicis æqualis. Quæ moles est Aeris vesica contentæ; & adjecta circiter parte duodecima, seu 3 unciis cubicis, ob Aeris condensationem à frigore Aquæ factam, cum tempestas fuerit hyemalis, efficiuntur 38 unciæ cubicæ. Præterea addendum est tantillum, tum propter Aquæ pressionem in vesicam, tum ob Vaporem qui cum halitu emittitur in humorem coactum; quod fiat necesse est ex frigore Aquæ, & vesicæ madidæ contactu. Æstimavi igitur Aeris copiam, leni exspiratione emissam tempore trium minutorum secundorum, numero rotundo 40 unciarum cubicarum.

In exspiratione validissima exspirabam uncias cubicas 125, tempore minuti secundi unius.

Hujusmodi autem exspiratione, cum vehementi Pulmonis contentione ad strangulatum fere continuata, 220 uncias cubicas ex Pectore emittebam. Unde patet, ut id obiter moneam, multo plus Aeris in Pectore superesse, quam unica exspiratione mediocri emitti.

Si ergo ponatur $l = 2$ pedes

$$q = 40 \text{ unciæ cubicæ}$$
$$Q = 125 \text{ unciæ cubicæ}$$
$$t = 3''$$
$$T = 1''$$

Aeris Gravitas Specifica ad Gravitatem Aquæ, ut 1 ad 1000.

Pes Aquæ cubicus = 1000 unc. *Avoird.*

Erit Motus mediocris Aeris Pulmone exeuntis æqualis motui ponderis Scrupulorum 4 & Granorum 9, quod percurrat unciam unam minuto secundo; vel motui

motui ponderis Grani 1 $\frac{1}{5}$, quod eodem tempore conficiat longitudinem 5 pedum & 7 unciarum. Quæ eſt velocitas Aeris per Laryngem effluentis, poſita Laryngis Sectione = $\frac{1}{5}$ unciæ quadratæ.

Motus maximus Aeris Pectore expulſi æquatur motui ponderis unciæ 1 $\frac{3}{4}$ circiter, percurrentis unciam unam minuto ſecundo; ſive motui ponderis grani 1 $\frac{1}{5}$ percurrentis eodem tempore 52 pedes. Quæ eſt velocitas Aeris in fortiſſima exſpiratione per Laryngem erumpentis.

Corol. 1. Data Aeris copia & longitudine Canalis aerei, motus Aeris eſt in ratione inverſa temporis exſpirandi.

2. Data mole Aeris & tempore, erit motus in ratione directa longitudinis.

3. Data longitudine & tempore, motus eſt ut Aeris copia.

4. Dato motu & Aeris copia, erit longitudo in ratione directa temporis.

5. Dato motu & longitudine, erit Aeris moles directe ut tempus.

6. Dato motu & tempore, erit Aeris moles reciproce ut longitudo Canalis Aerei.

7. Motus Aeris eſt in ratione compoſita ex ratione quadruplicata Diametri cujuſvis homologæ ipſius Animalis, & ratione inverſa temporis exſpirandi; vel in ratione compoſita ex ratione ponderis totius Animalis, ratione ejuſdem ponderis ſubtriplicata, & ratione temporis reciproca.

Nam pondus Animalis, Diametri cujuſvis homologæ Cubus & moles Aeris expulſi ſunt in eadem ratione. Ponitur autem Corpora Animalium Machinas eſſe ſimiliter factas.

Scholium. Longitudinem hic uſurpatam, vel ipſam eſſe concipies Canalis aerei longitudinem, ſi Rami omnes

Tra-

Trachææ longitudine æquales ponantur; vel mediam inter longitudines diverſas, ſi Rami ſint inæquales.

Problema II.

Determinare impetum, ſive impreſſionem quam excipit interna Pulmonum ſuperficies ab Aere exſpirando.

Cum actioni æqualis & contraria ſit reactio; neceſſe eſt, ut, quanto motu urgetur ab interna Pulmonum ſuperficie Aer exſpirandus, tanto viciſſim ab Aere repellatur ſuperficies Pulmonum.

Unde, per Problema ſuperius, impetus dictus in exſpiratione mediocri $= \frac{q\,l}{t}$

fortiſſima $= \frac{Q\,l}{T}$. *Q. E. I.*

Hinc poſitis iiſdem quæ in ſuperiore ponuntur, impetus mediocris Aeris in Pulmones æquales eſt motui ponderis drachmæ circiter 1 $\frac{1}{7}$, quod minuti ſecundi ſpatio percurrat unciam unam; vel motui ponderis 19 librarum conficientis eodem tempore $\frac{1}{1643}$ unciæ, quæ eſt velocitas Aeris in contactu ſuperficiei Pulmonis internæ. Ponimus autem cum Viro Doctiſſimo *Jacobo Keilio* ſuperficiem Pulmonis internam 21900 circiter unciis quadratis æqualem.

Impetus vero maximus Aeris in Pulmones æquatur motui ponderis unciæ circiter 1 $\frac{3}{4}$ moti unciam unam minuto ſecundo; vel motui ponderis 19 librarum, quod partem $\frac{1}{175}$ unciæ conficiat eodem tempore. Quæ eſt Aeris velocitas ad ſuperficiem Pulmonis in exſpiratione vehementi.

Corol. 1. Sequuntur ex hac Propoſitione Corollaria præcedenti ſubjuncta.

2. Impetus mediocris incumbens in partem ſuperficiei Pulmonis, quæ ſit ipſi Laryngis Sectioni æqualis, eſt motus

motus ponderis $\frac{1}{131}$ grani, conficientis unciæ ſpatium minuto ſecundo; vel motus grani 1 $\frac{1}{5}$ quod eodem tempore percurrat unciæ partem $\frac{1}{1651}$. Impetus autem maximus in parem ſuperficiem eſt motus ponderis $\frac{1}{161}$ partis grani quod unciam unam; vel motus ponderis grani 1 $\frac{1}{3}$ quod $\frac{1}{175}$ unciæ ſingulis minutis ſecundis conficiat.

3. Impetus Aeris in mediocri exſpiratione in Pulmones impreſſus, æquatur motui Columnæ aqueæ percurrentis unciam unam minuto ſecundo, cujus Columnæ baſis eſt ipſa Pulmonum ſuperficies interna altitudo autem eſt [illegible] unciæ. Eſtque Columnæ altitudo pars $\frac{1}{7100}$ unciæ, in exſpiratione omnium vehementiſſima.

4. Impetus incumbens in ſuperficiem parem circulo maximo Globuli Sanguinei, in leni exſpiratione, eſt pars $\frac{1}{14}$ ponderis Globuli Sanguinei; in exſpiratione vehementi $\frac{2}{7}$ ejuſdem ponderis, moti unciam unam minuto ſecundo. Qua autem ratione Diametros Globulorum Sanguinis dimenſus ſim, cum uſui eſſe queat ad aliorum Objectorum minimorum magnitudines definiendas, libet obiter exponere. Capillum tenuem, & ſatis longum aciculæ pluries circumvolvi, ut omnes convolutiones ſeſe invicem accurate contingerent, quod admotum ſubinde Microſcopium luculenter oſtendebat. Deinde cum intercapedinem inter extremas utrinque circumvolutiones Circino cepiſſem, eandem Scalæ, quam vocant Diagonali applicabam, ſpatiumque in Scala repertum per convolutionum numerum dividebam. Unde inventa eſt unius convolutionis latitudo, ſive ipſa Capilli Diameter. Poſtea Capillum eundem, in Segmenta minutula diviſum, plano Microſcopii, cui Sanguinis parum ita erat illitum ut Globuli conſpicerentur diſtincti, ſuperinſpergebam. Ea cum Microſcopio contuerer, reperiebam aliquibus in locis Capilli Segmenta ita commode diſpoſita, ut numerare liceret, quot Globuli Diametro Segmenti opponerentur. Erant autem Segmenta Diametro inæqualia,

quod

quod Capillus tenuior versus extremum fuerit, quam propius à Radice, adeo ut jam 7, vel 8, jam 12, 13ve Globuli transversæ Sectioni Capilli responderent. Utroque autem Experimento sæpius iterato, æstimavi tandem mediam Capilli Diametrum parte $\frac{1}{314}$ unciæ, & Diametrum Globuli Sanguinei parte decima Diametri Capilli, sive parte $\frac{1}{3140}$ unciæ.

5. Impetus, quem patitur interna Pulmonum superficies ab Aëre exspirando, minor est Motu lenissimi roris è Cœlo decidentis.

Scholium. Neglecta est in solutione Problematum duorum præcedentium impedimenti consideratio, quod Aëri ex Pulmone egredienti objicitur ex affrictu laterum Arteriæ Trachææ, ejusque ramorum; cum id perparvum sit, neque ullo experimento satis accurate æstimari posse videatur. Nec fuimus admodum solliciti in rationibus numerorum exquisitè servandis, cum id unum nobis propositum fuerit, ut methodum exponeremus æstimandi, aliquanto certius quam videtur antehac factum, vires eas, quibus agit Aër inter exspirandum in vasa sanguinea superficiem Pulmonis internam perreptantia. Unde dignosci potest, utrum pares sint hæ vires effectis istis producendis, quæ iisdem à Doctissimis quibusdam Scriptoribus Medicis tribuuntur. Quod liberum esto Lectoris Scientia Mechanica & Anatomica instructi Judicium.

Problema III.

Definire impetum Sanguinis in Vena Cava prope dextram Auriculam Cordis; sive motum Sanguinis per omnes Arterias & Venas fluentis, præter Pulmonares.

Sit q = Quantitas Sanguinis una Cordis Systole in Aortam projecti.

l = Longitudo media ductus integri Arterio-Venosi, ratione habitâ ramorum longiorum & breviorum

t =

$t =$ Temporis ſpatium inter binos Pulſus interceptum.

Inde, per Theorema 3. Caſ. 4. impetus quæſitus $= \frac{ql}{t}$.

Hoc eſt, Impetus Sanguinis in Vena Cava æquatur motui molis Sanguineæ, quæ una Syſtole in Aortam projicitur, cujus ea ſit velocitas, qua percurri queat integra Arteriarum & Venarum longitudo, temporis ſpatio inter binos Pulſus intercepto. *Q. E. I.*

Si in Corpore Humano ponantur

$q = 2$ unciæ *Avoird.*
$l = 6$ pedes
$t = \frac{3''}{4}$.

Erit impetus Sanguinis in Vena Cava æqualis motui ponderis 12 librarum, quod unciæ unius longitudinem conficiat ſingulis minutis ſecundis; ſeu motui ponderis 2 librarum, quod pari temporis ſpatio percurrat pedem $\frac{1}{2}$. Quæ eſt fere Sanguinis velocitas in Cava fluentis. Ponimus autem, ex dimenſione Viri Doctiſſimi ſupra dicti, Cavæ Sectionem dodrantem eſſe unciæ quadratæ.

Corol. Oriuntur ex hoc Problemate mutatis mutandis omnia Problematis primi Corollaria.

Problema IV.

Determinare motum abſolutum Sanguinis in Vena Cava; ſive motum Sanguinis, per omnes Arterias & Venas fluentis rd æter Pulmonales, ſublata Vaſorum reſiſtentia.

Sit velocitas Sanguinis Naturalis, ad eam velocitatem qua Sanguis flueret, dempta omni reſiſtentia, ut 1 ad x. Cumque per *Corol.* ſuperioris Problematis, & *Corol.* 1.

Probl. 1. Motus Sanguinis sit in ratione velocitatis, erit inde motus quæsitus $= \frac{x q l}{t}$. *Q. E. I.*

Quod si proportio per Experimentum à Viro Clarissimo supra laudato institutum inventa, ut veræ propinqua, admittatur, erit $x = 2 . 5$.

Unde, positis iisdem quæ in superiore ponuntur, motus absolutus Sanguinis in Vena Cava æquatur motui ponderis 30 librarum, quod minuto secundo longitudinem uncialem percurrat; sive motui ponderis 2 librarum percurrentis eodem tempore pedem 1 $\frac{1}{4}$. Qua fere velocitate Sanguis, omni resistentia liber, per Cavam deferretur.

Problema V.

Motum Sanguinis invenire in Vena Pulmonali prope sinistram Cordis Auriculam; sive motum totius Sanguinis per Pulmonem fluentis.

Præter notulas in *Probl.* 3. usurpatas, sit $\lambda =$ Canalis Arterio-Venosi Pulmonici media longitudo.

Unde, per *Theor.* 3. *Cas.* 4. invenitur motus quæsitus $= \frac{q \lambda}{t}$.

Hoc est, motus Sanguinis per Pulmonem fluentis æqualis est motui molis Sanguineæ, quæ una Systole in Arteriam Pulmonalem projicitur, obtinentis eam velocitatem, qua percurratur longitudo Arteriarum ac Venarum Pulmonalium, tempore inter duos Pulsus intercepto. *Q. E. I.*

Si ponatur in Corpore Humano $\lambda = 1 \frac{1}{2}$ pes.

Erit motus Sanguinis in Pulmone æqualis motui ponderis 3 librarum, percurrentis unciale spatium minuto secundo.

Problema

Problema VI.

Definire momentum Sanguinis abſolutum in Vena Pulmonali.

Eodem argumento, quod in *Probl.* 4. uſurpatum eſt, invenitur motus quæſitus $= 2.5 \times \frac{q\lambda}{t}$. *Q. E. I.*

Poſitis vero iiſdem quæ ſupra ponuntur, motus abſolutus Sanguinis Pulmonem præterfluentis æquatur motui ponderis 7 $\frac{1}{2}$ librarum, quod ſingulis minutis ſecundis unciæ unius ſpatium percurrat.

Scholium. Experimento *Keiliano* definita eſt proportio, quam obtinet Sanguinis per Aortam ejuſque ramos fluentis velocitas naturalis, ad eam velocitatem qua Sanguis per eoſdem flueret, ſublata reſiſtentia Arteriarum & Sanguinis præcedentis. Eam nos proportionem ad Sanguinem per Arteriam Pulmonalem fluentem tranſtulimus. Quia vel ſublata vel imminuta ſecundum quamvis rationem reſiſtentia, quæ Sanguini per utramque Arteriam fluenti objicitur, neceſſario Sanguis pariter acceleratur in utraque Arteria. Id enim niſi fiat, bini Cordis Ventriculi aut eodem tempore non contrahentur, aut eandem Sanguinis quantitatem non ejicient. Quorum utrumvis, abſque ſumma totius Machinæ perturbatione & diſcrimine, fieri omnino non poteſt.

Corol. Ad tria Problemata præcedentia.

Sequuntur hinc Corollaria Problemati quinto ſubjuncta, mutatis mutandis.

Scholium ad quatuor Problemata ſuperiora.

Notandum Sanguinis velocitatem, tum per Pulmonem, tum per reliquum Corpus fluentis, cum reipſa æquabilis non ſit, hic tamen talem fingi, ut motus Sanguinis medius inveniatur.

Scholium generale.

Si cui numeri minus accurati videantur, qui sparsim Characteribus speciosis apponuntur, poterit ille facili opera, inventis per experimenta numeris qui propius ad verum accedant, motuum exempla supra posita, vel Propositionum ipsarum vel Corollariorum ope, corrigere. Ignoscat autem nobis Lector ingenuus, si per viam incedentibus nullis præcedentium vestigiis tritam, adeoque Erroribus in omnes partes opportunam, Humani aliquid forte acciderit.

Damus hanc veniam, petimusque vicissim.

IV. *An Account of the Sinking of three* Oaks *into the Ground, at* Manington *in the County of* Norfolk. *Communicated by* Peter Le Neve, *Esq;* Norroy *King at Arms, and Fellow of the* Royal Society.

ON *Tuesday July* the 23*d*, of the last Year, 1717, in the Grounds, and near the Seat of Sir *Charles Potts*, Baronet, in the County of *Norfolk*, and Parish of *Manington*, (which lies about mid-way between the Market Towns of *Holt* and *Aylsham*, and about seven Miles from the Coast near *Cromer)* in the day time, to the great astonishment of those that were present; first, one single Oak, with the Roots and Ground about it, was seen to subside and sink into the Earth, and not long after, at about 40 Yards distance, two other Oaks that were contiguous, sunk after the same manner, into a much larger Pit; being about 33 Foot Diameter, whereas the former is not fully 18. These, as they sunk, fell a-cross, so that obstructing each other, only the

the Roots of one of them reaches the Bottom, whereas the first stands Perpendicular.

When the first Tree sunk, it was observed, that the Water boyl'd up in the Hole; but upon the sinking of the greater Pit, that Water drain'd off into it, from the former, which now continues dry. The depth thereof to the firm Bottom is nine Foot three Inches; and the Tree that stands upright in it, is 3 Foot 8 Inches in Girt, and its Trunk about 18 Foot long, half of which is now within the Pit. In the Bottom of the greater Pit, there is a Pool of Water about 8 Foot Diameter; whose Syrface is 11 Foot 3 Inches below the Ground, and the Trees that are in this Pit, are much of the same length with the other, but somewhat smaller, the one being in Girt 3 Foot 5 Inches, the other but two Foot 9 Inches.

The Soil on which these Trees grew, is Gravelly; but the Bottom is a Quick-sand over a Clay, upon which there are Springs, which feed large Ponds adjoyning to Sir *Charles Potts*'s House, at about a quarter of a Mile from these Holes.

The Nature of the Soil seems to afford us a reasonable conjecture at the Cause of this odd accident, which some perhaps may be apt to reckon as a Prodigy. The Springs running over the Clay at the bottom of a Bed of very minute Sand, such as your Quicksands usually are, may reasonably be supposed in many Ages to have washt away the Sand, and to have thereby excavated a kind of Subterraneous Lake, over which these Trees grew: And the force of the Winds, on their Leaves and Branches, agitating their Roots, may well have loosened the Sand under them, and occasioned it to fall in, more frequently than elsewhere: whereby in length of time the thin Bed of Gravel being only left, it might become unable to support its own weight and

that

that of the Trees it bore That this is not a bare conjecture, may appear fr m the boyling up of the Water at first in the lesser Ho e, and its standing in the bigger and lower. And if it shall be found that it was a very windy day whereon this accident happen'd, it will much add to the probability of this solution

An accident not unlike this lately happened in *Fleet-street, London* by the defect of the arched Roof of a very deep Common-Sewer. The Earth gradually falling into the Sewer, was carried away by it, so as not to obstruct the Water; and the continual tremour of the Ground, occasioned by the constant passing of Carts and Coaches, by degrees shook down the Earth, so as to leave a very great C vern, the Top whereof at length grew so very thin, that one day a weighty Cart having just past it, a great space of the Pavement sunk in, in the middle of the Street, not without hazard to a Coach then driving by.

V. *A Rectification of the Motions of the five* Satellites *of* Saturn; *with some accurate Observations of them, made and Communicated by the Reverend Mr.* James Pound. R.S. Soc.

IT is now above thirty Years since that great Astronomer Mr. *Cassini* communicated to the World his discovery of two new Satellites of *Saturn*, which made their number Five; and the account he gave of them to the *Royal Society* (of which he was a Member) is to be seen in N°. 187, of these Transactions. Much about the same time the excellent M *Christian Huygens* of *Zulichem*, made the Society a present of the Glasses of a

Telescope

Telescope of 125 Foot length, with the *Apparatus* for using them without a Tube; by help whereof we might have satisfied our selves of the reality of these Discoveries. But those here that first tried to make use of this Glass, finding, for want of Practice, some difficulties in the Management thereof, were the occasion of its being laid aside for some time. Afterwards it was designed for making perpendicular Observations of the fixt Stars passing by our Zenith, to try if the Parallax of the *Earths* annual Orb might not be made sensible in so great a Radius, according to what Dr. *Hook* had long since proposed: but in this we miscarried also, for want of a place of sufficient height and firmness, whereon to fix the Object Glass, so that it lay by neglected for many Years.

In the mean time we could not but remark a great reserve in the *French* Astronomers, in relation to these Satellites, of which they have given us in their Yearly *Memoirs* no Observations till very lately, nor have they seemed willing to shew them in their Glasses to such as requested it: so that it might possibly occasion in some Persons a suspicion of the reality of this Discovery: And the Reverend Mr. *William Derham* having borrowed of the Society their long Glass, could not thereby assure himself that the small Stars he sometimes found about *Saturn*, were really his Satellites, their situation not agreeing with their places derived from the Tables of their Motions exhibited in N°. 187. of *Phil. Transact.* besides that he wanted a sufficient height to raise the Object Glass, so as to view *Saturn* to advantage, above the Vapour of the Horizon. But in the *Memoirs* for 1714, published but about a Year since, M. *Cassini*, the worthy Successor of his great Father, has given us some Observations which clear up the Point, and by shewing the errors of those first Tables, has enabled us to be assured,

that

that we have seen the whole *Satellitium* of *Saturn* our selves.

The Substance of these Observations is as follows.

Anno 1714 *Maii* 6. *St. N* about Mid-night, *Saturn* being then Stationary in ♍ $4^{\circ}.27'$, the Fifth and outermost Satellite was in its superiour Conjunction with the Planet, and at the same time, the Earth was nearly in the Plain of this Satellit's Orbit, so that it appeared to pass very near the Center of *Saturn*: From hence and from some other preceeding Observations, Mr. *Cassini* concludes that the Nodes of this Satellit's Orb are in 4 degrees of ♍ and ♓, and that its Inclination to the Ecliptick is not much more than half that of the other Satellites. Hence it should follow that the Ellipses it describes by its apparent motion about *Saturn*, when in ♊ and ♐ are much flatter and nearer to his Body, than those of the other four, which he allows to move in the plain of the *Ring*, and to have their Nodes in 21^{gr} of ♍ and ♓, with an Inclination to the Ecliptick of 31 degrees. To confirm this discovery, he produces another Observation of his Fathers, near Thirty Years before, *viz.* that, *Anno* 1685, *Maii* 31. *St. N* about Noon, the same Satellite was observed in superiour conjunction with *Saturn*, with less than one Diameter of the Ring *North* Latitude, *Saturn* being then in ♍ $11^{\circ}.48'$. So that the Satellite wanted but $7^{\circ}.21'$ of compleating 134 Revolutions, in the Interval of time between them. From these *Data* it was easy to settle the Theory of this Satellite.

As to the *Fourth* or the *Hugenian* Satellite; in the *Memoirs* for 1715, but just now come to hand, we find a very curious Observation of it, and the first of its kind, *viz.* that *Mart.* 25°. *S. N.* about 11^{h} *P. M.* this fourth Satellite, then in *Apogeo*, did immerge behind the Body of

Saturn

turn. With this Emendation the place of this Satellite may for the future be computed with a sufficient exactness.

The Third Satellite, by an original mistake in the Letters in N°. 187, is all wrong; its dayly Motion being there printed 2s. 18°. 41'. 50" instead of 2s. 15°. 41'. 50"; as may be perceived by the Period thereof being determined, in the aforesaid *Memoirs* of 1714, to be 4d. 12h. 25'. 12". that is, that it makes 400 Revolutions in 1807 days. This Satellite was observed by Mr. *Cassini*, *April* 4d. *St. N.* 10h. *P. M.* to have newly past its inferior conjunction with *Saturn*, and a perpendicular from it fell on the extremity of the western *Anse*, so that at about 5h. *P. M.* it was with the center of the Planet then in ♍. 5°. 23'. and consequently in ♓ 5°. 23'. But *ineunte anno Gregoriano* 1686, the *Epoche* thereof was ♍ 9°. 39'. So that from the Noon of the last of *December* 1685, to *April* 4d. 6h. 18' *anno* 1714, that is, in 10320 Days 6h. 18', there have been made 2284½ Revolutions of this Satellite to the Equinoctial; from which *Data*, the Tables of its Motion are readily derivable

The Radix of the penintime or second Satellite, according to the aforesaid Letter, *ineunte anno Greg.* 1686. was in ♍ 9°. 10'. But by the Observations of Mr. *Cassini* made the Nights before and after, this Satellite was in its superior Conjunction *anno* 1714. *April* 4d. 21h ½. *St. N.* that is, in ♍ 5°. 21', where *Saturn* then was: So that *April* 4d. 22h. 12', an entire Number of Revolutions were performed since the *Epoche* of 1686, that is, in 10320 Days 22h. 12: which Number can be no other than 3771, according to the Period thereof given in this *Memoire*, *viz.* 2d. 17h. 41'. 22".

Lastly the innermost or first Satellite, at the same time, *viz.* 1714, *April* 4°. 21h. 30'. *St. N.* was in its in-

feriour conjunction *proxime*, and consequently in ♓ 5° 21'. But the *Epoche* thereof for 1686, is ♑. 24°. 50'. Which place the Satellite had past 40^{gr}. 31' at the time of the Observation. This Arch it moves in 5^{h}. 6': Wherefore from the time of the *Epoche* to *April* 4^{d}. 16^{h}. 24', 1714, or in 10320 Days 16^{h}. 24'. the Satellite has performed 5467 Revolutions, its Period being determined to be 1 Day, 21 hours, 18'. 27", in this *Memoire*.

Having by the help of these late Observations corrected the motions of the Satellites which it was not possible for their first Discoverer to settle truly, in the short interval before 1687; and having fixed their *Epoches* for the present Year, we were enabled to know where to expect them with more certainty, and to distinguish them one from another, and from the small fixt Stars appearing with them. And the Reverend Mr. *James Pound*, (whose indefatigable Industry is no way inferiour to his incomparable Skill in Astronomical matters) having, by means of his Steeple of *Wansted*, provided a *Gnomon* high enough for the purpose, and having fitted a very commodious *Apparatus* for using the Society's aforesaid long Telescope, soon discovered by it all these five Satellites; and lately communicated to them the following very curious Observations.

1718. *April* 21^{d}. 10^{h}. 40'. The third and fourth Satellits of *Saturn* were in *Apogæo*, a little past their Conjunction with *Saturn*: A perpendicular from the fourth to the Transverse *Axis* of the Ring (or Line of the *Ansæ*) fell a little without the Eastern *Ansa*; and a Line through the fourth and third touched the Eastern Limb of *Saturn*. *Fig*. 17.

The first was Northward of the Line of the *Ansæ* (and therefore in the *Apogæon* Semicircle also) distant from the said Line about as far as the end of the Conjugate Axis of the Ring was from the Center of ♄, *viz.* nearly

nearly $\frac{1}{7}$ of *Saturns* Semidiameter; and it was about a Semidiameter of the Ring from the Weſtern *Anſa*.

The ſecond was a very little Southward of the Line of the *Anſæ* (and therefore in the *Perigæon* Semicircle) above a Semidiameter of the Ring (or about the Semidiameter of the Ring + the Semidiam. of ♄) from the Weſtern *Anſa*. And the Third, Firſt and Second were in a ſtrait Line.

At 10^h. 50$'$. A Perpendicular from the 3*d* to the Line of the *Anſæ* fell almoſt on the middle of the bright part of the Eaſtern *Anſa*, but ſomewhat nearer the Center than the ſaid middle.

April 22^d. 11^h. 5$'$. The four innermoſt Satellits were all Eaſtward of ♄. The 2*d* and 4*th* in the *Apogæon*, and the 1*ſt* and 3*d* in the *Perigæon* Semicircle. A Line through the 2*d* and 4*th* touched the *South Eaſt* Limb of ♄. A Line paſſing through the 3*d* and the end of the Conjugate *Axis* of the Ring, was parallel to the Line of the *Anſæ*.

At 11^h. 10$'$. A Perpendicular from the firſt to the Line of the *Anſæ* fell on the Eaſtern Extremity of the Ring. *Fig*. 18.

Theſe Diſtances and Directions were taken only by Eſtimation, and not by any actual Meaſurement.

The fifth (or outermoſt) Satellite being at this time near its greateſt Elongation Eaſtward, among ſeveral very ſmall Teleſcopick Stars, he could not determine its Poſition. But by obſerving the Motion of this ſome other Nights before, he was now fully ſatisfied, from the Motions rectified as above, that there are five Satellits of *Saturn*, as Mr. *Caſſini* had long ſince aſſerted.

In the bright part of each *Anſa* was a darkiſh Ellipſe nearer to the out ſide than the in-ſide of the Ring, as if it was compoſed of two Rings near to one another.

On

On the Body of ♄, beside the Ring on the *South* there appeared on the *North*-side a Zone not so far the Center as the Ring, and not much unlike the f est of *Jupiter's* Belts. These appearances were first notice of by Mr. *Cassini*, as may be seen in *Phil. T* N°. 128 pag 690 *Vide Fig.* 13.

We shall in our next give the Publick Tables of Motions, corrected from the aforesaid Observati instead of those in N°. 137. But it is not to be ex ed that these Satellites, exceedingly minute in t selves, and so faintly illuminated, should appear w the Air is but ordinarily Serene, they requiring not the *Medium* to be *summo modo* defecate and limpid withal in perfect Darkness. For which reasons it well be understood why the Gentlemen of the *R* Observatory may have sometimes made a difficul undertake to shew them upon demand.

FINIS.

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *May*, and *June*, 1718.

The CONTENTS.

I. *Tabulæ Motuum quinque* Satellitum Saturni *ad fidem nuperarum Observationum correctæ, Cœloque conformes redditæ.*

CIrca finem Anni 1686. D. *Jo. Dom. Cassini,* Reg. Soc. Sodalis, & in Astronomicis nemini secundus, cum Societate nostrâ inventa sua de motibus quinque Satellitum *Saturni* communicavit, Epochasque singulorum ad annum ineuntem 1686. eorumque motus diurnos in Epistola N° 187. harum *Transact.* editâ exhibuit: E quibus datis motuum Tabulas concinnavimus, dictæque Epistolæ subjunctas una edidimus. Cum vero deinde per triginta fere annos nullas omnino Oservationes eorum tradiderint, qui soli poterant, Astronomi *Galli*; cumque aliunde, ob intervallum temporis nimis breve, non nisi laxè periodos Satellitum, præsertim interiorum, definire potuerit præclarissimus Inventor; non prius dictarum Tabularum defectus corrigere datum est, quam in nuperis Actis Academiæ Regiæ *Parisiensis* Physicis & Mathematicis, observata ea, quæ sub finem præcedentis *Transact.* N° 355. protulimus, prodiere.

Horum vero ope facta aliquali Motuum castigatione, tum demum Telescopio *Hugeniano* omne Saturni Satellitium ipsi agnovimus; adhibitisque accuratis Rev. D. *Jac. Poundi* observationibus, Tabulas subsequentes cælo satis consonas obtinuimus. Addendo sc. motui annuo *Interioris*, 2 $^{gr.}$ 9′; *Penintimi* vero 3°. 25′. retentis Epochis D. *Cassini* ad Annum 1686. Augendo etiam motum annuum *Extimi* 9 min. sublatis vero 16 grad. ab *Epocha*, quæ in Epistola dicta N° 187. perperam scribitur ♓ 16°. 19′. pro ♓ 0°. 16′. *Hugenianum* 6′ annuatim tardiorem invenimus. *Tertii* autem Tabellas, ob motum diurnum falsò in Epistola illa traditum, de integro recudere necesse habuimus, retenta saltem *Epocha*.

Tabulæ

Tabula Mediorum Motuum Intimi Satellitis Saturni à Cassino *detecti Anno* 1686.

Annis Christi ineunt.	Epochæ	Annis	Mot. Med.	Diebus	Mot. Med.	H M	Mot. Med Sex. ° ,	Min.	Motus Medi.
	s ° '		s ° '		s ° '		° ' ''		° '
1681	♒ 8 48	1	4 4 43	1	6 10 42	1	0 7 57	31	4 6
1686	♉ 13 4	2	8 9 25	2	0 21 24	2	0 15 53	32	4 14
1701	♍ 6 34	3	0 14 8	3	7 2 6	3	0 23 50	33	4 22
1714	♎ 9 57	4	10 29 33	4	1 12 47	4	0 31 47	34	4 30
1715	♒ 14 39	5	3 4 16	5	7 23 29	5	0 39 44	35	4 38
1716	♊ 19 22	6	7 8 59	6	2 4 11	6	0 47 40	36	4 46
1717	♉ 4 47	7	11 13 42	7	8 14 53	7	0 55 37	37	4 54
1718	♍ 9 30	8	9 29 6	8	2 25 35	8	1 3 34	38	5 2
1719	♑ 14 13	9	2 3 49	9	9 6 17	9	1 11 31	39	5 10
1720	♉ 18 55	10	6 8 32	10	3 16 59	10	1 19 28	40	5 18
Mens. Anni Com.	Mot. Med.	11	10 13 15	11	9 27 41	11	1 27 24	41	5 26
		12	8 28 40	12	4 8 22	12	1 35 21	42	5 34
	s ° '	13	1 3 23	13	10 19 4	13	1 43 18	43	5 42
Jan.	0 0 0	14	5 8 5	14	4 29 46	14	1 51 15	44	5 50
Febr.	5 1 38	15	9 12 48	15	11 10 28	15	1 59 11	45	5 58
Mar.	3 1 10	16	7 28 13	16	5 21 10	16	2 7 8	46	6 5
April	8 2 48	17	0 2 56	17	0 1 52	17	2 15 5	47	6 13
Maii	6 23 44	18	4 7 39	18	6 12 34	18	2 23 1	48	6 21
Junii	11 25 22	19	8 12 21	19	0 23 16	19	2 30 58	49	6 29
Julii	10 16 19	20	6 27 46	20	7 3 57	20	2 38 55	50	6 37
Aug.	3 17 57	40	1 25 32	21	1 14 39	21	2 46 52	51	6 45
Sept.	8 19 35	60	8 23 19	22	7 25 21	22	2 54 49	52	6 53
Octob	7 10 31	80	3 21 5	23	2 6 3	23	3 2 45	53	7 1
Nov.	0 12 9	100	10 18 51	24	8 16 45	24	3 10 42	54	7 9
Dec.	11 3 5	120	5 15 37	25	2 27 27	25	3 18 39	55	7 17
In Anno Bissextili post Februarium adde unius diei motum.				26	9 8 9	26	3 26 35	56	7 25
				27	3 18 50	27	3 34 32	57	7 33
				28	9 29 32	28	3 42 28	58	7 41
				29	4 10 14	29	3 50 25	59	7 49
				30	10 20 56	30	3 58 22	60	7 57

Tabula Mediorum Motuum Satellitis Saturni *Penintimi, à* Cassino *detecti Anno* 1686.

Annis Jul. ineunt	*Epochae* s	°	'	*Annis*	*Med. Mot.* s	°	'	*Diebus*	*Med. Mot.* s	°	'	H M	*Mot. Med.* °	'	"	*Min.*	*Motus Medii.* °	'
1681	♓	3	23	1	4	10	2	1	4	11	32	1	0	5	29	31	2	50
1686	♉	5	25	2	8	20	4	2	8	23	4	2	0	10	58	32	2	56
1701	♓	22	3	3	1	0	6	3	1	4	36	3	0	16	26	33	3	1
1714	♑	7	5	4	9	21	40	4	5	16	8	4	0	21	55	34	3	7
1715	♉	17	7	5	2	1	42	5	9	27	40	5	0	27	24	35	3	12
1716	♍	27	9	6	6	11	44	6	2	9	12	6	0	32	53	36	3	17
1717	♊	18	43	7	10	21	46	7	6	20	44	7	0	38	22	37	3	23
1718	♎	28	45	8	7	13	20	8	11	2	16	8	0	43	51	38	3	28
1719	♓	8	47	9	11	23	22	9	3	13	49	9	0	49	19	39	3	34
1720	♋	18	49	10	4	3	24	10	7	25	21	10	0	54	48	40	3	40
				11	8	13	26	11	0	6	53	11	1	0	17	4	3	45
Mens. An. Com.	*Med. Mot.* s	°	'	12	5	5	0	12	4	18	25	12	1	5	46	42	3	51
				13	9	15	2	13	8	29	57	13	1	11	15	43	3	56
Jan.	0	0	0	14	1	25	4	14	1	11	29	14	1	16	44	44	4	1
Febr.	3	27	34	15	6	5	6	15	5	23	1	15	1	22	13	45	4	7
Mar.	6	20	32	16	2	26	40	16	10	4	33	16	1	27	42	46	4	12
Apr.	10	18	6	17	7	6	42	17	2	16	5	17	1	33	11	47	4	17
Maii	10	4	7	18	11	16	44	18	6	27	37	18	1	38	39	48	4	23
Junii	2	1	41	19	3	26	46	19	11	9	9	19	1	44	8	49	4	28
Julii	1	17	43	20	0	18	20	20	3	20	41	20	1	49	37	50	4	34
Aug.	5	15	17	40	1	6	40	21	8	2	13	21	1	55	6	51	4	39
Sept.	9	12	51	60	1	25	0	22	0	13	45	22	2	0	35	52	4	45
Oct.	8	28	53	80	2	13	20	23	4	25	17	23	2	6	4	53	4	5
Nov.	0	26	27	100	3	1	4	24	9	6	49	24	2	11	32	54	4	56
Dec.	0	12	28	120	3	20	0	25	1	18	22	25	2	17	1	55	5	1
								26	5	29	54	26	2	22	30	56	5	7
								27	10	11	26	27	2	27	59	57	5	12
								28	2	22	58	28	2	33	27	58	5	18
								29	7	4	30	29	2	38	56	59	5	23
								30	11	16	2	30	2	44	25	60	5	29

In Anno Bissextili post Februarium adde unius diei motum.

Tabula Mediorum Motuum Satellitis Saturni *Medii*, *à* Cassino *detecti Anno* 1671.

Annis Julia. ineunt	Epochæ s	°	'	Annis	Med. Motus s	°	'	Diebus	Med. Motus s	°	'	H M	Med. Mot Sex. °	'	"	M	Med. Mot '	"
1681	♍	12	16	1	9	17	2	1	2	19	41	1	0	3	19	31	1	43
1686	♏	27	6	2	7	4	3	2	5	9	23	2	0	6	38	32	1	46
1701	♎	1	17	3	4	21	5	3	7	29	4	3	0	9	58	33	1	49
1714	♎	11	43	4	4	27	48	4	10	18	46	4	0	13	17	34	1	53
1715	♋	28	45	5	2	14	50	5	1	8	27	5	0	16	36	35	1	56
1716	♉	15	47	6	0	1	52	6	3	28	9	6	0	19	55	36	2	0
1717	♉	22	30	7	9	18	53	7	6	17	50	7	0	23	15	37	2	3
1718	♓	9	32	8	9	25	36	8	9	7	31	8	0	26	34	38	2	6
1719	♐	26	34	9	7	12	38	9	11	27	13	9	0	29	53	39	2	10
1720	♎	13	35	10	4	29	40	10	2	16	54	10	0	33	12	40	2	13
Mens. Anni Com.	Med. Motus s	°	'	11	2	16	42	11	5	6	36	11	0	36	31	41	2	16
				12	2	23	25	12	7	26	17	12	0	39	51	42	2	19
				13	0	10	26	13	0	15	59	13	0	43	10	43	2	23
Jan.	0	0	0	14	9	27	28	14	1	5	40	14	0	46	29	44	2	26
Febr.	10	10	24	15	7	14	30	15	3	25	21	15	0	49	48	45	2	29
Mar.	0	21	44	16	7	21	13	16	6	15	3	16	0	53	8	46	2	33
Apr.	11	2	9	17	5	8	15	17	9	4	44	17	0	56	27	47	2	36
Maii	6	22	51	18	2	25	16	18	11	24	26	18	0	59	46	48	2	39
Jun.	5	3	16	19	0	12	18	19	2	14	7	19		3	5	49	2	43
Julii	0	23	58	20	0	19	1	20	5	3	49	20	1	6	24	50	2	46
Aug.	11	4	23	40	1	8	2	21	7	23	30	21	1	9	44	51	2	49
Sept.	9	14	47	60	1	27	4	22	10	13	11	22	1	13	3	52	2	53
Oct.	5	5	30	80	2	16	5	23	1	2	53	23	1	16	22	53	2	56
Nov	3	15	54	100	3	5	6	24	3	22	34	24	1	19	41	54	2	59
Dec.	11	6	37	120	3	24	7	25	6	12	16	25	1	23	1	55	3	3
								26	9	1	57	26	1	26	20	56	3	6
								27	11	21	39	27	1	29	39	57	3	9
								28	2	11	20	28	1	32	58	58	3	13
								29	5	1	1	29	1	36	17	59	3	16
								30	7	20	43	30	1	39	36	60	3	19

In Anno Bissextili post Februarium adde unius Diei motum

Tabula Mediorum Motuum Penextimi Satellitis Saturni *ab* Hugenio *inventi Anno* 1655.

Annis Jul. ineunt	*Epochæ* s	°	'	*Annis*	*Med. Mot.* s	°	'	*Diebus*	*Med. Mot.* s	°	'	*H* *M*	*Mo.Me.* ° '	' "	*Min.*	*Motus Medi.* '	"
1641	♑	2	48	1	10	20	35	1	0	22	35	1	0	56	31	29	10
1661	♓	13	23	2	9	11	10	2	1	15	9	2	1	53	32	30	6
1681	♈	27	58	3	8	1	45	3	2	7	44	3	2	49	33	31	3
1686	♏	3	28	4	7	14	55	4	3	0	18	4	3	46	34	31	59
1701	♊	12	33	5	6	5	30	5	3	22	53	5	4	42	35	32	55
1714	♓	17	53	6	4	26	5	6	4	15	28	6	5	39	36	33	52
1715	♒	8	28	7	3	16	40	7	5	8	2	7	6	35	37	34	48
1716	♐	29	3	8	2	29	50	8	6	0	37	8	7	31	38	35	45
1717	♐	12	13	9	1	20	25	9	6	23	12	9	8	28	39	36	41
1718	♏	2	48	10	0	11	0	10	7	15	46	10	9	24	40	37	38
1719	♍	23	23	11	11	1	35	11	8	8	21	11	10	21	41	38	34
1720	♌	13	58	12	10	14	45	12	9	0	55	12	11	17	42	39	31
1721	♋	27	8	13	9	5	20	13	9	23	30	13	12	14	43	40	27
Mens. An. Co	*Med. Mot.*			14	7	25	55	14	10	16	5	14	13	10	44	41	24
	s	°	'	15	6	16	30	15	11	8	39	15	14	7	45	42	20
Jan.	0	0	0	16	5	29	40	16	0	1	14	16	15	3	46	43	17
Febr.	11	9	54	17	4	20	15	17	0	23	48	17	16	0	47	44	13
Mar.	8	12	2	18	3	10	50	18	1	16	23	18	16	56	48	45	10
Apr.	7	21	55	19	2	1	25	19	2	8	58	19	17	52	49	46	6
Maii	6	9	14	20	1	14	35	20	3	1	32	20	18	49	50	47	3
Junii	5	19	7	40	2	29	10	21	3	24	7	21	19	45	51	47	59
Julii	4	6	26	60	4	13	45	22	4	16	42	22	20	42	52	48	56
Aug.	3	16	18	80	5	28	20	23	5	9	16	23	21	38	53	49	52
Sept.	2	26	12	100	7	12	55	24	6	1	51	24	22	35	54	50	49
Oct.	1	13	30	120	8	27	30	25	6	24	25	25	23	31	55	51	45
Nov.	0	23	24	140	10	12	5	26	7	17	0	26	24	27	56	52	42
Dec.	11	10	42	160	11	26	40	27	8	9	35	27	25	24	57	53	38
In Anno Bissextili post Februa-								28	9	2	9	28	26	20	58	54	35
rium adde unius diei motum.								29	9	24	44	29	27	17	59	55	31
								30	10	17	18	30	28	13	60	56	27

Tabula Mediorum Motuum Satellitis Saturni Extimi, à Cassino detecti Anno 1671.

Annis Julia. ineunt	Epochæ s	°	'	Annis	Med. Motus s	°	'	Diebus	Med. Motus s	°	'	H M	Mo. Me. ° ' '	"	M	Med. Mot. '	"
1681	♈	8	40	1	7	6	32	1	0	4	32	1	0	11	31	5	51
1686	♈	15	50	2	2	13	3	2	0	9	5	2	0	23	32	6	3
1701	♉	11	53	3	9	19	35	3	0	13	37	3	0	34	33	6	14
1714	♓	20	20	4	5	0	39	4	0	18	9	4	0	45	34	6	25
1715	♎	26	52	5	0	7	10	5	0	22	42	5	0	57	35	6	37
1716	♊	3	23	6	7	13	42	6	0	27	14	6	1	8	36	6	48
1717	♑	14	27	7	2	20	13	7	1	1	46	7	1	19	37	7	0
1718	♌	20	58	8	10	1	17	8	1	6	18	8	1	31	38	7	11
1719	♓	27	30	9	5	7	49	9	1	10	51	9	1	42	39	7	22
1720	♏	4	2	10	0	14	20	10	1	15	23	10	1	53	40	7	34
Menſ.	Med. Motus			11	7	20	52	11	1	19	55	11	2	5	41	7	45
Anni				12	3	1	56	12	1	24	28	12	2	16	42	7	56
Com.	s	°	'	13	10	8	27	13	1	29	0	13	2	27	43	8	8
Jan.	0	0	0	14	5	14	59	14	2	3	32	14	2	39	44	8	19
Febr.	4	20	41	15	0	21	30	15	2	8	5	15	2	50	45	8	30
Mar.	8	27	46	16	8	2	34	16	2	12	37	16	3	1	46	8	42
Apr.	1	18	27	17	3	9	6	17	2	17	9	17	3	13	47	8	53
Maii	6	4	37	18	10	15	37	18	2	21	42	18	3	24	48	9	4
Jun.	10	25	18	19	5	22	9	19	2	26	14	19	3	35	49	9	16
Julii	3	11	27	20	1	3	13	20	3	0	46	20	3	47	50	9	27
Aug.	8	2	9	40	2	6	26	21	3	5	18	21	3	58	51	9	38
Sept.	0	22	50	60	3	9	38	22	3	9	5	22	4	9	52	9	50
Oct.	5	8	59	80	4	12	51	23	3	14	23	23	4	21	53	10	1
Nov.	9	29	41	100	5	16	4	24	3	18	55	24	4	32	54	10	12
Dec.	2	15	50	120	6	19	17	25	3	23	28	25	4	43	55	10	24
								26	3	28	0	26	4	55	56	10	35
								27	4	2	32	27	5	6	57	10	46
								28	4	7	5	28	5	17	58	10	58
								29	4	11	37	29	5	29	59	11	9
								30	4	16	9	30	5	40	60	11	21

In Anno Bissextili post Februarium adde unius Diei motum.

Motibus mediis Satellitum ad hunc modum constitutis, proveniunt Revolutiones eorum jam veris proximæ, scilicet

	D	h	′	″
Primi sive Intimi	1.	21.	18.	26 $\frac{1}{3}$
Secundi Penintimi	2.	17.	41.	10 $\frac{1}{2}$
Tertii sive *Medii*	4.	12.	25.	10
Quarti Hugeniani	15.	22.	41.	28
Quinti sive *Extimi*	79.	7.	46.	00

Posito autem, juxta regulam Naturæ (saltem in hoc nostro Systemate) universalem, quæque tam in Jovialium ac Lunæ motibus, quam Planetarum Primariorum circa Solem, obtinet, Vires centrum Saturni petentes esse in duplicata ratione distantiarum reciprocè, ac proinde Cubos distantiarum à centro esse ut quadrata Temporum periodicorum; ex data distantia & periodo *Hugeniani*, fiunt reliquorum distantiæ ut sequitur.

		Semidiam. Annuli	*Semidiam. Globi* ♄
Dist.	*Primi*	1.9289	4,3400
	Secundi	2.4708	5,5593
	Tertii	3.4508	7,7643
	Quarti	8.0000	18.0000
	Quinti	23.3146	52,4578

Quæ quidem distantiæ cum D. *Cassini* observatis satis quadrant. Quatuor autem interiores Satellites juxta planum Annuli Saturni orbitas suas describunt proximè; in plano, *sc.* Æquatoris nostri plano quoad sensum parallelo, quicquid in contrarium proferant nonnulli. Quintum vero orbem situ paulo diversum à cæteris describere, nuper deprehendit D *Jac. Cassinus* prioris filius & virtutum hæres, ut videre est in Actis Academiæ Scientiarum *Parisiensis* Anni 1714. Sed hæc [redacted] omnia propriis oculis contemplari, atque penitius introspicere jam ipsi accingimur.

II. *The*

II. *The rest of the Treatise of that Learned Antiquary Dr.* John Tabor *of* Lewes (*whereof the First Part is publish'd in* N° 351. *of these* Transactions) *concerning the Site of the ancient City of* Anderida, *and other Remains of Antiquity in the County of* Sussex.

THE *former* Part *of this* curious Treatise *having found a just Esteem among several worthy Members of the* Royal Society, *who are Lovers of Antiquity, at Their instance we have adventur'd to insert here the Remainder thereof; entreating our Pholosophical and Mathematical Reader, to indulge the Liberty we now take, of breaking in upon the usual Subject of these Papers.*

Where *Tacitus* speaks of *Britain* and its Affairs, his Descriptions are so lively deliver'd, that one would think himself had been here, with his Wife's Father *Agricola*; and where he mentions the *Irish* [1] Prince, the Expression by him us'd seems to give Strength to such a Supposition.

The gaining the Southern part of this Island, was the greatest, if not the only Acquisition, made to the *Roman* Empire, from the Death of *Tiberius* to the Sixth Year of *Claudius*; which we may well suppose was not pass'd over in silence by that excellent Historian *Tacitus*: But his Four Books of Annals, which contain'd the Transactions of those Nine Years, we have reason enough to fear, are irretrievably lost. From the mention *Suetonius* makes of *Claudius* his Expedition hither; 'tis

Hhhhhh commonly

[1] Tac. Agric. cap. XXIV.

commonly insinuated his Conquest here [2] cost no Blood. Our Countryman *Bede*, we may see, was of that opinion; because, in the Account given by him of *Claudius*, the Words of *Suetonius* [3] are copied. But *Dio Cassius*, from whom we have the most particular Information of that War, gives a quite different Relation of the Matter: He takes notice of at least Four Battels, fought with the *Britons* (before *Claudius* came over) by *Aulus Plautius*; who had *Flavius Vespasianus*, *Flavius Sabinus*, and *Hosidius Gæta*, that commanded under him: In the first Conflict, *Cataratacus* was defeated; in the second, *Togodumnus*, and, as may be inferr'd from his Words afterwards, slain. From the manner of his delivering the Story, all those Battels seem to have been fought, South of the River *Thames*, and North of the *Sylva Anderida*, except the last; and that in the first Campagne the Conquests of *Plautius* could not have extended beyond *Kent* and *Surry*: For it's likely [4] that the Two first Actions happen'd about the Skirts of the *Sylva Anderida*, Eastward of the River *Medway*; and the Third, which held Two Days, on the Banks of that River; because, from the River, where they were routed Two Days successively, the *Britons* retiring, assembled [5] their Strength again before their Fourth Overthrow,

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[2] Suet. Claud. cap. 17. *Ac sine ullo prælio aut sanguine, intra paucissimos dies parte insulæ in deditionem recepta, sexto quam profectus erat mense Romam rediit* [3] Beda Eccles. Hist. Gent. Angl. Lib. I. cap. 3.

[4] Dionis Cassii Hist. Rom. Lib. LX. Claud. V p. 768. A. Οἱ γὰρ Βρεττανοὶ μὴ προσδοκήσαντες αὐτοὺς δι' ἅπερ ἐπυνθάνοντο ἥξειν, οὐ προσυνελέγησαν, οὐ μὴν οὐδὲ τότε ἐς χεῖρας αὐτοῖς ἦλθον, ἀλλ' ἔς τε τὰ ἕλη καὶ ἐς τὰς ὕλας κατέφυγον, ἐλπίσαντες σφᾶς ἄλλως κατατρίψειν, ὥσθ' ὥσπερ ἐπὶ τοῦ Καίσαρος τοῦ Ἰουλίου ἐγένετο, διακενῆς αὐτοὺς ἀναπλεῦσαι. Ὁ οὖν Πλαύτιος πολλὰ μὲν πράγματα ἀναζητῶν σφᾶς ἔσχεν· ἐπεὶ δὲ εὗρέ ποτε, πρῶτον μὲν Καράτακον, ἔπειτα Τογόδουμνον, Κυνοβελλίνου παῖδας, ἐνίκησεν.—— φυγόντων δὲ ἐκείνων —— [illegible] ἐπὶ πολέμῳ τινὶ ἐγένοντο. [5] Pag. 678. D. Ἀναχωρησάντων δ' ἐντεῦθεν τῶν Βρεττανῶν ἐπὶ τὸν Ταμέσαν ποταμὸν, καθ' ὃ ἔς τε τὸν ὠκεανὸν ἐκβάλλει, πλημμυροῦντός τε αὐτοῦ λιμνάζει.

throw, in that part of *Kent* which borders on the *Thames*, not far from its entrance into the Sea; and having pass'd it, were follow'd by *Plautius* his *Germans*, and on the other side put to flight; which was the Fourth Action mention'd by *Dio*. *Claudius* having been sent for, comes the Second Year with powerful Succours to the Assistance of *Plautius*; who with his Forces waited his Arrival near the *Thames*, not unlikely still where he quarter'd in the Winter; which perhaps was in that large strong Camp, as yet to be seen [6] not far from *Bromly* in *Kent*, on the River *Ravensbourn*. The Emperour joining him [7], immediately cross'd the *Thames*; overthrew the *Britons* posted on the other side to resist him; advanced to *Cynobelin*'s chief Residence *Camalodunum*, and took it: Then receiving Homage of some States, return'd to *Rome*.

Considering therefore that *Claudius* staid but Sixteen Days [8] in this Island, we must conclude his Dispatch was great; and that his Progress could not have been through more Parts than *Kent*, *Essex*, *Hertfordshire*, *Middlesex*, and *Surry*. As to what else relates to the *British* War in the time of *Claudius*, save that Three Years after *Titus* rescued his Father *Vespasian* when in great danger, we have no Account from *Dio*. But where *Suetonius* [9] treats of *Vespasian*'s Life; we are told, when that Emperour commanded in *Britain* for *Claudius*; that he fought Thirty Battels, subdu'd Two of the

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[6] Camden Brit. Edit. 1695. Col. 213. c. [7] Dion. Cassii. Hist. Rom. Lib. LX. pag. 679. B. Κἀντεῦθεν τὰ μὲν πεζῇ, τὰ δὲ καὶ διὰ τῶν ποταμῶν πορευόμενος, πρός τε τὸν Ὠκεανὸν ἀφίκετο καὶ περαιωθεὶς ἐς τὴν Βρεττανίαν, συνέμιξε τοῖς στρατοπέδοις πρὸς τῷ Ταμέσᾳ ἀναμένουσιν αὐτόν. καὶ παραλαβὼν σφᾶς, ἐκεῖνόν τε ἐπεδιέβη, καὶ τοῖς βαρβάροις πρὸς τὴν ἔφοδον αὐτοῦ συνεστραμμένοις ἐς χεῖρας ἐλθὼν, μάχῃ τε ἐνίκησε [8] Dion. Lib LX pag. 680. B. Ἦλθέ τε ἐς τὴν Ῥώμην ὁ Κλαύδιος, ἓξ μῆνας ἀποδημήσας (ἀφ' ὧν ἑκκαίδεκα μόνας ἐν τῇ Βρεττανίᾳ ἡμέρας ἐποίησε.) [9] Suet. Vespasian. cap. 4.

most powerful Nations, won Twenty Towns, and brought the Isle of *Wight* under the *Roman* Obedience. Of which Actions, besides what might have been said in the lost Books of Annals; *Tacitus*, in other Pieces of his, largely [1] hints, that when *Claudius* rul'd, *Vespasian's* Behaviour and Success in this Island, shew'd to the World his Conduct and Courage in the Affairs of War: The same is also taken notice of [2] by *Dio*. From his Conquest of the Isle of *Wight*, it may be imply'd, the Stage of his Actions here, was in those Countries which border on the South Channel rather than in the North: Since therefore the Clime, the Soil, and the more ready Conveniencies for foreign Trade and Correspondence, might entitle this Part of the Land, to sustain as numerous, as stout, and as experienc'd a People as any other (because *Cæsar* [3] takes notice they not only lent Aids to the *Veneti* in their Revolt, but were wont to assist the *Gauls* in most of their Wars against [4] the *Romans*:) And whereas no Historian afterwards mentions any Disturbance given to the *Romans* from the Southern Parts; we may conclude, *Vespasian* entirely subdu'd them; and that before he left the Island, the Methods he establish'd for securing Peace, were no way inferior to those he had shewn in making War.

The

[1] Tacit. Agricol. cap. xiii. *Divus Claudius auctor operis transvectis legionibus auxiliisque, & assumpto in partem rerum Vespasiano; quod initium venturæ mox fortunæ fuit, domitæ gentes, capti Reges, & monstratus fatis Vespasianus.* Tacit. Hist. Lib. III. cap. xliv. *Et Britanniam inclytus erga Vespasianum favor, quod illic secundæ Legioni à Claudio præpositus, & bello clarus egerat, non sine motu adjunxit cæterarum.* [2] Dion Cass. Hist. Rom. Lib. LXV. p. 736. C. ἥ τε γὰρ τῶν ἀνθρώπων εὔνοια πολλὴ ἦν πρὸς αὐτὸν —— ἥ γὰρ ἐκ τῆς Βρεττανίας δόξα, καὶ ἐκ τῶν ἐν χερσὶ πολέμων συνέλεγε.
[3] De Bello Gal. Lib. III. *Socios sibi ad id Bellum Osismios, Lexobios, &c auxilia ex Britannia, quæ contra eas Regiones posita est, accersunt.*
[4] Idem Lib. IV. *Tamen in Britanniam proficisci contendit, quod, omnibus ferè Gallicis Bellis, hostibus nostris inde subministrata auxilia intelligebat.*

The *Romans* well knew, that those who were Strangers to Civility, could not without great Difficulty be kept in Obedience: As soon therefore as the Countries they had conquer'd, were reduced to some degree of Quiet; they endeavour'd to make the People in love with their Government, by introducing their Arts and Customs among them: From that inconsiderable Instance recorded [5] by *Pliny*, we may see, how ready the *Romans* were, to oblige the People under their Power, with any Curiosity that might entertain their Sences, in order to endear them to the Authority they had over them. (He tells us, Cherries were not known in *Italy*, till the 680th Year of *Rome*, when *L. Lucullus* first brought them thither from *Pontus*; and that in a Hundred and Twenty Years, they were so increas'd, that not only many other Countries, but *Britain* also was supply'd with them; which must have been about Three Years after *Claudius* himself had been here. The usual Landing from *Rome* being then in the County of *Kent*; that Fruit without question was there first planted; and the Soil well agreeing with it, may be the reason that the best and greatest Quantity of it is yet there to be had.)

Agricola, in the Second Year of his Lieutenancy here, when in Winter-Quarters, pursu'd the same Maxims (which *Tacitus* terms *Saluberrima Consilia*; and, as it may be inferr'd from an Expression of [6] *Cæsar* conducive to the same End) to gain the *Britons*, by making them acquainted with the *Roman* Manners: He not only in private persuaded, but publickly help'd and

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[5] Plin. Lib. XV. cap xxv.

[6] De Bell. Gallic. Lib. I. *Horum omnium fortissimi sunt Belgæ: Propterea quod à cultu atque humanitate Provinciæ longissime absunt, minimeque ad eos Mercatores sæpe commeant, atque ea quæ ad effeminandos animos pertinent, important.*

and incourag'd them to build Temples, Places for common Aſſemblies, and private Houſes after the *Roman* Mode: He took care to have the principal Youth inſtructed in the Liberal Arts: He allur'd them to affect the Habit of the *Romans*: And laſt of all, to engage them the more firmly, help'd them to a Taſt of the *Roman* Luxury and Goodfellowſhip, by introducing the Uſe of ſhady Piazzas and Baths [7], and their way of Banqueting. But here, *Tacitus* may be underſtood to ſpeak of what was done in order to civilize the Northern Parts of this Nation, where *Agricola*'s Preſence was required: The Southern was, we may ſuppoſe, ſoftned and quieted by the ſame Methods near Forty Years before, when reduced by *Veſpaſian*.

From hence it may be inferr'd; that ſhould never any other Tokens of the Antiquity of theſe Works be found; yet would the Bath denote the Age of the Pavement, and ſet it near as high as the moſt early Time, that the *Romans* had any real Authority in this Iſland.

As by the Loſs of ſome of the Annals of *Tacitus*, we may have been depriv'd of the moſt early Hiſtory of this County; ſo likewiſe, for want of antient Religious Houſes; there has been little or no Accounts left of its Circumſtances, in the Times next after the *Roman* Authority expired here. *Malmsbury* [8] ſays, that in his Time, there were here only the Abbies of *Battell* and *Lewes*, and thoſe not long erected. The earlieſt Mention made of it, is by [9] *Bede*, who informs us, that Biſhop *Wilfrid*, in the Year 678. being thruſt out of his Province of *Northumbria* by King *Ecgfrid*, ſetled at *Selſey* in 680. and ſtaid Five Years, labouring in

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[7] Tacit. Agricol. cap. xxi. [8] Gul. Malmsb. de geſtis Pontific. Angl. Lib. II. [9] Bedæ Hiſt. Eccleſ. Lib. IV. cap. xiii.

in the Conversion of the neighbouring Parts; but of what else relates to the County, save the miserable Ignorance of the Inhabitants, and the Number of Families, he has left no Account. *Bede* spent most of his Time in the Monasteries of *Wiremouth* and *Jarrow*, and travel'd little; so, that considering the Distance from thence to this County, and the different Governments and Interests that lay between, he may well be excus'd for the few Particulars he has left us of it.

The next Records we have to view are those of *Ethelwerd*, the *Chronicon Saxonicum*, and *Henry* Archdeacon of *Huntingdon*. But that you may the more clearly apprehend the antient State of this County; look into the best Map of it you can get. At the West End, you will find *West-Harting* and *Stansted*, distant from each other Six or Seven Miles; imagin a streight Line to be drawn from *Harting* to *Bourne* near *Pevensey*, and another to be drawn from a Point which must be little South of *Stanstead* to *Brighthelmstone*; What lies North of these Lines is the Weald or Lowlands, formerly the *Sylva Anderida*; that which is comprehended between these Lines, and bounded by the Sea, from *Brighthelmstone* to *Bourne*, is the *Downs*, so famous for their pleasant Situation and Fruitfulness. The Part South of these Lines, is a flat champain Ground, ending like a Wedge at *Brighthelmstone*. These two last Parts were those only that were inhabited in *Bede*'s Time; they contain not more than Two Fifths of the whole County; which must be the reason why *Bede* said, *Sussex* [1] consisted not of more than 7000 Families or Farms; whereas in another place he computes *Kent* to have 15000 Families.

In

[1] Bedæ Hist. Eccles. Lib. IV. cap. xiii.

In the three Accounts [1] above-mention'd 'tis agreed, that in the Year 477. *Ella*, with his Three Sons *Cymen*, *Wlencing*, and *Cissa*, landed his Forces at *Cymenes-Ora* (which from a Charter of King *Cedwallas* to the Church of *Selsey* the learned [2] *Cambden* proves to be about *Wittering* near *Selsey*;) not far from which he routed the *Britons*, and drove them into the Weald (Andredesleige): Their farther Progress is most distinctly and naturally deliver'd by the Archdeacon of *Huntingdon*, in these Words; *Saxones autem occuparunt littora Maris in* Sudsexe, *magis magisque sibi regionis spatia capessentes, usque ad nonum annum adventus eorum. Tunc verò cum audacius regionem in longinquum capesserent; convenerunt Reges & Tyranni Brittonum apud* Mercredesburne, *& pugnaverunt contra* Elle *& filios suos, & fere dubia fuit victoria. Uterque enim Exercitus valdè læsus & minoratus, alterius congressum devovens, ad propria remearunt. Misit igitur* Elle *ad compatriotas suos auxilium flagitans.*

This County having been invaded in the most Western part of it by the *Saxons*; if what they did afterwards, was to possess themselves of it; their Progress must have been from West to East. And so much *Henr. Huntingdon*'s Words plainly imply. He says farther, they were Eight Years about it; which, if we consider the Circumstances of the Country, 'twill be no great wonder it should take up so much Time; unless their Forces had been very great, which we have no warrant from any History to suppose: For the *Weald* then uncultivated, must have been most difficult to pass, even in the driest Summers. The *Downs*, like a Wall (with a Terras-Walk on the top) have a very steep Descent

[1] Ethelward Hist. Lib. I Cap. 5. Chronic. Saxon. Ann. CCCCLXXVII. Hen. Hunt. Hist. Lib. II. [2] Camden Brit. Sussex.

Deſcent into it, their whole Length; excepting, that every Ten Miles, or thereabouts, they have deep Channels through them to afford Paſſage for the Rivers into the Sea: Therefore, what was then habitable, being thus canton'd out into ſo many Parcels by the Rivers; nothing could be more difficult to gain, than thoſe Cantonments; were there any Forces to defend the Paſſes that ſhould have been attempted; the Rivers being deep and muddy, and the Moraſſes on each ſide broad and boggy: Hence we may conceive, 'twas no very difficult Task for the *Britons* to defend, nor an eaſy one, for the *Saxons* to gain the Country. And indeed, the many old Camps, ſtill to be ſeen on the *Downs*, are an Evidence that ſcarce any part eſcaped being a Scene of War. Mr. *Camden* mentions but two, *Ciſsbury* and *Chenkbury*. In the new Edition of his Works Dr. *Harris* has added Three more; a *Roman* Camp at the *Brile* near *Chicheſter*, St. *Rooks-hill*, and *Gons-hill* near the Weſt Limits of the County. It may not be improper here to inſert an Account of the reſt; in which, I ſhall firſt take notice of thoſe that are on the North Edge of the *Downs*, and overlook the *Weald*.

Firſt, *Chenkbury*, mention'd by Mr. *Cambden*, Two Miles Weſt of *Steyning*, and about Three Miles North of *Ciſsbury*; 'tis circular; its Circumference about two Furlongs. From *Chenkbury* Eight Miles Eaſt, over *Poynings*, is a very large one, an Oval, not leſs than a Mile round; acceſſible at one narrow Neck only, and that fortify'd, with a deep broad Ditch, and a very high Bank: I could never learn any other Name it has gone by, than *Poor-Mans Wall*; perhaps from its having been a Security to the diſtreſſed *Britons*. About Three Miles Eaſt from thence, is *Wolſenbury*, on a Hill, projected beyond the reſt of the *Downs*, like a Baſtion; it comes near a Circle in ſhape; its Diameter a little

more than a Furlong. Near Three Miles East of *Wolsenbury*, on the highest part of the *Downs* in that Quarter, is a Camp, near square, about 60 Rods long, and 50 broad; much like a *Roman* Camp; the side next the North is secur'd by the Precipice of the Hill, which is both very deep and steep; the other Three Sides have each their *Porta* after the *Roman* manner still very visible; the Ditch seems to have been not less than Eleven Foot broad; but the Ground having been plough'd, the Bank is but low: This is call'd *Ditchling*, as is the old Town under it. Near Seven Miles farther East, and a Mile and half East of *Lewes*, is the last on the North Edge of the *Downs*; it goes by the Name of *Caburn*; which perhaps is but a Corruption of the *British* Word *Cadir*; the Parish below it still retains its *British* Name *Glynd*: This is a round Camp, scarce Three Furlongs in Circuit; its Ditch very broad and deep, and the Rampart within very high; the Places where the Tents were pitch'd are yet visible; which, from the Strength of the Out-Works, intimates that those within held it no small time. Near a Quarter of a Mile West of it, there is a strong Work much larger, but not so perfect; yet enough to shew, it was made to secure a Power, that might lie there to bridle those in the strong Camp, and prevent their making Excursions towards *Lewes*.

The Camps on the Southern Limits of the *Downs*, are St. *Rooks* near *Chichester*. *High-Down*, a small Square, Four Miles East of *Arundell*, and in the Parish of *Goring*: *Cissbury*, Four Miles South-West of *Steyning*. *Hollingbury* is the only one in the middle of the *Downs*, Two Miles North of *Brighthelmstone*, and Three Miles South of *Ditchling*; 'tis a Square; the *Porta* still remaining; it contains about Five Acres. A Mile East of *Brighthelmstone* on the top of a Hill, half a Mile from the Sea,

Sea, is a Camp, which has a triple Ditch and Bank; this also is a Square, only the Corners are rounding; the outmost Trench measures about three Quarters of a Mile. In the Parish of *Telscomb*, about Five Miles East of the last, are two, but both imperfect; the Cliff is a South-Fence to One; the Other is a Mile distant from it; their West Sides are both finish'd with very able Works; they were design'd for Squares, and to contain 12 or 15 Acres. At *Meeching* or *Newhaven*, on the Point of the Hill, which overlooks the Harbour's Mouth from the West, is a Fortification which they call *the Castle*; its Banks are very high, the Shape near half oval, containing about Six Acres; formerly it might be much more, because the Cliff, which forms the Diameter, every Year more or less moulders away, and falls into the Sea. Near a Mile East of *Seaford* is another call'd also *the Castle*, bounded by the Cliff on the South; its Figure almost semicircular, the Trench and Rampart large, inclosing Twelve Acres. Three Miles East of *Cukmere* Haven is the last, near a narrow Pass coming up from the Sea call'd *Burling-gap*; it incloseth a Hill nam'd *Belltout* of a half oval Shape; the Works have the same Figure, and measure about three Quarters of a Mile; the Cliff here also makes the Diameter.

Though neither History nor Tradition, has handed to us any Relation, when either of these Works were made or by whom us'd (except *Cissbury* by *Cissa*) yet from this View we may conceive, the Calamity of War once rag'd in all these Parts: that the Ground was disputed inch by inch: that in the Attack, as well as Defence of it, the Pick-Axe and Spade, were as much made use of, as the Sword: and lastly, that, unless the Aggressors were very numerous, eight Years was no long time taken up, in dispossessing the Inhabitants of this fast Country.

Some may imagine, many of these Camps were made by the *Danes*; but by what may be observ'd from the History of those Times, that People seem'd not to be so formal an Enemy, as to prolong War by Encampments: Their Refuge was in their Fleets that always attended them; so that, when likely to be vigorously oppos'd, they betook themselves to their Ships, and suddenly invaded another Part where was less Opposition: and what they could not carry with them, consum'd with Fire and Sword. Thus continually harassing the Nation by their hasty and rapacious Visits, they exhausted it of its Riches and Strength, and as it were imitating the Quality of the Faulcon their Ensign, they flew the Prey to a Stand, and then seiz'd it.

The Archdeacon of *Huntingdon*, in the Prologue or Dedication of his Annals, to *Alexander* Bishop of *Lincoln*, assures his Diocesan, that he compil'd his History from Chronicles reserv'd in ancient Libraries: no question therefore, when speaking of the *Saxons* here, he had good Authority to say (as above cited), *magis magisque sibi Regionis spatia capessentes*; and that no other Meaning could belong to it; than that they carried their Cônquest from West to East, *in longinquum*, lengthways. Had they entirely made themselves Masters of the Country, 'twould have been too late: But before they had wholly gain'd it, the *Britons* assembled against them; the *Saxon* Chronicle says **neah**, i. e. *propè*; *Ethelwerd*, *juxta*; or, as *Huntingdon* has it, *apud* **Mercredesburne**; where a Battle was so hard fought, that each Side had enough on't, and retir'd. The *Saxons* were so diminish'd, that *Ella* was oblig'd to send for more Forces. This Action was in the Ninth Year after *Ella*'s first footing here, Three Years before *Hengist*'s Death, *Ann. Dom.* 485. It so weaken'd *Ella*, that we hear no more of him till he receiv'd his Supplies

from

from *Germany*; which came not, according to *H. Huntingdon*, till the first Year of the Emperour *Anastasius*. Three Years after *Hengist*'s Death, and Six Years after the hard Battel, *viz. An. Dom.* 491.

Being thus strengthned, *Ella* mov'd again, besieg'd *Anderida* (in *Huntingdon*'s Words, *Urbem munitissimam*) at last forced the Place; and by reason of the stout Resistance the Defendants made, Savage like, left not a Soul alive, and raz'd the City, which in *Huntingdon*'s Time remain'd desolate.

As to the Field where the Battel was fought; the *Saxons* extending their Power Eastward, the Check that was given them, in all probability must have been where they push'd on their Victories; and it being near Mercredesburn, this *Bourne* near *Pevensey* may be the Place meant, since it sounds like the latter part of that Name (for there not being a West *Bourne* that it relates to, the Name of it may rather be *Esbourne* than East-*Bourne*;) and likewise that *Anderida*, the *Britons* last Stake and Support, was not far from it. 'Tis probable therefore the Battel was fought on the *Downs*, between the Camp last mention'd at *Burling-Gap* and East *Bourne*; for there are no where on the *Downs*, that I have seen (and there are few Parts of them that I have not often view'd), Marks of a greater Battel than there; because, from the top of that very high Cliff, by the Inhabitants call'd *The Three Charles* (and by Mariners *Beachy-Head*) to *Willington Hill*, which is four Miles, the Ground is full of large *Tumuli* or Places of Burial; and in many parts within that Tract, where the Position of the Ground seems to offer, there are deep Trenches and Banks, which one would imagin were Breast-Works made to defend the Front of an Army; and the *Tumuli* on each side of them seem to shew, there was no small Struggle, in forcing as well as defending them.

The Learned and Judicious Mr. *Somner* [1] dislikes, that the Site of *Anderida* should be fix'd at *Newenden*, and is inclin'd to assign some Place in *Sussex* for it: But from a modest Deference to the Opinions of the Learned *Camden* and *Selden*, he drops the matter.

But let us see, what our more elder Historians say of it; *Henry* of *Huntindon*'s Words are, *Et quia tot ibi damna toleraverant Extranei, ita Urbem destruxerunt, quod nunquam postea readificata est. Locus tantum, quasi nobilissimæ urbis, transeuntibus ostenditur desolatus.* *Mathew* of *Westminster* says, *Locus autem Civitatis usque hodie transeuntibus ostenditur desolatus. Mansit ergo ibidem* Ella *cum tribus Filiis suis, & Regionem illam, quæ usque hodie Anglice* Suthsex, *Latine autem Regio Australium Saxonum dicitur, colere cœpit.* From the Expressions above cited, it may be suppos'd the Ground where that City stood was not quite forgot, in either of those Historian's Days. *Henry* of *Huntindon* being the elder by 200 Years (had *Newenden* been the Place), his Words might have been true, in saying it was desolate: But 'tis very improbable *Mathew* of *Westminster* should have said so likewise; or at least, not taken notice of the Act of Piety and Charity of Sir *Thomas Albuger*, who, in his Time, had newly erected a Monastery at *Newenden* [2] for the *Carmelites* who came from *Palestine*: But let that pass: what Authority Mr. *Camden* had for saying [2] *Hengist* sent for *Ella* out of *Germany*, to help him reduce *Anderida*, is not to be found. From the Accounts above stated, and others that might be produced, it is clear, that *Hengist* was dead Three Years before the Siege was laid to *Anderida*. In the Time of *Hengist*'s Life, we find, for Eight Years

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[1] Somner's *Roman Ports and Forts in* Kent, p. 106. [2] *Camd. Brit. Kent Edit.* 1695. *Col.* 211.

Years *Ella* had enough to do in *Sussex*; and the Blow he had given him the Ninth Year at Mearcredsburne, oblig'd him to be quiet the other two Years of *Hengist*, and till his Succours (as above-mention'd) came to him from *Germany*: Besides, we have not the least Hint from any of our Historians, that *Anderida* was an Eye-sore, either to *Hengist* or his Son *Esk* after him; or that *Ella* assisted the *Kentish Saxons*, or the *Kentish Saxons Ella* in reducing it: Therefore this must be a Supposition only of Mr. *Camden*, in order to give Strength to the Notion of *Anderida's* being at *Newenden*. Taking no notice therefore of that Supposition, we may consider *Newenden* is on the *Kent* side of the *Limen* (for so is the River *Rother* call'd [3] in the *Saxon* Annals, and by *Mathew Westminster*; and the Mouth of it nam'd *Portus Limeneus*, and Limene by *Ethelwerd* [4] and *Henr. Huntindon*;) and that *Kent* having been subdued by *Hengist* and his *Saxons*, near Forty Years before; the Town at the Mouth of the *Limen*, and the rest, if any, up the Stream on the side of *Kent*, were also part of their Conquest.

Furthermore, after it had cost *Ella* so much Time, and no doubt Pains too, in reducing the plain Ground of *Sussex*, 'tis not likely he should call more Forces out of *Germany*, that he might lead them Thirty Miles, through the Difficulties of the great Wood (which he must have done if *Newenden* were the Place,) to besiege a City, so far from his own, and within the *Kentish-Saxon* Limits, especially if there's any heed to be given to the Words of *Math. Westminster* before cited; who, after relating the sad Fate of the Inhabitants and City

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[3] Chron. Sax. A. Dom DCCCXCIII. Mat. Westm. Fl. Hist. A. Dom. DCCCXCII. [4] Ethelwerd. Lib. III. cap. iii. A. D. DCCCXCIII. Hen. Hunt. Hist. Lib. V. Alfr. Reg. an. 19.

City of *Anderida*, immediately subjoins, *Mansit ergo*, &c. Ella *and his Sons resided there* (*i. e.* in that part of *Sussex* where *Anderida* was), *and began to cultivate and improve the Country*.

In the last place, from the Use made of *Anderida* by the *Romans*, 'tis not likely (as Mr. *Somner* [5] judiciously hints) its Place was at *Newenden*; for being one of the Stations, under the *Praefectus littoris Saxonici*, where Forces were quarter'd, to have a watchful Eye on the Sea, when ever the *Saxon* Pyrats came to infest the Coast: We may suppose it, like the rest of the Garisons under that Officer, conveniently situated for the same purpose; as were *Branodunum* [6] *Brancaster* at the North Point of *Northfolk*: *Gariannonum*, North-*Yarmouth*, or very near it; *Othona*, *Ithanchester* in *Dengy* Hundred, in *Essex*, some Ages since swallowed up by the Sea; *Regulbium*, *Reculver* in *Kent*; *Rutupis*, *Richborow*; *Dubris* *Dover*; *Lemannis* (which from the *Saxon* Chronicle [7] we must look for, Four Miles East of *Appledore*) probably New *Romney*, all situate near the Sea, on Ground which had a full Prospect of the Sea: whereas *Newenden* lies low, at least Eight Miles within *Appledore*, on a turning of the River, where the Land Eastward must have cut off any Prospect of the Sea. To all this may be added, that the *Romans* having a *Numerus*, Cohort, or Battalion of the *Turnacenses*, in Garison at the *Portus Lemanis* on the Mouth of the Haven, we may suppose they knew how to husband their Strength to better purpose, than to place another

[5] Somner Rom. *Ports and Forts*, pag. 103. [6] Not. Imperii à Pancirol. cap. lxxiii. pag. 162. [7] Chron. Sax. A. Dom. DCCCXCIII. *Tum appulerunt* (*sc. Dani*) *in Limeni ostium, cum* CCL. *Navibus. Super eum Fluvium traxerunt suas Naves usque ad sylvam, quatuor millariis ab exteriore parte aestuarii; ibique expugnarunt quoddam munimentum* (*sc. Apuldre.*)

another Garison to watch the Motions of the *Saxon* Rovers, Twelve Miles up the little River, quite out of sight of the Sea, where they could be of no Service.

Those who would have the Seat of *Anderida* to have been at *Hastings*; let them look on these Words of *Henr. Huntindon* [1] (*Haraldus rex Anglorum, eadem die reversus ad* Eouirwic *cum summa lætitia, dum pranderet, audivit nuntium dicentem sibi, Willielmus dux Normanniæ littora Australia occupavit, & castellum construxit apud* Hastings,) and they will conclude *Hastings* was not a desolate place, in the Ages of the Historians, who affirm *Anderida* was: If at *Pevensey*; that Place was so far from being raz'd by *Ella*, that even after the *Norman* Conquest it remain'd a strong Castle, where *Odo*, Bishop of *Bayon* and his Forces sustain'd a Six Weeks Siege; and for want of Provision were oblig'd to surrender to K. *William* II. At this time there is so much of *Pevensey* standing, that perhaps 'tis the greatest and most entire Remain of *Roman* Building, any where to be seen in *Great Britain*.

From the Arguments on the foregoing Authorities, *Anderida* must have been somewhere in *Sussex*, not in the West but East part of it, and not far from the East End of the *Downs*, near the Sea. From the Bath, Pavement, Coins, and Bricks, 'tis sure the *Romans* had once an Abode, and not a short one, at this Place near *East-Bourne*: From the large Extent of Foundations about the Place where these were discover'd; that there was a large Town or City there: From the common Height those Foundations bare under the Surface of the Ground; that the Buildings they sustain'd were effectually levell'd or raz'd: And from the Coals dug up amongst the Rubbish, 'tis evident that Part was burnt; all which Circumstances

[1] *Henr. Huntindon*, Hist. Lib. VI.

cumſtances well enough agree with the Account given us of *Anderida*.

The Situation likewiſe of a Town here, gives reaſon enough to ſuppoſe, it was a Place of Importance, and whence it had its Name; no Part hereabouts being any way ſo convenient, for a ſecure Settlement; or for ſuch a uſe as the *Romans* might have occaſion to make of it. We are inform'd by *Cæſar*, that the Maritime Parts of *Britain* (ſpeaking of what he ſaw, which was the South-Eaſt) were inhabited by People from *Belgium*; and [1] that they call'd their Settlements by the Name of the Places from whence they came. It was the Opinion of *Tacitus* alſo, that [2] thoſe who inhabited next to *Gaule*, came from *Gaule*. And *Bede* ſays, the Tradition in his Time was, that the Southern Part of the Iſle was peopled [3] from *Bretaign*. In the Third and Seventh Books of *Cæſar*'s Commentaries, mention is made of the *Andes*, a City and a People belonging to it among the *Celtæ*, inhabiting on the Sea-Coaſt. Time varying the Names of Things, near Two Hundred Years after *Cæſar*, *Ptolemy* calls the City *Anderidum*: And near 250 Years after him, when the *Notitia Imperii*, now extant, was in uſe, the *Claſſis Anderetianorum* [4] is regiſter'd; and the Reſidence of their Admiral fix'd at *Paris*. From whence 'tis to be inferr'd, that tho' the Capital of the *Andes* might have been *Angers* near the *Loyre*, yet their Country had on the North the *Britiſh* Channel; and on the Eaſt the *Seine* for its Bounds. The *Britiſh* Coaſt about *Eaſt Bourne* is the neareſt of any to the Mouth of the *Seine*: Therefore, according

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[1] De Bell. Gal. Lib. V. *Qui omnes, fere iis nominibus civitatum appellantur, quibus orti ex civitatibus eo pervenerunt.* [2] Tac. Agric. cap. xi. *In univerſum tamen æſtimanti, Gallos vicinum ſolum occupaſſe credibile eſt.*

[3] Bedæ Hiſt. Eccleſ. Gent. Angl. Lib. I. cap. 1. *In primis hæc Inſula Britones ſolum, à quibus nomen accepit, incolas habuit; qui de tractu Armoricano (ut fertur) Britanniam advecti, Auſtrales ſibi partes illius vindicarunt.* [4] Pancirol. Comm. in Notit. Imp. Cap. XC. pag. 179, 180.

according to the Usage before *Cæsar*'s Time, the Name of *Anderida* there, is readily accounted for. Moreover, this Place seems most naturally seated, for giving an Appellation to the great Wood, to which it adjoin'd: For, as it self is on the Shoar, so also the *Sylva Anderida* here, came very near the Shore; and a large part of it might be seen from the Sea before it: Indeed, on the Sea off of *Romney*, it might be discover'd; but then the Distance was great: At all other parts of the Coast, the Sight of it from Sea, is hinder'd by Hills, or high Cliffs.

Setting aside the want of a navigable River, the Spot of Ground where this old Town stood, yields to none in the County for Importance and Pleasure: For here, like a Wedge, ends the firm Soil of the *Downs*; Nature has shap'd it like an Equilateral Triangle, having each side half a Mile in Length: Towards the Sea, on the Southern side, 'tis fenc'd by a low Cliff, of 12, 15, and in some Places 20 Foot high (in which Cliff is now to be seen a strong Foundation, that has acute Angles, which shews it to have been for a Fort rather than a Dwelling-House.) On the Northern side is a Morass, with a large Rivulet of very good Water. Between the West side and the *Downs* lies a small Valley, by which Advantage, there was formerly a Harbour, capable of a small Fleet; the Banks on each side of it are an Evidence it was sunk by Industry; but by Weeds and Gravel from the Sea, and by Mould annually added, as is observable [1] in Valleys, it is now so rais'd, that 'tis never flow'd but at high Spring-Tydes, when a strong Wind forceth the Waves into it. This Harbour must have been a good Security to part of the West side; what other Works might have been to guard it, from the

[1] Philos. Transact. An. 1701. No. 274. Pag. 926.

the end of the Harbour to the Morafs, cannot be faid; by reafon the Ground between has for many Ages been in Tillage. It is eafy to imagin of what Importance a Town fortified at this Place muft have been in thofe Ages, when the only Pafs by Land from the Weft to the Eaft End of the County was through it; for other there could not be, in many Miles North; unlefs the Lands in that Tract, which are ftill very owzy and tender, had been well drain'd.

As the Situation defcrib'd, render'd this Place ftrong; it is very pleafant withal; for the Ground is high enough for a good Profpect of the Low Lands adjoining, and the Country towards *Battell*; befides, it has a commanding View over that Bay, which is between *Beachy-Head* and *Haftings*. If the Ufe made of it by the *Romans*, was to guard the Coaft, there was this Advantage belonging to it; that a Centinel on the top of *Beachy*, not Two Miles from it, in a clear Day, without turning his Body, might fee the *Ifle of Wight*, the Hills in *France* near *Bologn*, and the *Nefs* in *Kent*; fo that from the *Nefs* to *Selfey* it muft have been a fmall Sail that could efcape his Eye. It was my purpofe to have added a Defcription of *Pevenfey*-Caftle; together with an Account of fome Remains of Antiquity, difcover'd laft Summer towards the Weft End of the County: But having been too tedious already, muft defer that for the prefent, and fubfcribe my felf

Your moft humble Servant,

Lewes, Jan. 26. $17\frac{16}{17}$.

JOHN TABOR.

III. *Tractatus*

III. *Tractatus de* Curvarum *Constructione & Mensura; ubi plurimæ series Curvarum Infinitæ vel rectis mensurantur vel ad simpliciores Curvas reducuntur. Autore* Colin Maclaurin, *in Collegio novo* Abredonensi *Matheseos Professore.*

EXimiæ Matheseos Theoriæ, ob infinitam Propositinum Universalitatem, æternam ac necessariam Veritatem, Evidentiam omni dubitatione majorem, Idearum claritatem luculentissimam, Demonstrationum elegantiam, Theorematum nexus & mutuas dependentias, pulcherrimis certè ac summis humani intellectus repertis sunt annumerandæ; inter eas vero eminent summorum hujus sæculi Philosophorum de Curvarum Longitudinibus & areis mensurandis ardua Theoremata. Ad hos diffusos cognitionis campos diu altè latentes tandem eruendos infinitæ scientiæ portiunculam mutuari, vix sibi temperare posset quin pronuntiaret, qui Arithmeticæ Infinitorum vires in immenso elegantissimarum Veritatum abysso eruendo, & humani intellectus Horizontem infinite ferè extendendo, paucis præteritis annorum decadibus, amplè satis comprobatas, animo perpenderit; Hujus vero methodi (sicut nunc aucta & exculta est) ope, incidi in rationem mensurandi infinitas Curvarum series, quam paucissimis explicabo.

Cum in omni linea curva sit aliqua curvaturæ regularitas licet fortè implicata, secundum quam figura determinatur; ideo Geometræ varias Curvarum characteres ex Æquatione Ordinatarum relationem ad abscissas axis alicujus exprimente definiunt. Cum verò idem fieri possit ex consideratione Curvarum respectu unius dati centri, imo

imo simplicissima Naturæ uniformitas in ejus indagine id fieri sæpe postulet, ideo hanc Curvas considerandi Methodum impræsentiarum usurpabimus, & imprimis ostendemus qua facillima ratione (secundum hanc Methodum Curvas determinandi) ex simplicibus complexiores construi possint.

§ I. Sint L & *l* puncta quamproxima in Curva B*l*L; sit *l o* arcus centro S descriptus perpendicularis in SL; & erit L*l* ut momentum Curvæ & L*o* momentum Radii SL: Ac si detur ratio L*l* ad L*o*, vel ad *lo* in distantia SL, dabitur æquatio Curvæ ad centrum S. Sint LP, *lp* Tangentes Curvæ in punctis L & *l*, in quas ex S demittantur normales SP, S*p* iis occurrentes in punctis P & *p*; similiter in omnes Curvæ Tangentes demittantur perpendiculares ex dato puncto S, & construetur Curva transiens per omnes Tangentium & perpendiculorum intersectiones. Hujus triangulum elementare P*np* simile erit triangulo L*ol*, quæ proinde dabitur ex data Curva B*l*L. Quippe ob æquales S*np*, P*n*L, & rectos S*pn*, SPL æquiangula erunt triangula S*pn*, P*n*L, & proinde P*n* : *pn* :: L*n* : S*n* :: L*o* : *lo*, adeoq; ob angulos P*np*, S*n*L, L*ol* æquales, erunt triangula P*np*, S*n*L, L*ol* similia. Cum igitur eadem sit ratio L*l* ad *lo* quæ P*p* ad *pn*, & SL ad SP, manifestum est, datâ ratione L*l* ad *lo*, & rectâ SL, dari rationem P*p* ad *pn* & rectam SP, adeoque Curvam DP*p*. Eadem ratione ex DP construi potest Tertia, & ex ea dein Quarta, & progrediendo prodibit series Curvarum infinita, quæ omnes ex uno dato innotescunt. Quod si erigantur LN

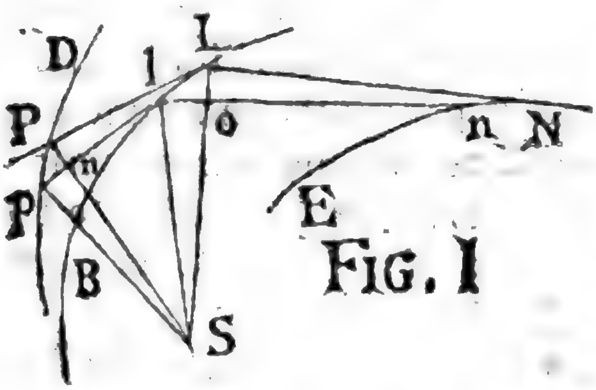

FIG. I

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& $l n$ perpendiculares in radios S L, S l, sibi mutuo occurrentes in n; & per omnia similiter definita perpendicularium concursuum puncta describatur Curva E N: ea ipsa erit Curva ex qua deduci potest B L, eadem ratione qua construximus D P ex B L. Ex E N similiter construi potest alia Curva, atque ex hac quoque parte Series infinita Curvarum construi poterit.

§ II. Curvarum verò hac ratione consideratarum simplicissimæ sunt quarum L l est ad L o in ratione potestatis alicujus Radii, ita ut, si a sit data quantitas, r denotet Radium Curvæ, n numerum quemcunque, sit L l ad $l\,o$ ut a^n ad r^n æquatio earum generalis. Omnes verò hæ Apsidem habent cum $r = a$, quoniam in eo casu L $l = l\,o$. Ut investigem æquationem Curvæ D P, cum in B L est ut L l ad $l\,o$ ita a^n ad r^n, ita r ad S P$=\frac{r^{n+1}}{a^n}$, ita $\frac{a^{\frac{n}{n+1}}}{a^{n+1}} \times$ S P$^{\frac{1}{n+1}}$ ad S P, ita $a^{\frac{n}{n+1}}$ ad S P$^{\frac{n}{n+1}}$, ita P p ad $p\,n$. Proinde si $\dot{s}$ repreſentet momentum Curvæ, $\dot{y}$ arcum circularem radio descriptum à centro S, & r radium correspondentem, quæcunque sit Curva cujus Æquatio investigatur, erit Æquatio Curvæ B L, $\dot{s} : \dot{y} :: a^n : r^n$; Æquatio verò Curvæ D P, $\dot{s} : \dot{y} :: a^{\frac{n}{n+1}} : r^{\frac{n}{n+1}}$. Angulus autem P S p erit ad Angulum L S l ut $\frac{p\,n}{SP}$ ad $\frac{l\,o}{SL}$, sive ut $\frac{P\,n}{S\,P}$ ad $\frac{L\,o}{SL}$, vel (si S P dicatur x & S L, r) ut $\frac{\dot{x}}{x}$ ad $\frac{\dot{r}}{r}$, hoc est, (ob $x = \frac{r^{n+1}}{a^n}$) ut $\frac{n+1}{r}\dot{r}$ ad $\frac{\dot{r}}{r}$, sive ut $n+1$ ad 1. Hinc (*vid. Fig.* II.) B S P est ad B S L ut $n+1$ ad 1; unde facilius absque Tangentium ope duci potest Curva B P. Si sumatur angulus B S P ad B S L in ratione $n+1$ ad 1, & in S P demittatur perpendicularis ex L, erit occursus perpendiculi cum S P, in Curva B P prius Tangentium ope descripta.

§ III. Osten-

§ III. Ostendimus quo pacto ex unâ series Curvarum infinita deducitur; quo verò pacto singularum longitudines ex illius & unius alterius longitudinibus datis innotescant pergo demonstrare. Cum angulus $SPp = SLl$,

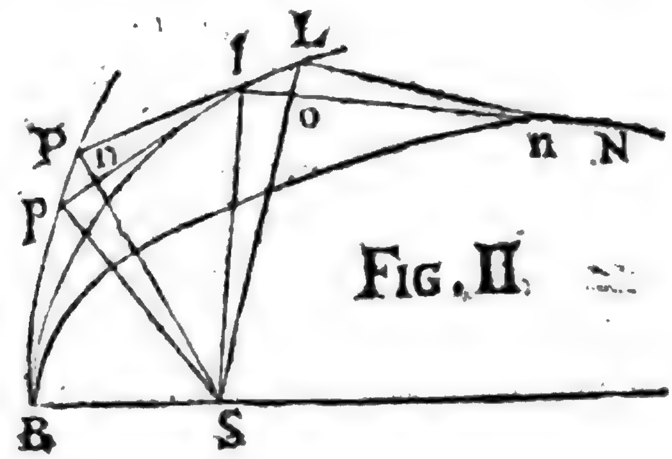


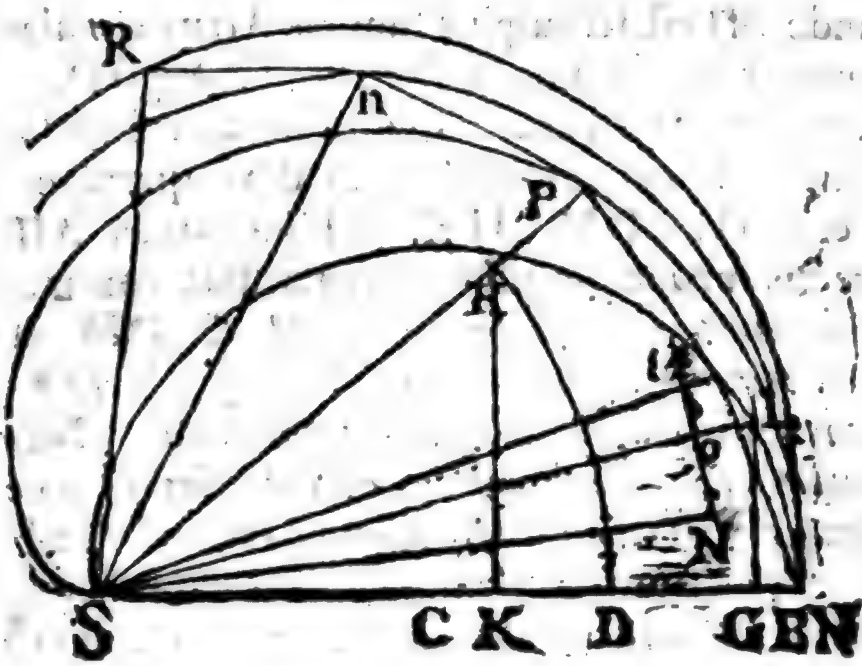
FIG. II.

atque LSl sit ad PSp ut 1 ad $n+1$, erit Ll ad Pp ut SL ad $\overline{n+1}\,SP$, sive (ob $SL : SP :: Ll : lo$) ut Ll ad $\overline{n+1}\,lo$, ac proinde $Pp = \overline{n+1}\,lo$: sed $lo = \overline{ln - on} = ln - LN + Nn$; ergo $Pp = n+1 \times \overline{ln - LN + Nn}$. Sed $ln - LN$ est momentum rectæ LN normalis in SL, Pp momentum Curvæ BP, & Nn momentum Curvæ BN: Cumque BP, BN, BL simul evanescant in B, erunt in ratione momentorum, adeoque $BP = \overline{n+1} \times \overline{BN + \text{vel} - LN}$. Unde Curva BP est ad summam vel differentiam Curvæ penultimæ in Serie ejusque Tangentis ab intermedia interceptæ, ut $n+1$ ad 1; sive, si m sit Index æquationis Curvæ BP (quoniam $m = \frac{n}{n+1}$) ut 1 ad $1-m$.

Hinc 1^mo in serie Curvarum infinitâ supra descriptâ, si dentur Longitudines duarum proximarum, dabuntur longitudines omnium; quippe mensura cujusvis pendet à mensura penultimæ semper in serie, & proinde unum par

par omnibus mensurandis sufficiet: Si una Curva sit rectis commensurabilis vel incommensurabilis, erit integræ seriei pars dimidia rectis commensurabilis vel incommensurabilis. Hinc 2°. Licet Curvæ B P & B N essent rectis incommensurabiles, differentia tamen Curvæ B P ab $n+1$ Curvæ B N esset æqualis assignabili rectæ. 3°. Si Curva transit per S, rectâ L N evanescente in S, erit $BPS = \frac{BNS}{1-m}$.

§ IV. Curvarum de quibus egimus, quarum nimirum $\dot{s} : \dot{y} :: a^n : r^n$, maxime insignis est Circulus, existente S in circumferentia, cujus æquatio est $\dot{s} : \dot{y} :: a : r$, ut ex similitudine triangulorum *L o l*, B L S (*Fig.* III.)



manifestum est, adeoque $n=1$; & proinde $m = \frac{n}{n+1} = \frac{1}{2}$. & æquatio Curvæ B P erit $\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$, quæ ipsa est æquatio *Epicycloidis* revolutione Circuli super basim sibi æqualem revolventis descripti, ad punctum ubi punctum describens tangit basim, quæ D^no^ *Paschal* dicitur *la Li-*

mation de M. Roberval, quamque *M. De la Hire* considerat ut *Conchoidem* Basis Circularis, in Actis Academiæ Parisiensis Anni 1708. Perpendiculares omnes L N, *l n* concurrunt in puncto B, adeoque B N $=0$: unde B P $= \frac{BN+NL}{1-m} = 2$ B L: Hinc Curva tota B P S $= 2$ B S, ac longitudo *Epicycloidis* semper dupla est chordæ arcus in circulo correspondentis. 2*do*. Ex *Epicycloide* describatur Curva B Π S, eadem ratione qua *Epicycloidem* ex Circulo descripsimus: In hoc casu $n = \frac{1}{2}$, & $m = \frac{n}{n+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3}$, ac proinde æquatio Curvæ B Π S erit $\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$. Longitudo Curvæ erit $\frac{BL+LP}{1-m} = \frac{3}{2}\overline{BL+LP} = \frac{3}{2}\overline{BL+LG}$, & proinde BΠ est sesquiplus summæ Arcus circularis ejusque Sinus recti. Quod si sumatur C D $=$ B D, & radio S D centro S describatur Circulus occurrens rectæ S P in H; & sit H K perpendicularis in B S; quoniam D H $= \frac{3}{2}$B L, erit BΠ $=$ D H $+$ H K. Hinc arcus B Π neque sunt rectis neque arcubus circularibus commensurabiles, differentia tamen arcuum B Π & D H est recta H K. In puncto S evanescit L G, adeoque B Π S $= \frac{3}{2}$ B L S, unde tota Curva est sescupla semicirculi. Nulla vero pars hujus Curvæ assignabilis commensurari potest toti, nec integra Curva in data quavis ratione secabilis est, ita ut portiones rationem assignabilem habeant ad se mutuo aut ad totam. Si hæc curva in data aliqua ratione Geometricè secari posset, constaret Quadratura Circuli, nam si *e.gr.* esset B Π ad B Π S ut 1 ad *m*, & B L ad B L S ut 1 ad *n*, esset BΠ $= \frac{B\Pi S}{m} = \frac{3\,BLS}{2\,m} = \frac{3n\,BL}{2\,m} = \frac{3}{2}\overline{BL+LG}$, unde esset B L $= \frac{m\,LG}{n-m}$ & B L S $= \frac{n\,m}{n-m}$ L G. 3*tio*. Ex B Π S construatur explicata methodo Curva B R, & quoniam

$n = \frac{1}{3}$

$n=\frac{1}{3}$ erit $m=\frac{n}{n+1}=\frac{1}{4}$, atque æquatio Curvæ BR erit $\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$. Hinc longitudo Curvæ fiet $\frac{1}{3}\overline{2BL+PT}$, totalis verò Longitudo Curvæ BRS $=\frac{8}{3}$ diametri SB. Si harum Curvarum Constructiones continuentur, prodibit hujusmodi series Æquationum quæ facile producitur ad libitum.

Æquatio Circuli		1.	$\dot{s} : \dot{y} :: a : r$
	Epicycloidis	2.	$\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$
	Secundi	3.	$\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$
	Tertii	4.	$\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$
	Cujusvis	n.	$\dot{s} : \dot{y} :: a^{\frac{1}{n}} : r^{\frac{1}{n}}$, &c.

Observare licet in genere, omnes quarum Indicum denominatores sunt Numeri pares, perfectæ rectificationis esse capaces; cumque quævis sit ad penultimam ut 1 ad $1-m$, perpendenti manifestum erit Curvæ cujusvis longitudinem fore $=\frac{1}{1-m}\times\frac{1-2m}{1-3m}\times\frac{1-4m}{1-5m}\times\frac{1-6m}{1-7m}$, &c. $\times SB$ continuando seriem donec ad nihilum reducatur Fractio. Quod si Indicis denominator sit Numerus impar, Curvæ erunt perfectæ rectificationis incapaces, & earum arcus quicunque erunt sibi mutuo, ipsis totis, rectis quibusvis & arcubus Circularibus incommensurabiles: exprimi verò possunt omnes arcubus circularibus & rectis: At Curvæ cujusvis totalis Longitudo erit ad Semicirculum ut $\frac{1}{1-m}\times\frac{1-2m}{1-3m}\times\frac{1-4m}{1-5m}$, &c. ad unitatem. Denique si Arcola à Corpore in harum quavis revolvente sumatur constans, hoc est si $r\dot{y}=1$, subtensa anguli contactus, cui semper (ob datum datâ areâ tempus) proportionalis est *Vis Centripeta* tendens ad S, erit reciproce ut potestas distantiæ cujus Index est $2m+3$; atque hoc est non con-

temnendum harum Curvarum privilegium, quod in iis omnibus Vis centripeta tendens ad S sit ut aliqua reciproca distantiæ dignitas, quæ simplicissima est, & utilissima in Naturæ indagine, Virium Centripetarum lex.

§ V. Curvarum quarum $\dot{s} : \dot{y} :: \dot{a}^{n} : \dot{r}^{n}$ proxime considerandа venit (quæ Curva quidem improprie dicitur) ipsa Linea recta, existente S extra rectam. In hac linea, ob similia triangula Ppn, PBS erit (si BS $= a$ & SP $= r$) $\dot{s} : \dot{y} :: r : a$. Ex linea recta methodo directâ

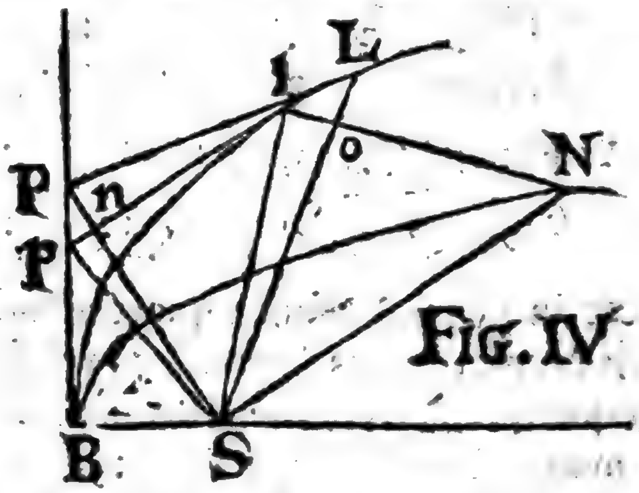


FIG. IV

nihil nisi punctum B construi potest, Methodo vero inversâ, perpendicularium nimirum P L, pl concursu, construi potest Curva, cujus Index (si m sit Index Curvæ BP) æqualis erit $\frac{m}{1-m}$; nam si Index Curvæ B L sit n, erit $m = \frac{n}{n+1}$, ac proinde $n = \frac{m}{1-m}$. Unde in hoc casu, cum $m = -1$ erit $n = \frac{-1}{2}$, & æquatio Curvæ B L erit $\dot{s} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$, quæ æquatio est Parabolæ ad Focum. Ex hac construe aliam, constituendo angulum L S N = L S B & erigendo L N normalem in S L occurrentem ipsi S N in N. Quoniam vero $m = \frac{-1}{2}$ erit $n = \frac{-1}{3}$, & æquatio Curvæ $\dot{s} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$ & B P $= \frac{BN-LN}{1-m} = \frac{2}{3}\overline{BN-LN}$, ergo

BN =

BN$=$2BP$+$LN; proinde hæc Curva eſt rectificabilis. Si Series continuetur, prodibunt ut prius æquationes in hoc ordine.

Æquatio Rectæ	$\dot{s} : \dot{y} :: r : a$
Parabolæ	$\dot{s} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$
Secundæ	$\dot{s} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$
Tertiæ	$\dot{s} : \dot{y} :: r^{\frac{1}{4}} : a^{\frac{1}{4}}$
Cujuſvis	$\dot{s} : \dot{y} :: r^{\frac{1}{m}} : a^{\frac{1}{m}}$

In hac Serie primæ ſunt Recta & Parabola, unde patet dimidiam hujus ſimiliter ac prioris Seriei eſſe rectis menſurabilem: alia vero dimidia pars in rectis & arcubus Parabolicis exhiberi poteſt. In his omnibus Vis centripeta ad S eſt reciproce ut poteſtas diſtantiæ cujus Index $3-2m$, ac proinde ſemper inter duplicatam & triplicatam rationem diſtantiæ reciproce.

§ VI. Æquatio Hyperbolæ æquilateræ ad centrum eſt $\dot{s} : \dot{y} :: r^2 : a^2$, ex qua deducitur methodo directa Series hujuſmodi,

$$1.\ \dot{s} : \dot{y} :: r^2 : a^2$$

$$2.\ \dot{s} : \dot{y} :: a^2 : r^2$$

$$3.\ \dot{s} : \dot{y} :: a^{\frac{2}{3}} : r^{\frac{2}{3}}$$

$$4.\ \dot{s} : \dot{y} :: a^{\frac{2}{5}} : r^{\frac{2}{5}}$$

$$5.\ \dot{s} : \dot{y} :: a^{\frac{2}{2n-1}} : r^{\frac{2}{2n-1}}$$

Ex his Curvæ, quarum Indicum denominatores ſunt in progreſſione -1, 3, 7, 11, &c. exhiberi poſſunt in rectis & arcubus Hyperbolicis; reliquæ verò in rectis & arcubus Curvæ, cujus æquatio ad axem SB (ſi x ſit abſciſſa, y verò Ordinata) eſt $\overline{xx+yy}|^2 = a^2x^2 - a^2y^2$, quæque conſtruitur (*vid. Fig.* III.) biſecando angulum BSL & ſumendo

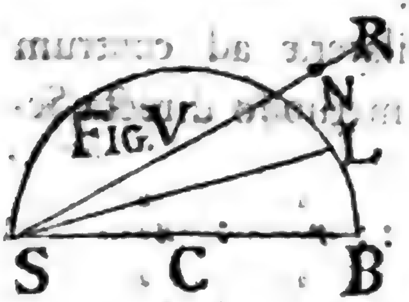
fumendo SN mediam proportionalem inter SB & SL.

Curvæ quæ ex Hyperbola methodo inversa construi possunt progrediuntur in hac Serie,

$$\text{Hyperbolæ } 1.\quad \dot{s} : \dot{y} :: r^{2} : a^{2}$$
$$2.\quad \dot{s} : \dot{y} :: r^{\frac{2}{3}} : a^{\frac{2}{3}}$$
$$3.\quad \dot{s} : \dot{y} :: r^{\frac{2}{5}} : a^{\frac{2}{5}},\ \&c.$$

Ubi Curvæ quarum Indicum denominatores sunt in progressione 1, 5, 9, 13, &c. exprimi possunt in rectis & arcubus Hyperbolicis; reliquæ vero in rectis & arcubus Curvæ modo explicatæ.

Si aliæ Curvæ desiderentur quæ alias exhiberent Series, id facillime fieri potest ope vel Circuli vel Rectæ; quippe ex earum aliqua omnes, in quibus $\dot{s} : \dot{y} :: a^{n} : r^{n}$, construi possunt, sumendo, si ope Circuli Problema sit solvendum, BSR ad BSL ut 1 ad n, & SN in ipsa SR $= a^{\frac{n-1}{n}} \times SL^{\frac{1}{n}}$; quippe Curvæ per omnia puncta N ductæ æquatio erit $\dot{s} : \dot{y} :: a^{n} : r^{n}$. Similiter ope Rectæ construi possunt quarum æquatio est $\dot{s} : \dot{y} :: r^{n} : a^{n}$.



Duas exhibuimus Series infinitas Curvarum rectis commensurabilium; aliam arcubus circularibus, aliam Parabolicis, aliam Hyperbolicis una cum rectis mensurabiles demonstravimus; eæ vero ad rectarum mensuram arte sola infinita reduci posse videntur, sicut æquatione sola infinita in rectis exprimuntur.

Hæc Cl. Author brevitati studens paucis tradit, illum autem plenius rem pro dignitate ejus illustraturum speramus.

IV. *Remarks*

IV. *Remarks on a Fragment of an old* Roman *Inscription lately found in the North of* England, *and transcribed by the Curious and Learned* Dr. James Jurin, *M. D. and* Reg. Soc. S.

OUR worthy Member, Dr. *Jurin*, having resided for some time at *Newcastle* upon *Tine*, had the Curiosity to travel the Country between that and *Carlisle*, in order to observe what might occur worth notice in the Remains of the Ruins of the famous *Picts-Wall*, built by the *Romans* to secure themselves, against the Incursions of the Natives of that part of *Britain* they cared not to conquer. In this Perambulation, besides many other valuable Observations which in time he may be prevail'd with to bestow on the Publick, Dr. *Jurin* saw and transcrib'd no less than Twenty *Roman Inscriptions*, some of which we had formerly receiv'd from others, but many of them wholly new; among them the following, which, tho' broken and in great part illegible, suffices to fix the Name of one of the Ancient Nations of *Britain*, that has hitherto been greatly miscall'd. 'Tis thus,

CIVITATE CAT
VVILLAVA'
ORVM· L· OL S
C *D I O*'

and is to be seen on the *Wall*, about two Miles West from *Lenercrofs-Abby*, near the Confines of our two Northermost Counties.

Here

Here 'tis observable, that the last A of the second Line has a Mark that follows it, not unlike to the last Stroak of an *N*; and if instead of A' we put *N*, we shall read it CIVITATE CATVVELLAVNORVM, which we cannot doubt to have been the true Name of that People which *Dion. Cassius, Lib.* LX. calls Καἰυελλανοὶ, and *Ptolomy*, in his Geography, *Lib.* II. *cap.* 3. more falsly, Καἰυευχλανοὶ; the first λ by producing the transverse Stroak having been mistaken for χ. This Nation appears by *Dion* to have been more potent than their Neighbours the *Dobuni* (whom he calls *Boduni*) and had, according to *Ptolomy, Verolamium* for their Capital, which 'tis most probable, was the *Cassivellauni oppidum* of *Cæsar*. So that it should seem *Cassivellaunus* King of these *Catuvillauni* when *Cæsar* invaded *Britain*, either gave his Name to his People, or took theirs. But he was no doubt the most potent Prince at that time in *Britain*, since by common Consent of the rest, he was made General of their united Forces, in defence of their Country's Cause against the *Romans*.

FINIS.

ERRATUM, N° 355.

Page 770. lin. 22. for *Maii* 31. lege *Martii* 31.

Printed for W. and J. INNYS, Printers to the *Royal Society*, at the *Princes-Arms* at the West-End of St. *Paul's* Church-Yard. 1718.

PHILOSOPHICAL TRANSACTIONS.

For the Months *July*, *August*, and *Septemb.* 1718.

The CONTENTS.

I. *Cometæ* Berolini *nuper visi observationes, ut & Eclipseos Solaris* Feb. 19^{no} *mane,* Noribergæ *&* Berolini *habitæ, è* Novis Litterariis Berolinensibus, *hoc anno primum edi cœptis, desumptæ.*

CL. *Christfridus Kirchius,* motus Corporum cœlestium, ut munere sibi à Societate Scientiarum Regia (*Berolini*) demandato rectè fungeretur, sedulò observans, a. d. XV. Kal. Febr. (*Jan* 18. *st. n.*) anni præsentis, vesperi dimidiâ Septimâ, versus Septentriones fortuitò Cometam conspexit. Vicinus erat ad dextram (stellarum) γ & β *Bayeri* in *Ursa minore,* nudoque oculo longè distinctius apparebat, quam β *Ursæ minoris,* licet ea insignis sit Stella secundæ magnitudinis, cum longè pallidior quidem majore tamen diametro, atque satis clara luce maxime circa centrum, conspiceretur. Per Tubum visus lucidam rotundamque referebat nubeculam; Caudæ autem nullum observari potuit vestigium, neque Nucleus dignosci. Motu celerrimo ab hora VII. ad XI. processit, gradusque quatuor cum dimidio absolvit, ut ex observationibus colligitur.

Die 19^{no} & 20^{mo} *Januarii* cœlum nubibus fuit obductum. Die vero 21^{mo} Cometa longè recesserat à loco suo nupero, atque in *Cassiopea* deprehendebatur, ubi cum stellis ϵ & δ triangulum conficiebat (*an æquicrure?*) scil. Hora 5^h $45'$ in $17°$ $34'$ ♉, sub latitudine Boreali $49°$ $54'$ hærebat: Deinde 9^h 15, in $16°$ $38'$ ♉ sub Lat. Bor. $49°$ $2'$ conspiciebatur. Cæterum multum decreverat atque à celeritate sua remiserat; præterquam enim quod pallidior quam ante apparebat, stellas etiam quartæ dignitatis magnitudine haud superare nudo oculo

lo conspectus videbatur, inque orbita sua, quatuor cum dimidia horis, non ultra sesquigradum processerat: Tubi autem beneficio diameter ejus 7 min. inveniebatur.

Jan. 23. hora IV. mat. Cometa cum δ & φ *Cassiopeæ* triangulum æquicrurum efficiebat, cùm ab utraque 2° 41 $\frac{1}{2}$ abesset. Hoc mane duarum horarum spatio vix dimidium gradum absolvit; hora decima vespertina cum δ *Cassiopeæ* & φ *Persei* in linea recta cernebatur, atque à priori 3° 38′, à posteriori 3° 9′ distabat. Diameter ejus erat 5 min. nudoque oculo conspectus stellam quintæ magnitudinis referebat.

Die 24to *Jan.* hora VI. mat. nondum attigerat φ *Persei*, sed cùm υ & *g* ejusdem Asterismi triangulum æquicrurum sistebat, & ab utraque non planè 3 $\frac{1}{2}$ grad. aberat. Plura ex observationibus docebit Astrophilos Vir accuratissimus, in pleniori quam parat hujus Cometæ Historia.

Hactenus Nova Litteraria *dicta* pag. 43, 44. *ubi desiderantur Observationes Diei* 18^{væ}, *cum Cometa velocissime motus terræ proximus erat, unde certius de Via ejus tam vera quam apparente judicium ferre possemus. Manifestum autem est eum Polo Æquatoris Boreo vicinissimum die* Januarii 19no *transiisse. Quod si cui libeat has Observationes ad examen revocare, calculoque accurato subjicere; in illius gratiam, loca Stellarum fixarum, quarum hic fit mentio, ex Catalogo* Britannico *excerpta, subnectuntur: Unde etiam patebit nonnulla in hac motus Cometæ descriptione haud ritè se habere; quæ tamen à Cl.* Kirchio *corrigi, in pleniori ejus quam promisit historia, spes est.*

Stellarum

Stellarum fixarum Loca ineunte Anno 1718.

BAYERO		Long. °	′	″	Lat. Bor. °	′	″
Ursæ minoris	β	♌ 9	18	0	72	58	10
	γ	♌ 17	35	15	75	13	15
Cassiopeæ	δ	♉ 14	00	35	46	23	25
	ε	♉ 20	50	8	47	31	50
	φ	♉ 11	36	35	45	4	5
Persei	υ	♉ 8	32	0	35	23	45
	φ	♉ 10	41	35	36	49	15
	g	♉ 12	15	20	36.	18	37

Ex iisdem Novis *etiam obtinuimus duplicem observationem Eclipseos exiguæ Solaris, currentis anni* Feb. 19no ſt. vet. *mane celebratæ; alteram* Noribergæ *à Cl. D.* Wurtzelbau, *alteram à præfato D.* Kirchio Berolini *habitam.*

Noribergæ *autem Sol ortus est aliquantulum deficiens in limbo superiore, qui quidem defectus ad tres plene digitos accrevit; Desiit Eclipsis* 8^h 8′ 48″ *circa* 60 *grad. à Vertice Solis ad Sinistras.* Berolini *vero Sol statim ab ortu cæpit deficere, Hora* ſcil. 6 49′ *vel* 49′ $\frac{1}{2}$. *Circa medium Eclipseos, nempe* 7^h 35′, *erant Partes lucidæ in Sole residuæ* 24′ 40″, *unde digiti obscurati* 2$^{dig.}$ 50′. *Finis autem incidit in* 8^h 28′ 10″. *Qui plura de his cupit, adeat* Nova *ipsa* Berolini *edita.*

II. *A*

II. *A Discourse occasion'd by an Inscription found, about Three Years ago, at* Langchester *in the Bishoprick of* Durham, *and communicated to the* Royal Society *from Dr.* Hunter *by Dr.* Woodward, *as it is printed in the Philosophical Transactions,* N° 354. *By* Roger Gale, *Esq;* R. S. S.

DR. *Hunter,* who communicated this Inscription, having only given us his Conjectures as to the first fortifying the Place where it was found, and the Time of its Repair after it had been destroy'd, but said nothing relating to the Explanation of the Inscription itself, tho' extremely curious; it will not, I hope, be taken amiss, if I offer some Thoughts that occurr'd to me at first sight of it, and afterwards induced me to put together what follows upon that Subject. I shall not in the least dispute or call in question the Time of its Foundation, as fix'd by the Doctor, but begin with the Place where it was discover'd, namely *Langchester* or *Lancaster,* in the Bishoprick of *Durham,* which I am, [1] with him, fully persuaded was the *Longovicus,* where the *Notitia Imperii* places the [2] *Numerus Longovicariorum.*

This place is seated upon a great Military Way, about 12 Miles distance from *Binchester,* and 7 from *Ebchester,* the one the *Vinovia,* and the other the *Vindomora* of *Antoninus,* as the Correspondence of the Numbers may evince; *Binchester* being 19 *Roman* Miles from *Ebchester,* as that is 9 from *Corbridge,* the exact Numbers the *Itinerary* gives us between *Vinovia, Vindomora,* and *Corstopitum.* What is very strange is, that the *Itinerary,* which must go upon the great Road directly thro' this

† Nnnnnn Town

[1] *Philosoph. Trans.* N° 266. p. 657. [2] *Not. Imp.* fol. 176.

Town of *Longovicus* betwixt *Vindomora* and *Vinovia*, takes not the least Notice of it, but measures the Way at the whole Length and Number of Miles, from the first to the latter of those Stations. If *Longovicus* was founded, as Dr. *Hunter* supposes [3], so early as the Time of *Julius Agricola*, and if that *Itinerary* was composed by any of the Emperors that bore the Name of *Antoninus*, this Station might have been destroy'd or deserted during the Wars with the *Britains* and not being repair'd till the Reign of *Gordian* III. was pass'd over by the Author of the Itinerary, as a Camp not then in being, or of no use to the *Roman* Armies; and this would be no weak Argument for the Antiquity of that Work: And perhaps some Parts of it may have been described as early as the Reigns of those Emperors, or earlier, and such Names of more modern Places as are found in it, may have been afterwards added as Occasion requir'd. As a farther Confirmation of this Conjecture, I beg leave to observe, that this Place, after it was repair'd by *Gordian*, subsisted even to the Ruine of the *Roman* Empire in *Britain*, as is evident by the Mention of it in the *Notitia Imperii*; so that had this [4] Journey which carries us from *Vindomora* to *Vinovia* been compos'd after the Reign of *Gordian*, it would be very hard to account for the Omission of this remarkable Station and Town, as it appears to have been from this, and many other Inscriptions found there.

Having this Opportunity of doing it, I am unwilling to let it slip without rectifying a Mistake in the *Essay towards the Recovery of the* Roman *Highways thro'* Britain, printed *in the 6th Volume of Mr.* Hearne's Itinerary *of* Leland [5], which having brought the *Ermingstreet* (not the *Watlingstreet*, as Dr *Hunter* and the Country call it) a little beyond *Cattarick* in *Yorkshire*, divides it there into two Branches,

[3] *Ph. Transf.* N° 354. p. 702. [4] *Iter.* I. *a Limite Prætor.* *usq;* [5] P. 111, 114.

Branches, tracing one of them to *Tinmouth*, and the other to *Carlisle* but omits the main Stemm of it, that runs almost directly Northward to *Piercebridge*, so to *Denton*, *Broughton*, *Binchester*, *Langchester*, *Ebchester*, *Corbridge*, and through the Heart of *Northumberland* into *Scotland*, about a Mile and a half to the West of *Berwick*. It is in several places very intire and fair, especially between *Corbridge* and *Binchester*, the Ridge of it there being for the most part two Yards in Height above the Level of the Soil, no less than Eight Yards broad, and all pav'd with Stones, that are as even as if new laid: as I am inform'd by the ingenious Mr. *Warburton*, who has often view'd it, and to whom we are obliged for the most accurate and useful Map of the County of *Northumberland* that was ever yet publish'd. This Digression, if it may be so call'd, I hope will be excus'd, since it not only sets right an Error, but acquaints you with a noble *Roman* Way, scarcely yet known or observ'd by any body.

Having fix'd the Seat of this *Long vicus*, where the Inscription was found, let us consider next what sort of a Place it was; and upon due Enquiry it will appear to have been one of the most ancient and eminent Stations the *Romans* were possess'd of in these Parts. As to its Antiquity, Dr. *Hunter* has made it probable, that we ought to look for it as high as *Julius Agricola*'s commanding under *Domitian*, in this Island: As to its Eminency, the Inscription that came last from him to the Society, as well as several others found there, is an undeniable Evidence of its being a Place of great Consideration; but nothing can put that more out of Dispute than the first which was some Years ago transmitted by the same Hand [e], which therefore I beg leave to insert here with that which came last from him, and the rather because little or nothing has ever been said upon it, and that they will give great Light one to the other.

[e] *Phil. Trans.* N° 266.

IMP·CÆS·M·ANT·GORDIA
NVS·P·F·AVG·BALNEVM·CVM
BASILICA·A[illegible]LO·INSTRVXIT
PRENEGLVCILIANVM·LEG·AVG
PR·PR·CVRANTE·M·AVR
QVIRINO·PRECOHILGR

II.

IMP·CÆSAR·M·ANTONIVS
GORDIANVS·P·F·AVG·
PRINCIPIA·ET·ARMAMEN
TARIA·CONLAPSA·RESTITV
IT·PER·MAECILIVM·FVSCVM·LEG
AVG·PR·PR·CVRANTE·M·AVR
QVIRINO·PR·COH·I·L·GOR·

The Stone whereon the first is cut has been broke in two, whereby some of the Letters are defaced, however, it may be very well read as follows; the Letters PRE in the fourth Line I take to be a Mistake of the Workman, having seen several Copies, where they are so transcribed; that they should be PER is evident from the fifth Line of the second Inscription.

I. *Imperator Cæsar Marcus Antonius Gordianus*
Pius Felix Augustus Balneum cum
Basilica à solo instruxit
Per Cneium Lucilianum Legatum Augustalem
Proprætorem Curante Marco Aurelio
Quirino Præfecto cohortis primæ Longovicariorum; or rather, *Legionis Gordianæ.*

The second can be read only after the following manner.

II. *Imperator Cæsar Marcus Antonius*
Gordianus Pius Felix Augustus
Principia & Armamentaria
Conlapsa restituit
Per Macilium Fuscum Legatum
Augustalem Proprætorem curante Marco Aurelio
Quirino Præfecto Cohortis primæ Legionis Gordianæ.

From these Two Inscriptions compar'd together, it will be apparent that they were not only erected under the same Emperor, but by the Care of the very same Person *Aurelius Quirinus*, tho' not in the same Year. The Emperor can be no other than *Gordianus* the youngest, or third of that Name; the two former having been slain so very soon after they had assumed the Purple, that it is improbable they should have gi-

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ven any Orders or Commands for the erecting of new, and repairing of antient Buildings, in so remote a Province as *Britain* was from *Africa*, where they were murder'd after a short joint Reign of scarce seven Weeks.

[1] Dr. *Hunter* tells us, that that which was first discover'd (and which I shall therefore always distinguish by the Name of the first) was dug up about a Hundred Yards East from a great Square, which had been fortified with a thick, strong Wall, faced with hewen Stone, within which, and without, especially towards the East, are nothing but ruinous Heaps of Stone, and thinks the Lodging of the Garison only to have been included within those Walls. His Conjecture is very much confirmed by the [2] Account he gives us of the finding the last Inscription within that square Inclosure; so that there seems to have been at this *Longovicus* a large Town, and one of those Camps call'd *Castra stativa*, where the Legions lay in Quarters during the time of Peace and Quiet.

The first Inscription tells us, that the Emperor *Gordian* built the *Balneum* and *Basilica* from the Ground, *à Solo*; whereas, by the second he appears to have been only the Repairer of the *Principia* and *Armamentaria*. Perhaps therefore here might be no Town, till the *Romans* thought fit to repair their old deserted Camp at this Place, and then the Emperor might also build the *Bath* and Palace for the Residence of the *Proprætor*, when in these Parts of *Britain*; the Word *Basilica* importing both a Palace, and an Edifice for hearing of Causes, and transacting all publick Affairs. As this eminent Building was erected by the Emperor's Command, it is an undeniable Argument of the Splendor of this Town, as are the great Heaps of Rubbish, and Ruines, where this

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1 *Phil. Trans.* N° 266. p. 658. 2 *Phil. Trans.* N° 354.

this Inscription was found, of its Largeness and Extent.

The second equally puts the being of the *Castrum stativum* out of dispute, when it acquaints us with the Rebuilding of the *Armamentaria* and *Principia* there, that is the *Arcenals* and *Quarters* either of the Legionary Soldiers, that were call'd the *Principes*, or the place where the Eagles and other military Ensigns were kept. It is probable they did not belong to one particular Legion, but to several, as they had occasion to be employ'd here; [1] tho the *Legio sexta Victrix* seems to have the best Title to them, as being constantly quarter'd in the North; whereas, the [2] *Legio Secunda*, and [3] *Vicesima* were generally garrison'd, the first at *Caerleon* in *Wales* and *Richburrow* in *Kent*, and the other at and about *Chester*; so that the [4] Monuments they have left in the North were erected by them, when the Wars, and other Works, as particularly the *Wall* carry'd cross the Island, call'd them thither; which being finish'd, they returned home to their more Southern Quarters, and continu'd in them till commanded Abroad upon new Services. I will not pretend to determin when these *Armamentaria* and *Principia* first fell to ruin; perhaps it might be when *Hadrian*, *Lollius Urbicus* and *Severus* had carried their Conquests farther into the Enemy's Country, and having built those famous *Walls*, the Relicks of which we still see in the Shire of *Sterling* in *Scotland*, and in *Northumberland* and *Cumberland* in *England*, that this Camp might be thought useless, the *Roman* Forces being drawn nearer to, and quarter'd upon the Frontiers; and so this Fortress abandoned and suffer'd to fall into decay, as the Word *conlapsa* implies: and not that it was

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[1] *Ptol. Leg.* VI. *Niceph. Ebor locat.* [2] *Anton. Itin.* XII. *Not Imp.* p. 161. [3] *Anton. Itin.* II. [4] *Camd.* p. 835, 920. *Phil. Transf.* N° 269.

was destroy'd by any Fire, War, or other Enemy than Age and Neglect.

Tho' the Word *conlapsa* is wrote here with an *N*, there can be no doubt but the Pronunciation of it was as we usually find it spelt, *collapsa*; a certain Argument of the Letter *N*'s being silent in the middle of a Word, before two Consonants, especially NS, and NT, when the T was pronounced like an S. To omit what [1] *Quintilian* says to this purpose, it is confirm'd by the Absence of that Letter in numberless Inscriptions in *Gruter*, *Reinesius*, &c. and no wonder, since the Workmen in those Days, as well as ours, usually wrote as they spoke their Words. I shall not trouble you with Quotations of any of them to this end, but as a Proof of what I say, only assure you from ocular Inspection and a most accurate Examination, that there is no transverse Line over the Letters I S belonging to the Word FABRICESIS in the Inscription of IVL. VITALI at *Bath*, whatever has been affirm'd to the contrary, but that the Letter N is totaly omitted there. You will also pardon my Endeavours, before I leave this Subject, to explain a short Inscription belonging to some of our Countrymen, tho' found at *Amerbach* in *Germany* [2], since it will be a new Proof of the foregoing Assertion.

III.

NYMPHIS ·	*Nymphis.*
N · BRITTON	*Numerus Brittonum*
TRIPVTIEN	*Triputiensis*, or ——*enus*
SVB CVRA ·	*Sub cura*
M · VLPI	*Marci Ulpii*
MALCHI ·	*Malchi*
⊳ LEG XXII	*Centurionis Leg.* 22.
PR · P · F ·	*Trimigeniæ, Piæ, Felicis.*

There

[1] *Quintil. Instit.* Lib. I. c. 7. [2] *Gruter.* p. xcIII.

There is no Difficulty but in the Word TRIPVTIEN, and that will presently vanish if you insert the Letter N, and read it TRIPVNT, *i. e. Tripontienus* or *Tripontiensis*, the Mutation of the O and V being so frequent, that no body is ignorant of it. This will bring you to *Tripontium* [1] or *Dowbridge* in *Northamptonshire*; tho' that excellent Antiquary Dr. *Battely* [2], in his *Antiquitates Rutupinæ*, would read it RIPVTIEN, and fix'd the Place whence this *Numerus* took its Appellation at *Richburrow* in *Kent*.

But to return where we left the Camp at *Longovicus*, it will be as difficult to assign a Reason for its being repaired, as it was for its being deserted; unless that the *Proprætors* might judge it advisable about the Time of *Gordian* III. to fix their Residence there, and consequently refortify the old Camp for their State and Security. And that it was not refortify'd upon any sudden Emergency, but for Time and Duration, is evident both from the strong Stone-Works that encompass'd it, and a Body of Forces lying here, even at the Expiration of the *Roman* Empire and Authority in this Island, which from its Continuance in the same Station, had got the Name of the *Longovicarii* [3].

The Person that under the Emperor gave Direction for these Repairs, was *Mæcilius Fuscus*: As *Mæcilius* is a Diminutive of *Mæcius*, it is not unlikely that he was the Son of *Mæcius Fuscus*, who was Consul with *Turpilius Dexter*, *A. D.* 225. in the Reign of *Alexander Severus*: By this Inscription it appears that this *Mæcilius* was the Emperor *Gordian*'s *Lieutenant* here and *Proprætor*; For tho' in *Phil. Transact.* N° 354, by the Inadvertency of the *Engraver*, we read only PR. instead of PR. PR; it is right in the Original, and in the Tran-

Oooooo script

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1 *Antonin. Itin.* VI. 2 p. 21. 3 *Notit. Imp.* fol. 176. b.

fcript fent up by Dr. *Hunter*, and accordingly in *pag.* 826. the Fault is amended. And as the Name of *Fuſcus* ſtands in the ſame Place in the ſecond as that of *Lucilianus* does in the firſt, and with the ſame Adjuncts both before and after, we may fairly conclude he was either his Predeceſſor or Succeſſor, but which, it is impoſſible to determin.

And here, perhaps, it may not be amiſs to remark, we never meet with a *Legatus Auguſtalis* in any Inſcription in this Iſland, without the joint Title of *Proprætor*; and [1] *Tacitus* himſelf either makes them the ſame Office, or at leaſt unites them in the ſame Perſon, when he tells us, *In Britanniâ P. Oſtorium Proprætorem turbidæ res excepere*; and having preſently after related the manner of the Fight with the *Iceni*, ſtiles him *Legatus*, *Quâ pugnâ filius Legati*, *M. Oſtorius*, *ſervati civis decus meruit* [2]; and a little after he gives both the ſame Titles to *A. Didius* the Succeſſor of *Oſtorius*.

We are indebted therefore to theſe two Monuments, not only for the Account they have preſerved of the *Roman* Arms and Magnificence at *Longovicus*, but for the indiſputable Records of the Names of two *Legates* and *Proprætors* of *Britain*, that would otherwiſe have been buried in Oblivion, *viz*. *Cneius Lucilianus* and *Mæcilius Fuſcus*: For from *Virius Lupus* (who was *Proprætor* and *Legate* here about the Year 208, under *Severus*, and juſt before that Emperor's coming into this Iſland repaired a Bath burnt down at *Lavatræ*, or *Bowes* [3], in *Yorkſhire*) we have no where extant the Name of one of thoſe Officers, till we come to *Nonnius Philippus*, whom I take to have ſucceeded the laſt of theſe; the [4] Stone which was found at *Old Carliſle* in

[1] *Tac. Lib.* Ann. XII. c. 32. [2] Ibid. c. 39. [3] *Camd.* p. 762. Edit. 1695. [4] *Camd. Britan.* p. 830.

in *Cumberland*, and has preserv'd his Memory, setting forth that he was *Legate* and *Proprætor* when *Atticus* and *Prætextatus* were *Consuls*, which was *A. D.* 242. the very Year that our *Gordian* went upon his *Persian* Expedition, from which he never return'd. And as that Emperor left *Nonnius Philippus* in that Post, when he march'd into the East, where he was murder'd about two Years after, it is highly probable that he was the last *Proprætor* of his appointing, and consequently, that *Macilius Fuscus* was his Predecessor, and the Repairs begun at *Longovicus* before the Year 243 [1]. I would not have troubled you with this Inscription, but that it is faultily transcribed in *Camden*, and that I shall have occasion by-and-by to refer to it again, upon a material Point, which therefore I hope will plead my Pardon.

IV. I. O. M.

PRO. SALVTE. IMPERATORIS
M. ANTONI. GORDIANI. P. F.
INVICTI. AVG. ET. SABINAE. FvR
IE. TRANQVILE. CONIVGI. EIVS. TO
TAQVE. DOMV. DIVIN. EORVM. A
LA. AVG. GORDIA. OB VIRTVTEM
APPELLAT. POSVIT. CVI. PRAEEST
AIMILIVS. CRISPINVS. PREF
EQQ. NATVS. IN. PRO. AFRICA DE
TVSDRO. SVB. CVR. NONNII. PHI
LIPPI. LEG. AVG. PROPRETO.
ATTICO. ET. PRETEXTATO COSS.

[1] *Philos. Transact.* N° 354. p. 702.

The Person who had the Care of these Repairs both in Town and Camp, was *Marcus Aurelius Quirinus, Præfect* or *Commander* of a Company of Foot; another Argument for the Proprætors *Lucilianus* and *Fuscus* succeeding immediately one the other, he serving in the same Post under both. I must observe however, that altho' the two first Inscriptions have been cut very near the same time, and by the same Hand, as appears by the Form of the Letters, and Manner of the Abbreviations in each of them, yet the Office that this *Quirinus* bore is something differently express'd in the first from what it is in the second, if they have been accurately transcribed; the First shewing, after *QVIRINO* the Letters PRE. CH. I. LG. R, which, before I had seen the Latter, I was induced to read *Præfecto Cohortis primæ Longovicariorum*, the [1] *Notitia Imperii* placing the *Præfectus numeri Longovicariorum Longovico*. That *Numerus* and *Cohors* were the same thing, [2] *Pancirollus*, in his Notes upon that Book, quotes St. *Chrysostome* to prove, and some others, *Cohors erat qui vocatur Numerus*; but I rather take it to be an indefinite Number of Men, which might comprize several Companies, independent of any Legion. [3] *Vegetius*, speaking of the *Legati Imperatoris*, says, *in quorum locum nunc illustres viros constat Magistros Militum substitutos, à quibus non tantum binæ Legiones, sed plures Numeri gubernantur*; by which it is plain, the *Numeri* were no *Legionary Cohorts*. Neither was the Name so modern as from the *Notitia Imperii* and *Chrysostome* it might appear to be; for we meet with a [4] *Numerus Britonum* upon an Altar found in *Transylvania*, dedicated to the *Nymphs*, when the Emperor *Commodus* and *Glabrio* were Consuls, *A. D.* 186. And [5] another *Numerus* upon an Altar erected to *Hercules* for the Prosperity of

[1] fol. 166. b [2] fol. 161. b [3] Lib. II. c. 9. [4] *Gruter*. p. 94. 2. [5] id. 46. 9.

of *Septimius Severus*, when *Lateranus* and *Rufinus* were Consuls, *A. D*, 197. But after I had review'd the Letters at the end of the second Inscription, which are plainly transcribed PR. CoH. I. L. GOR. I could read them no otherwise than *Præfecto Cohortis primæ Legionis Gordianæ. Gordian* III. was so beloved of the Soldiery, that several Legions complimented him by honouring themselves with his Name, as the [1] *Legio tertia Italica*, which took the Addition of *Gordiana*; and the [2] *Legio decima gemina*, and [3] *Decima tertia gemina* did both give themselves the same Appellation. But which of the Legions quartered in this Island so stiled itself is not determin'd by this Inscription or any other that I know of. However, as the *Legio sexta Victrix* was all along quarter'd in the Northern parts of this Kingdom (as I observed before) where these Inscriptions were erected, I make no doubt but it was that which call'd it self *Gordiana*, tho' the numeral Distinction of VI is omitted, only perhaps for want of Room on the Stone. We find by several Inscriptions in *Camden*, that there was an *Ala* in those Parts which prided it self upon its Valour, and was therefore call'd the *Ala Augusta*; of the many Memorials it has left us of its Title, I shall only mention [4] one found at *Old Carlisle*, and which is the ancientest of them all, by any certainty of Date.

I. O. M.
AL. AVG. OB
. . . RTVT. APPEL. CVI
PRAEEST. TIB. CL. TIB. F. P.
LING · · N. IVSTINVS.
PRAEF. FVSCIANO.
II. SILANO. II. COS.

that

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[1] *Velf. Monum. Augustæ Vindel.* p. 431. [2] *Grut.* p. 80. [3] *Gruter.* p. 433. 1. [4] *Camd.* p. 827.

that is,

Jovi Optimo Maximo, Ala Augusta ob Virtutem appellata, cui præest Tiberius Claudius, Tiberii filius, provinciâ Lingonensi, Justinus præfectus, Fusciano secundò, Silano secundò Consulibus.

This Altar was dedicated when *Fuscianus* and *Silanus* were the second time *Consuls*, that is, in the Year 188. under the Reign of *Commodus*, and Fifty Years before our *Gordian* came to the Empire. At the same place was also discover'd the Fourth Inscription by me quoted, where we find this same *Ala Augusta* stiling itself also *Gordiana*; from whence I think it is not a little probable that the Legion to which this Wing appertain'd was the *Legio Gordiana* mention'd in the Inscriptions found at *Langchester*; and that Legion to have been the *Legio sexta Victrix*, from the long Continuation of this *Ala Augusta* in these Northern parts of the Nation, the constant Quarters of that Legion.

July 10.
1718.

III. A

III. *A Letter of that curious Naturalist Mr.* Henry Barham, *R. S. S. to the Publisher, giving a Relation of a fiery* Meteor *seen by him, in* Jamaica, *to strike into the Earth; with Remarks on the Weather, Earthquakes,* &c. *of that Island.*

SIR,

ACcording to your Request I have collected what I can remember, relating to a *Meteor* I saw in *Jamaica* about the Year 1700, as I was riding one Morning from my Habitation situated about Three Miles North-West from *St. Jago de la Vega*: I saw a Ball of Fire, appearing to me of the Bigness of a Bomb, swiftly falling down with a great Blaze. As I thought it fell into the Town; but when I came within a quarter of a Mile of the Town, I saw many People gather'd together a little to the Southward in the *Savanna*, to whom I rode up, where they were admiring at the Ground's being strangely broke and plough'd in by a Ball of Fire, which, as they said, fell down there. I observed there were many Holes in the Ground, one in the middle of the Bigness of a Man's Skull, and five or six smaller Holes round about it, of the Bigness of a Man's Fist, and so deep (especially the biggest) as not to be fathom'd by what long Switches or Sticks they had at hand. I did not hear that any was so curious as to make any farther Search: It was observ'd, that the green Grass was perfectly burnt near the Holes, and a strong Smell of Sulphur remain'd thereabouts for a good while after.

Note that we had a terrible rainy Night before, with much Lightning and great Thunder-Claps, which we have

have very frequently in *Jamaica*, often killing Cattle in the Fields. Mr. *Henry Lord*, who lives at *Dry-River*, had two Sons (big Boys) ſtruck dead with Lightning, in 1716, without any Wounds or Appearance of Hurt found about them. And as theſe Claps are much louder and ſtronger than any I ever heard in *Europe*, ſo are our Showers of Rain, pouring down in a moſt violent manner. We have Lightning all the Year round, but our great Rains are in the Months of *May*, *Auguſt*, and *October*. I knew *May* for two or three Years without Rain, which was lookt upon as a great Wonder; and we paid dear for it in our *Indigo*; for a Catterpillar appear'd and wove a fine Silk about the *Indigo-Plant*, and deſtroy'd it all, hurting nothing elſe. *May*-Rains us'd to deſtroy theſe Worms. *Auguſt* and *October* never go out without a Flood, we having then univerſal Rains, all over the Iſland, coming from the Sea: For we have often Rains in the Mountains from the Clouds lodging there, when we have none in the Lowlands.

Our Iſland is full of Mines, and, if ſearch'd into, I queſtion not but very rich. We are very ſubject to Earthquakes, ſeveral happening every Year, eſpecially after great Rains, which fill up all our great Cracks in the Surface of the Earth: For in a very dry Time, we have them ſo very large, deep, and gaping ſo open and wide, that it is dangerous to ride over ſome Parts of the *Savannaes*, for fear a Horſe ſhould get his Legs into them. Our Earthquakes make a Noiſe or Rumbling in the Earth, before we feel the Shake; and ſeem to run ſwiftly to the Weſtward. This is all I can inform you of this kind at preſent, relating to the Iſland of *Jamaica*, being ever ready to ſhew how much I am, *&c.*

Decemb. 19.
1717.

Henry Barham.

IV. *An Attempt to prove the Antiquity of the* Venereal Diseaſe, *long before the Diſcovery of the* Weſt-Indies; *in a Letter from Mr.* William Beckett, *Surgeon, to Dr.* James Douglaſs, *M.D. and* R. Soc. Soc. *and by him communicated to the* Royal Society.

SIR,

THE Undertaking I am at preſent engaged in, has unavoidably obliged me to conſult, among others, a great Number of ancient Phyſical and Chyrurgical Books, written by my own Countrymen: From theſe I took the Hint, that the *Venereal Diſeaſe* was known among us, much earlier than the *Æra*, which has been generally aſſign'd for its Riſe by modern Authors; for it's believed it was not known, at leaſt in *Europe*, till about the Year 1494. Notwithſtanding which, I determin, in the following Papers, to make it evident, it was frequent among us ſome Hundreds of Years before that Date. I could mention ſeveral Phyſicians and Surgeons of Eminence, who have been of the ſame Sentiments with me, particularly, the Learned Dr. *Charles Patin*, who has written a curious Diſſertation to prove the Antiquity of this Diſeaſe, which is ſufficient to excuſe me from the Imputation of having ſtarted a Novelty, or being at the trouble of quoting ancient Authorities before taken notice of, from the moſt ancient Writers of Medicine; as the Great *Hippocrates*, *Galen*, *Avicen*, *Celſus*, &c. and even the *Holy Scriptures*. I ſhall in theſe and ſome following Papers, lay aſide all thoſe foreign Aids and Aſ-

ſiſtances, and trace out the Symptoms of the Diſeaſe, as they naturally ariſe, from the firſt Infection to the laſt deſtructive Period, and ſhew that, by ſearching into our own Antiquities, we may be furniſhed with Inſtances of the Frequency of the Diſtemper among us, in all its reſpective Stages, before ever our Modern Authors dream it had its Appearance in *Europe*.

I ſhall begin with the firſt Degree of this Diſeaſe, and prove from authentick Evidences, it was anciently call'd the **Brenning** or **Burning**; and that this Word has been ſucceſſively continu'd for many Hundreds of Years, to ſignify the ſame Diſeaſe we now call a *Clap*; and that it was not diſcontinu'd till that Appellation firſt began to have its Riſe. The moſt likely Method to accompliſh my Deſign, will be firſt to examine thoſe Records that relate to the *Stews*, which were by Authority allowed to be kept on the *Bank-Side* in *Southwark*, under the Juriſdiction of the Bp. of *Winchester*, and which were ſuppreſſed the 37*th* of *Hen.* VIII. For it's impoſſible but, if there were any ſuch Diſtemper in being at that Time, it muſt be pretty common among thoſe lewd Women who had a Licence for entertaining their Paramours, notwithſtanding any Rules or Orders which might be eſtabliſh'd to prevent its Increaſe: But if we ſhall find that there were Orders eſtabliſh'd to prevent the Spreading of ſuch a Diſeaſe, that Perſons might be ſecure from any contagious Malady after their Entertainment at thoſe Houſes (which were anciently Eighteen in Number, but in the Reign of *Hen.* VII. reduced to Twelve) we may then ſecurely depend upon it that it was the Frequency of the Diſeaſe that put thoſe that had the Authority, under a neceſſity of making ſuch Rules and Orders. For the ſame Powers that granted a Liberty for keeping open ſuch lewd Houſes, muſt find it their Intereſt to ſecure,

as

as much as possible, all Persons from receiving any Injury there; lest the Frequency of such Misfortunes should deter others from frequenting them, and so the original Design of their Institution cease; from the entire sinking of the Revenues. Now I find that, as early as the Year 1162, divers Constitutions relating to the Lordship of *Winchester*, (being also confirmed by the King) were to be kept for ever, according to the old Customs that had been time out of mind. Among which these were some, *viz.* No *Stew-holder* to take more for a Woman's Chamber in the Week than 14 *d.* Not to keep open his Doors upon Holy Days. No single Woman to be kept against her Will, that would leave her Sin. No single Woman to take Money to lie with any Man, except she lie with him all Night till the Morning. No Stew holder to keep any Woman that hath the perilous Infirmity of **Burning**. These and many more Orders were to be strictly observed, or the Offenders to be severely punished. Now we are assured there is no other Disease that can be communicated by carnal Conversation with Women, but that which is Venereal, by reason that only is contagious; and it's evident the **Burning** was certainly so: For, had it been nothing else but some simple Ulceration, Heat, or Inflammation, there would have been no Contagion; and that affecting only the Woman, could not be communicated by any Venereal Congress, and so not inferr a Necessity of her being comprehended under the restraining Article. These Orders likewise prove the Disease was much more ancient than the Date above-mentioned; because they were only a Renewal of such as had been before established time out of mind.

But to confirm this farther, I find that in the Custody of the Bishop of *Winchester*, whose Palace was

situated on the *Bank-side*, near the *Stews*, was a Book written upon Vellum, the Title of which runs thus; Here begynne the Ordinances, Rules, and Custumes, as well for the Salvation of Mannes Lif, as for to ascheme many Myschiefs and Inconvenients that dayley be lik there for to fall owte, to be rightfully kept, and due Execution of them to be don unto any Personne wythin the same. One of the Articles begins thus; *De his qui custodiunt Mulieres habentes Nephandam infirmitatem.* It goes on, *Item*, That no Stew-holder keep noo Woman wythin his Hous that hath any Sycknesse of BRENNING, but that she be putte out upon the peyne of makeit a fyne unto the Lord of a hundred Shylyngs. This is taken from the Original Manuscript which was preserv'd in the Bishop's Court, suppos'd to be written about the Year 1430. From these Orders we may observe the Frequency of the Distemper at that Time; which, with other Inconveniencies, was dayley lik there for to fall owte: and the Greatness of the Penalty, as the Value of Money then was, that is laid on it, proves it was no trifling or insignificant thing.

But the bare Proof of there having been anciently such a Disease as was called the Burning, may be thought to be insufficient, unless we were perfectly assured what it was, and how it was in those Times described: I shall therefore do it from an unquestionable Authority, which is that of *John Arden*, Esq; who was one of the Surgeons to our King *Richard* II. and likewise to King *Henry* IV. In a curious Manuscript of his upon Vellum, he defines it to be, a certain inward Heat and Excoriation of the *Urethra*; which Description gives us a perfect Idea of what we now call a *Clap*; for frequent Dissections of those that laboured under

under that Diſeaſe, have made it evident, that their *Urethra* is excoriated by the Virulency of the Matter they receive from the infected Woman; and this Excoriation or Ulceration is not confined to the *Oſtiola* or Mouths of the *Glandulæ Muſcoſæ*, as has been lately thought, but may equally alike attack any part of the *Urethra* not beyond the reach of the impelled malignant Matter. The Heat before deſcribed, which theſe Perſons are ſenſible of, as well now as formerly, is a Conſequent of the excoriated *Urethra*; for the Salts contained in the Urine muſt neceſſarily prick and irritate the nervous *Fibrillæ*, and excite a Heat in thoſe Parts of the *Urethra* which are diveſted of its natural Membrane; which Heat will always be obſerved to be more or leſs, as the Salts are diluted with a greater or leſs Quantity of Urine; a thing I have often obſerved in Perſons that have laboured under this Infirmity in hot Weather, when the perſpirable Matter being thrown off in greater Quantities, the Salts bear a greater Proportion to the Quantity of Urine, and thereby make its Diſcharge at that Time ſo much the more painful and troubleſom.

Thus we ſee this very early and plain Deſcription of this Diſeaſe among us, to be entirely conformable to the lateſt and moſt exact Anatomical Diſcoveries. Here is no Tone of the *Teſticles* depraved, according to *Trajanus Petronius*; no Exulceration of the *Paraſtatæ*, according to *Rondeletius*; no Ulceration of the *Seminal Veſſels*, according to *Platerus*; no Seat of the Diſeaſe in the *Veſiculæ Seminales* or *Proſtatæ*, according to *Bartholin*; nor in thoſe Parts and the Teſticles at the ſame Time, according to our Countryman *Wharton* and others, who have falſly fixed the Seat of this Diſeaſe, and whoſe Notions, in this reſpect, are now juſtly exploded; but a ſingle and true Deſcription of it, and

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its Situation, about an Hundred and Fifty Years before any of those Gentlemen obliged the World with their learned Labours.

Having, I hope, sufficiently made it appear, the **Burning** was a Disease very early among us, and given the Description of it, I shall proceed to say something of the ancient Method that was made use of to cure it. We are not to expect the Measures our Predecessors, in those early Times, made use of, should be calculated for the removing any Malignity in the Mass of Blood, or other Juices, according to the Practice in Venerial Cases at this Time; because they looked upon the Disease to be entirely local, and the Whole of the Cure to depend upon the Removal of the Symptoms: Hence 'twas they recommended such Remedies as were accommodated to the taking off the inward Heat of the Part, and cure the Excoriations or Ulcerations of the *Urethra*. The Process for the accomplishing of this, I shall set down from the beforementioned *John Arden*, who wrote about the Year 1380. his Words are as follow, *Contra Incendium. Item contra incendium Virgæ Virilis interius ex calore & excoriatione, fiat talis Syringa* (i. e. *injectio*) *lenitiva. Accipe Lac mulieris masculum nutrientis, & parum zucarium, Oleum violæ & ptisanæ, quibus commixtis per Syringam infundatur, & si prædictis admiscueris lac Amigdalarum melior erit medicina.* There is no doubt but this Remedy, being used to our Patients at this Time, would infallibly take off the inward Heat of the Part, and cure the Excoriations or Ulcerations of the *Urethra*, by which means what issued from thence would be entirely stopt; and this was all they expected from their Medicines, forasmuch as they were entirely unacquainted with the Nature of the Distemper; and did not in the least imagine, but if the Symptoms that first attack'd the Part were removed, the Patient was entirely cured. I

I ſhall now, as a farther Confirmation of what I have advanced, proceed to prove, that by this **Brenning** or **Burning** is meant the Venereal Diſeaſe, by demonſtrating that ſucceeding Hiſtorians, Phyſical and Chirurgical Writers, and others, have all along with us in *England* uſed the very ſame Word to ſignify the Venereal Malady. In an old Manuſcript I have by me, written about the Year 1390. is a Receipt for **Brenning of the Pyntyl, yat men clepe ye Apegalle**; **Galle** being an old Engliſh Word for a running Sore. They who know the *Etymologie* of the Word *Apron*, cannot be ignorant of this. And in another Manuſcript, written about 50 Years after, is a Receipt for **Burning** in that Part by a Woman. *Simon Fiſh*, a zealous Promoter of the Reformation in the Reign of *Hen.* VIII. in his Supplication of Beggars, preſented to the King in 1530. ſays as follows, *Theſe be they* (ſpeaking of the *Romiſh Prieſts*) *that corrupt the whole Generation of Mankind in your Realm, that catch the Pockes of one Woman and bear them to another; that be* **Burnt** *with one Woman and bare it to another; that catch the Lepry of one Woman and bare it unto another.* But to make this Matter ſtill more evident, I am to obſerve, that *Andrew Boord*, a Doctor in Phyſick, and Romiſh Prieſt, in the Reign of *Henry* VIII. in a Book he wrote, entitul'd *The Breviary of Health*, printed in 1546. ſpeaks very particularly of this ſort of Burning; one of his Chapters beginneth thus, **The 19th Chapiter doth ſhew of** BURNING **of an Harlotte**; where his Notion of communicating the *Burning* is very particular. The ſame Author adds, that if a Man be **Burnt** with an *Harlot*, and do meddle with another Woman within a Day, he ſhall **Burn** the Woman that he ſhall meddle withal; and as an immediate Remedy againſt the **Burning**, he recommends the waſhing the *Pudenda* two or three

three times with White Wine, or else with Sack and Water; but if the matter have continued long, to go to an expert Chirurgeon to have Help. In his 82d Chapter, he speaks of two sorts of Burning, the One by Fire, and the Other by a Woman through carnal Copulation, and referrs the Person that is Burnt of a *Harlot* to another Chapter of his for Advice, what to do, yf he get a Dorser or two, so called from its Protuberancy or bunching out: For I find about that Time the Word *Bubo* was mostly made use of, to signify that sort of Swelling which usually happens in pestilential Diseases.

From hence it appears, the *Burning*, by its Consequents, was *venerial*, since every Day's Experience makes it evident, that the ill Treatment of the first Symptoms of the Disease, either by astringent Medicines, or the removing them by cooling and healing the excoriated Parts, will generally be attended with such Swellings in the Groin, which we rarely observe to happen from any other Cause whatsoever.

I shall give a few more Instances of this Disease being call'd the *Burning*, and conclude. In a Manuscript I have of the Vocation of *John Bale* to the Bishoprick of *Ossory* in *Ireland*, written by himself, he speaks of Dr. *Hugh Weston* (who was Dean of *Windsor* in 1556. but deprived by Cardinal *Pole* for Adultery) as follows, " At this Day is lecherous *Weston*, who is more " practised in the Art of Brech Burning than all " the *Whores* of the *Stews*. And again, speaking of the same Person, he says, " He not long ago brent a *Beg*-" *gar* in St. *Botolph*'s Parish. The same Author says of him elsewhere, " He had been sore Bitten with a *Win*-" *chester Goose*, and was not yet healed thereof; which was a common Phrase for the Pox at that Time, because the *Stews* were under the Jurisdiction of the Bishop of

of *Winchester*. *Mich. Wood*, in his *Epistle* before *Steph. Gardiner*'s Oration *de vera Obedientia*, printed at *Rhoan*, 1553. gives another Evidence of the **Burning**. And *William Bullein*, a Physician in the Reign of Queen *Eliz.* in a Book he publish'd, call'd *The Bulwark of Defence*, *&c.* printed in 1562. bringing in *Sickness* demanding of *Health* what he should do with a Disease call'd the *French Pockes*. *Health* answers, " He would not that a-" ny should fishe for this Disease, or to be bold when " he is bitten to thynke thereby to be helped, but ra-" ther to eschewe the Cause of thys Infyrmity, and " filthy rotten **Burning** of *Harlots*.

I believe, by this time, I have sufficiently prov'd what I proposed, that the first Degree of the Venereal Disease was very anciently known among us, under the Title of **Burning**; and that you may lose no more Time at present upon this Subject, I shall reserve my Collections, which shew that the Disease, when it came to be confirmed, was no Novelty here in those early Times, for a further Opportunity, and detain you no longer than to express my Pleasure in professing my self, *Yours*, &c.

London, Feb. 4.
1717-18.

Will. Beckett.

V. *Accuratarum Observationum Astronomicarum, anno superiore & currente, cum Reg. Societate communicatarum Sylloge.*

INterest sanè Scientiæ ne pereant Observata Astronomica, debita curâ fidisque Instrumentis ab Artificibus idoneis cælitus deprompta: Hoc enim solo fundamento nititur Urania practica. Itaque in his *Trans-*

actionibus, per plusquam quinquaginta annorum curriculum, passim sparguntur hujus generis Notæ. Ausim tamen spondere vix ullas unquam reperiri posse Observationes quæ certitudine eas quas nunc damus vincant, ne dicam quæ pares sint, utpote Tubis prælongis ac Micrometris præter solitum affabre factis mensuratæ. Cape igitur primo.

Planetarum Observationes.

Anno 1717. *Aprilis* 15°. 9^h 49′ T. æq. observavit D. *Pound* apud *Wansted*, *Jovem* jam reversum ad stellam illam, quam *Novemb.* 22°. 1716. manè corpore suo texerat, de qua vide N° 350. *Phil. Transact.* pag. 508. *Jovis* autem centrum tum temporis distabat ab ea Stella (quæ tertia est Geminorum in *Catalogo Britannico*) 23′ 22″ boream versus; simulque ab alia vicinâ, quæ quarta est Geminorum in dicto Catalogo, 27′ 11″. atque huic fere conjunctus erat planeta.

Aprilis 25to sequente, eodem observatore ac loco, 10^h 3′ T. æq. *Jupiter* apud quatuor Fixas exiguas visus est, eas omnes præcedens, & in ipso quasi principio Cancri. Centrum autem planetæ distabat ab *e* 13′ 00″, ab *h* 13′ 50″, ab *f* 19′ 53″, & à *g* 9′ 27″.

Postridie vero *Apr.* 26°. 9^h 7′ *Jovis* centrum distabat ab *e* 8′ 35″, ab *f* 9′ 00″, à *g* 4′ 5″, & ab *h* 13′ 50″. Jamque præterierat omnes præter *f* ad quam tendebat, quamque parum admodum die crastino infra se relinquere debuit.

Eodem fere momento, horâ scil. nonâ, *Londini* visa est stella *g* in vertice Trianguli Isoscelis ac fere Isopleuri cum *Jovis* centro ac tertio Satellite, tum sex *Jovis* diametris ad occasum distante, nisi quod parum admodum base longiora erant crura; ac intra quadrantem horæ,

horæ, angulus ad *Jovis* centrum, qui prius major erat angulo ad Satellitem, factus est eo sensibiliter minor.

Tres autem Stellæ *h*, *g*, *e*, sunt 10^{ma}, 11^{ma}, & 12^{ma}. Geminorum in *Catal. Britan.* juxta quem tum temporis situm habuere, *h* in ♋ 0° 22′ 55″, cum Latit. Borea 0° 11′ 25″. Et *g* in ♋ 0° 28′ 25″. Lat. Bor. 0° 3′ 40″: *e* vero in ♋ 0° 29′ 20″ cum Lat. Aust. 0° 8′ 05″. Distat autem quarta *f* à Stella *g* 11′ 40″, ab *e* 12′ 50″, ac denique ab *h* 20′ 36″, unde constabit locus ejus. Ex his manifestum est *Jovem* Latitudinem habuisse parvam admodum Borealem, nec semiminuto majorem, saltem si dictis stellarum locis habenda fides. Hæc posteris usui esse possunt definiendo Nodorum *Jovis* motu, si quem habeant.

Ejusdem anni *Junii* 18^{vo} 10^{h} *Londini*, in ædibus Societatis Regiæ, visus est *Saturnus* Stellæ fixæ Telescopicæ admodum propinquus, à qua vix distabat ad Austrum una Annuli diametro, & normalis in lineam Ansarum à Stella demissa incidebat in medium Ansæ orientalis. Fixa hæc parvula nullique Catalogo adscripta tunc habuit ♎ 12° 58′ $\frac{1}{2}$ cum Lat. Bor. 2° 33′ proximè; comitemque habet sibi adjunctam ac luce æqualem, quatuor minutis ad ortum distantem, ac paulò australiorem, unde facile dignosci poterit, locusque ejus si cui libeat verificari.

Eadem nocte 10^{h} 30′ *Mars* visus est prope Stellam quæ præcedit 35. *Scorpii*, à qua distabat Tubo 24 pedum mensurata 7′ 16″; idque in recta per claram in pede Ophiuchi θ & dictam Stellam producta. Hæc autem Stella præcedit 35. *Scorpii* 30′ 27″ Asc. Rect. eâque Australior est 2′ 28″, unde fit locus ejus tum temporis *Sagitt.* 15° 24′ 20″ Lat. Aust. 3° 59′ 25″. Sed θ Ophiuchi tunc habuit *Sagitt.* 17° 28′, & Lat. Aust. 3° 47′ 38″. *Mars* itaque Stellam præcedebat Longitudine 4′ 58″, australior ea 5′ 30″.

Deinde *Sept.* 13° 8^h 5'. T. æq. *Mars* visus est à Dom: *Pound* præcedere claram in humero *Sagittarii* σ 11' 54" Asc. Rect. simulque borealior erat Stellâ 22' 56". Hora 8^h 25' erat distantia Planetæ à Stella 25' 00" accuratè.

Decemb. 5. 18^h 30'. T. æq. consensu sæpius repetitarum observationum, invenit D. *Pound Saturnum* præcedere Telescopicam claram sibi vicinam 27' 19" Asc. Rect. Stellaque australiorem esse 1' 59". Simul *Saturnus* præcedebat κ in Syrmate *Virginis* 1° 25' 21", eâque australior erat 4' 05". Hinc *Saturni* locus *Libra* 29°. 16' 21". Lat. Bor. 2° 22' 21". Telescopica autem tunc habuit *Libr.* 29° 40' 56". Lat. Bor. 2° 33' 43".

Anno 1718. *Jan.* 7. 5^h 30'. T. æq *Venus* apud duas Stellas in *Catal. Britan.* omissas observata est. Erat autem Planeta utrâque Fixâ Borealior, distans à præcedente 32' 30", à sequente 17' 30". Stella præcedens tunc habuit *Pisc.* 14° 42' 20", cum Lat. Aust. 0° 40' 10"; altera vero sequens *Pisc.* 15° 21' 55", Lat. Austral. 0° 27' 15", prout ex observationibus D. *Flamstedii* colligere licet.

Jan. 15. 8^h 00', T. æq. *Jupiter* præcedebat κ in pectore *Cancri* 3° 30' 50" Asc. Rect, fixâque Australior erat 14' 15". Hinc provenit *Jovis* locus *Canc.* 28° 20' cum Latitudine Borea 0° 36' 45".

Martii 11. 10^h 36', T. æq. *Saturnus* præcedebat κ in Syrmate *Virginis* 18' 51", eâque Fixâ australior erat 5' 23. Hinc fit Locus *Saturni Scorp.* 0. 18' 34" cum Lat. Bor. 2. 44' 8". Posito scilicet, juxta *Catal: Britan.* κ *Virginis* occupare ♏ 0. 34' 10", cum Lat. 2. 55' 40". Eadem nocte 17^h 00' *Westmonasterii* observarunt DD. *Desaguliers* & *Gray Saturnum* præcedere Stellam 19' 00", cum declinatione majore in Austrum 4' 45".

April. 8. 11^h 30' *Londini* visus est *Saturnus* nuper Acronychus parum admodum occidentalior Telescopicâ clarâ, eâdemque 5 minutis borealior. Unde Fixæ locus

Libræ

Libræ 28. 18′ 30″ Lat. Bor. 2. 41′. Circulus autem magnus per hanc Stellam & *Saturnum* ductus dirigi videbatur ad Stellam $5^{tæ}$ magnitudinis in *Catal. Brit.* omiſſam, ſed quæ *Hevelio* eſt *in cuſpide Alæ Boreæ Virginis*, cuique locum aſſignat *Libr.* 26. 10′, cum Lat. 14. 43′ Bor.

Eadem nocte 13^h 20′, apud *Wanſted*, perpendiculum à dictâ Stella Teleſcopicâ in lineam Anſarum *Saturni* demiſſum præcedebat centrum planetæ quaſi ſeſquialtera diametro annuli; aberat autem Stella ad Auſtrum ab Anſarum axe 4′ 30″. Simul Anſæ orientalis extremitas deprehenſa eſt in linea recta inter hanc Stellam & aliam eidem quaſi longitudine conjunctam, quæ tunc à *Saturno* diſtabat 24′ 48″ verſus Boream. Locus autem prioris Stellæ tunc fuit *Libr.* 28. 18′ 30″ cum Lat. Bor. 2. 41′ proxime.

Sept. 7. circa meridiem incidit conjunctio *Jovis* & *Veneris* arctiſſima, cujus quidem ſpectaculum Aſtronomis noſtris inviderunt Nubes. Die autem ſexto præcedente mane, vel 5^d 22^h 57′ 30″ T. æq. apud *Wanſted*, *Venus* occidentalior diſtabat à *Jove* 1. 3′ 28″. Die autem 7. 17^h 21′, *Venus* jam facta orientalior à *Jove* aberat 43′ 18″; ac 17^h 34′, *Venus* auſtralior erat *Jove* differentiâ declinationum 14′ 23″. Et 17^h 39′ capta eſt diſtantia Planetarum 44′ 4″. Hinc calculo accuratiſſimi Obſervatoris conjuncti ſunt *Sept.* 7. 0^h 9′ T. æq. *Veneris* centro tum *Jovis* auſtraliore non niſi 1′ 42″.

Denique *Sept.* 18. mane, apud *Wanſted*, *Jupiter* viſus eſt prope *Cor Leonis*, quocum die præcedente conjunctus fuerat. *Sept.* 17. 16^h 51′ T. æq. *Jovis* centrum aberat à *Corde Leon.* 24′ 22″; & 17^h 6′ 20″ erat diff Declin. 12′ 43″. Dein poſt Horam, nempe 17^h 54′, facta eſt diſtantia 24′ 44″; ac 18^h 7′ differentia Declinationum inventa eſt 12′ 35″. Hinc ſupputante Dom. *Pound*, fit *Sept.* 17. 18^h 00′ T. æq. *Jovis* locus ♌ 26 11′ 7″ cum Lat. Bor. 45′ 39″.

Observationes Lunæ & Eclipsium:

Anno 1717. *Jan.* 12. *Westmonasterii* observavit Dom. *Stephanus Gray* Lunæ appulsum ad quatuor Stellas contiguas sub cornu Austrino *Tauri*, apud quas observata est Luna Anno 1683. *Mart.* 23. st. v. ab *Hevelio* & *Flamstedio.* Itaque $9^h\ 45'$ T. app. Luna gibba visa est quasi conjuncta cum Stella è quatuor præcedente, quæ est *Tauri* 107. *Catal. Brit.* quæque tunc Australior erat Lunæ limbo Aust. sesquialtero minuto. $11^h\ 29'$ altera, quæ minor est, & ideo in Catalogo omissa, occultabatur paulo infra medium obscuri limbi. Ad $12^h\ 24'$ Tertia & clarior (110. *Tauri*) in ipsa fere conjunctione sex minutis distabat à limbo boreo. Denique $12^h\ 54'$ sequens è quatuor (111. *Tauri*) limbo Boreo superior erat $3'\ 30''$. Locus autem præcedentis, sive 107. *Tauri*, ex dicto Catalogo tunc erat *Gemini* 18. 12. Lat. Aust. 5. 18′; *Tauri* autem 110 habuit *Gem.* 19. 26′ $\frac{1}{4}$ cum Lat. Aust. 4. 44′: Sequens vero, sive 111 *Tauri*, erat in *Gem.* 19. 45′. Lat. Aust. 4. 48′ $\frac{1}{2}$. Secunda parvula, ut ex aliis observationibus constat, Locum tunc habuit *Gem.* 19. 17′. Lat. 5. 5′ ferè.

Eodem anno *Mart.* 16. mane, erat Eclipsis Lunæ partialis, apud nos ob cœlum nubilum inconspicua. At apud *Cambridg Nov-Anglorum*, Dom. *Robie* Astronomiæ peritissimus vidit Eclipseos initium circa horam nonam. Finem vero, juxta *Paludem Maotida*, ad $11^h\ 42'\ 30''$ sat accurate. Est autem *Cambridg* sub altitudine Poli 42. 25′, *Londino* 71 grad. sive $4^h\ 44'$ occidentalior, ut ex pluribus antea observatis constat.

Dein *Sept.* 9. vesperi, in ædibus Societatis Regiæ *Londini*, observarunt nonnulli è Sociis finem Eclipseos Lunaris $7^h\ 26'$. Luna autem orta est juxta medium

Eclipseos,

Eclipſeos, nec niſi paulo ante finem è nubibus horizontem obſidentibus ſeſe extricaverat.

Sept. 14 Veſperi, hac prima vice poſt longum intervallum rediit Luna ad occultandum *Palilicium*. Favit autem admodum cœlum *Londini* præter ſolitum purum, ita ut Luna & Stella exorientes in ipſo quaſi Horizonte ſimul conſpicerentur. Incidit Immerſio Stellæ 9^h $6'$ $20''$, Lunâ nondum $3°$ altâ, in ipſo quaſi medio Limbi orientalis, è regione ſcilicet Boreæ partis maculæ illius exiguæ quam *Hevelius* Stagnum *Mæridis* vocat, quamque *Ricciolus* ſui ipſius nomine inſignivit. Emerſit autem paulo infra medium limbi obſcuri ad 9^h $58'$ $20''$, in ictu oculi tota ſua claritate effulgens; unde etiam in tam illuſtri Stellâ quaſi nullitas diametri demonſtratur.

Septembris 23. veſperi, incidit Eclipſis Solis vix ullibi in *Europâ* conſpicua. Ex *America* autem noſtrâ duplicem obtinuimus ejus obſervationem; alteram ex literis illuſtris Viri D. *Keith* Provinciæ *Penſylvaniæ* Præfecti digniſſimi, qui *Philadelphiæ*, ſub altitudine Poli $40°$ $00'$ fere, vidit Eclipſin jam cœptam (ſed quæ ante minutum temporis nondum inceperat) ad 11^h $55'$. Circa medium Digiti erant quaſi decem. Finis autem viſus eſt accurate ad 2^h $46'$ $35''$.

Altera autem hujus obſervatio habita eſt ad *Cambridg. Novæ Angliæ* Academiam, à Dom. *Robie*, de quo ſupra: Initium Eclipſeos ibi obſervatum eſt 0^h $23'$ $00''$ poſt meridiem. Ad 1^h $47'$ defecêre IX Digiti. Ad 3^h $5'$ $10''$ deſiit Eclipſis, Sole integro per Tubum 24 pedum conſpecto. Hæc ex literis accurati Obſervatoris communicavit cum Reg. Societate Reverendus Vir D. *Guil. Derham*, R.S.S. Eccleſiæ apud *Windſor* Canonicus, &c.

Dec. 5. Luna paulo ſupra *Palilicium* invecta eſt: Tranſitum autem ſatis arctum obſervavit D. *Jac. Bradley*, A.M. eruditus Juvenis, qui ſimul ingenio & induſtria pollens

his

his ſtudiis promovendis aptiſſimus natus eſt, idemque Reverendi D^{ni} *Pound* ex ſorore nepos. Hic, cum Luna jam propemodum plena eſſet, Stellam contulit cum inſigni illa Macula quam *Ricciolus Tychonem*, *Hevelius Sinam* appellat, & ex pluribus æqualibus diſtantiis Micrometro ante & poſt captis, Stellam dictæ maculæ centro proximam apparuiſſe concluſit ad $11^h\ 15'\ 8''$ T. æq. apud *Wanſted*. Ad $11^h\ 15'\ 42''$ diſtabat Palilicium à limbo Lunæ proximo & Auſtrino $5'\ 55''$. Macula autem *Tycho* ab eodem limbo aberat $4'\ 16''$. Ad $11^h\ 18'\ 42''$ Stella erat in lineâ rectâ cum maculis *Tychonis* & *Copernici*, ſive *Sinæ* & *Ætnæ*; & $11^h\ 25'\ 27''$ T. æq. erat in rectâ cum *Tychone* & *Keplero*. Inter hæc obſervata eſt Lunæ diameter $32'\ 45''$.

Anno 1718. *Jan.* 29. veſperi, DD. *Deſaguliers* & *Gray*, *Weſtmonaſterii* alteram *Palilicii* Occultationem præſtolabant; ſed nubium interventu impediti, viderunt ſaltem quod $5^h\ 52'$ nondum immerſerat Stella; attenuatis autem poſtea nubibus concluſa eſt Emerſio ad $7^h\ 20'$, è regione Promontorii *Sarmatiæ Aſiaticæ Hevelii*.

Feb. 19. manè. Iidem obſervatores ibidem variè cum nubibus colluctati Eclipſin Solis ægre conſpexerunt: Horâ tamen 6. $59'$ viſi ſunt deficere duo Digiti, & poſt unum temporis minutum chorda inter Cuſpides viſa eſt æqualis ſemidiametro Solis.

Apud *Wanſted* autem D. *Pound* notavit ad $6^h\ 54'\ 7''$ T. app. chordam inter Cuſpides $18'\ 30''$. Ad $7^h\ 17'\ 00''$ erat $10'\ 18''$. Ad $7^h\ 19'\ 30''$ eadem inventa eſt $8'\ 05''$. Deſiit autem Eclipſis ad $7^h\ 23'\ 20''$.

Feb. 25, veſperi, $6^h\ 44'$ T. app. *Weſtmonaſterii*, Stella prima *Hyadum* in Naribus *Tauri* (γ *Bayero*) viſa eſt in rectâ per cuſpides Lunæ, adeoque propemodum conjuncta; diſtabat autem à limbo Lunæ Auſtr. $5'\ 51''$. Diameter Lunæ $31'\ 45''$ menſurata Micrometro.

Feb. 28,

Feb. 28. 8^h 36′ T. app. etiam *Westmonasterii*, visa est Immersio Stellæ in Poplite *Pollucis* (λ Geminorum *Bayero*) sub limbi *Lunæ* obscuri ea parte, quæ paulo Borealior erat macula quam *Hevelius Cretam* vocat. Emersio ipsa ob cœlum minus purum non conspecta est: sed ad 9^h 51′ egressa erat Stella è limbo lucido, à quo distabat 3 min. circiter, è regione Boreæ partis Insulæ Majoris *Cassii*.

Aug. 8. Luna orta est paulo infra *Palilicium*, cum quo tamen ob nubes conferri non potuit. Apud *Wansted* autem 13^h 2′ 00″ T. app. visa est Præcedens contiguarum ad σ *Tauri Bayero*, (sive Penultima in nostro *Hyadum* Catalogo, in Num° 354. *Transact.* litera *q* notata) in linea recta per cuspides *Lunæ*, distans ab Austrino 4′ 36″. Ad 13^h 7′ 25″ Stella *p* ejusdem Catalogi emersit paulo infra medium obscuri limbi. Ad 13^h 19′ 4″ emersit Sequens contiguarum dictarum, tantum distans à Cornu Austrino quantum contiguæ illæ inter se, hoc est 7 min.

Aug. 29. Vesperi, Luna fere Apogæa passa est deliquium totalem ac fere centralem: orta autem est Eclipsi jam cœptâ. Hujus observationes maximè luculentas Regiæ Soc. exhibuit toties laudatus Rev. D. *Pound*, eo ordine quo notatæ sunt, nempe

Observ.	*Tempus apparens* h	′	″	*Eclipsis* Lunæ *observata apud* Wansted, 29. *Augusti*, 1718.	′	″
1	6	53	38	Chorda inter Cuspides Micrometro	22	37
2		55	8	Eadem repetita (mensurata	21	14
3		56	31	Repetita —— —— —— ——	19	51
4		57	49	Iterum —— —— — ——	18	28
5		59	38	Denuo—— —— ——	15	00

Obs.	T. appar. h	′	″	*Eclipsis* Lunæ, *Aug.* 29. 1718.	′	″
6	7	2	41	Immersio Totalis in Umbram —		
7	8	36	13	Stella clara in Catalogis omissa occultata est à *Lunâ*, infra *Paludem Mareotida Hevelii* — —	10	2
8	8	48	18	*Luna* cœpit emergere ex Umbra —		
9		51	13	Terminus Umbræ per med. *Mareotidis*; simul Chorda inter Cuspides	15	0
10		53	7	Chorda inter Cuspides — —	18	28
11		54	16	Eadem repetita — —	19	51
12		54	59	Iterum — — —	21	14
13	8	56	18	Denuo — — —	22	37
14	9	0	48	*Porphyrites* emersit ex umbra.		
15		8	3	Mons *Sinai* incepit emergere.		
16		9	17	Umbra per medium *Sinæ.*		
17		10	[illegible]	Jam totus *Sinai* extra Umbram.		
18		11	20	Umbra per medium *Ætnæ.*		
19		17	23	Per medium *Corsicæ.*		
20		20	0	Per medium *Lacus Nigri majoris.*		
21		27	54	Per medium *Besbici.*		
22		28	45	Emersit Stella prædicta.		
23		32	34	*Byzantium* & *Horminius* simul emergunt.		
24		33	58	Stella eandem habuit Declinationem cum Cuspide Aust. Eclipseos.		
25		43	28	Chorda inter Cuspides — 18′ 28″.		
26		47	2	Eadem repetita — — 15 00		
27	9	53	[illegible]	Desiisse videbatur Defectus.		

$10^h\ 30'$, Capta est Lunæ diameter 29′ 45″. Collatis autem inter se Observationibus, ubi Chordæ partis deficientis æquales deprehensæ sunt, provenit Eclipseos medium.

Ex Observ-

		Medium.		
		h	′	″
Ex Obſerv. prima & decima tertia	—	7	54	58
Ex ſecunda & duodecima	—	7	55	3
Ex tertia & undecima	—	7	55	24
Ex quarta & decima	—	7	55	28
Ex quinta & nona	—	7	55	25
Ex ſexta & octava	—	7	55	29
Quorum omnium Medium fit	—	7	55	18

Non minore cum curâ eandem Eclipſin, *Londini* in vico *Fleetſtreet*, inſtrumentis & Teleſcopio optimo D. *Geo. Graham* Automatopœi præſtantis, obſervavit D. *Martinus Folkes* Armig. cum aliis quibuſdam Regiæ Societatis Sodalibus, ut ſequitur,

h	′	″	
6	38	0	Luna per fumum Urbis & Vapores ægre viſa.
6	54	13	Chorda inter Cuſpides utcunque, 21′ 27″.
7	2	0	Immerſio Totalis in umbram.
7	42	15	Stella fixa ſatis clara diſtabat in limbo Lunæ orientali 19′ 21″.
8	35	18	Eadem fixa occultata eſt, 10′ circiter minutis centro Lunæ Auſtralior.
8	45	50	vel, ut quibuſdam viſum eſt, uno minuto tardius Luna cœpit emergere.
8	49	38	Palus *Mareotis* primo margine emerſit.
8	50	14	Integra Palus extra Umbram.
9	0	5	Montis *Porphyritidis* medium emerſit.
9	7	39	Primus margo *Sinæ* emerſit.
9	9	8	Mons *Sinai* totus extra umbram.
9	10	35	Umbra per medium *Ætnæ*.
9	12	0	Totus mons *Ætna* extra umbram.
9	18	51	Umbra per medium *Lacus Nigri* majoris.
9	27	35	Inſula *Beſbicus* tota emerſit.

9 42 21

9 42 21	Chorda inter Cuſpides 19′ 9″.
9 51 25	Finis Eclipſeos, ut quibuſdam viſum eſt.
9 52 45	Finis ex præcedente diſtantia Cuſpid[illegible] concluſa.
9 56 45	Lunæ diameter 29′ 45″, iterumque 29′ 4[illegible]

Erat autem Umbra admodum diluta, unde orta [illegible] difficultas in dijudicandis Emerſionis & Finis momen[illegible] Atque Maculæ etiam obſcuriores clarè conſpectæ, [illegible] pluribus minutis antequam Umbræ marginem attin[illegible]rent. Stella vero quæ durante Eclipſi occultata eſt, [illegible] cum tunc habuit ♓ 17° 16′ ½ cum Lat. Auſt. 1° 6 [illegible] proximè.

Recepimus etiam Obſervationes hujus Eclipſeos à R[illegible] D^no^ *Derham*, apud *Upminſter* in agro *Eſſexienſi* habitas [illegible] D^no^ *Wright* apud *Crew* in agro *Ceſtrienſi*; & à D [illegible] *Har*[illegible] apud *Wakefield* in *Eboracenſi*, cum præmiſſis ubique [illegible] conſentientes, ſi adhibeantur meridianorum differen[illegible] poſito ſcil. quod *Upminſter* ſit 1 ½ min. *Londino* orie[illegible]lius, *Crew* vero 10 min. & *Wakefield* 5 min. occidental[illegible]

Denique Coronidis loco obſervationem adjiciamus [illegible]ximiam quidem, ſuique generis, quod ſcimus, ab inv[illegible]to Teleſcopio primam; quamque indefeſſæ D. *Jac. Bra*[illegible] debemus diligentiæ. Quinto enim *Septembris* manè, S[illegible] jam fere 30 *gr.* alto, vidit apud *Wanſted* arctiſſimum [illegible]næ infra *Palilicium* tranſitum, cujus diſtantiam à lim[illegible] proximo, ad 7^h^ 59′ 00″ T. æq. Micrometro invenit 5′ [illegible] Ad 8^h^ 17′ 5″ diſtabat à limbo 1′ 25″. Stella autem [illegible] 8^h^ 33′ 15″ erat in linea recta per Lunæ Cuſpides tun[illegible]tuſiuſculas, nec niſi 0′ 13″ diſtabat à Boreâ. 8^h^ 41′ [illegible] jam Cuſpidem illam reliquerat 3′ 42″. Et 8^h^ 45′ 37″ [illegible] eadem diſtabat 5′ 36″. Lunæ diameter ad 8^h^ 58′ [illegible] eſt 31′ 7″.

Printed for W. and J. INNYS, Printers to the R[illegible] Society, at the *Princes-Arms* at the Weſt-End [illegible] St. *Paul*'s Church-Yard. 1718.

Fig. 1.

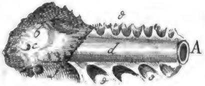

Fig. 11.

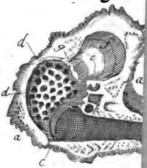

Fig. IIII

V

VI

IX.

X

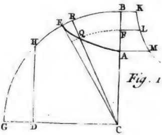

Fig. 1.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Octob. Nov.* and *Dec.* 1718.

The CONTENTS.

I. *Inventio Curvæ quam Corpus descendens brevissimo tempore describeret; urgente Vi Centripetâ ad datum punctum tendente, quæ crescat vel decrescat juxta quamvis Potentiam distantiæ à Centro; dato nempe imo Curvæ puncto & altitudine in principio Casus.* *Per* Joh. Machin, Astron. Profess. Gresh. *&* Reg. Soc. Secret.

SIt centrum Virium C, (*Fig.* 1. 2), quo centro ad distantiam CB æqualem altitudini unde Corpus casurum est, describatur Circulus B E G, & fiat angulus B C G rectus. Ponatur A punctum Curvæ infimum, ubi axi C B occurrit ad datam distantiam C A. Oportet invenire punctum Q, ubi Curva celerrimi descensûs E Q A occurrit circulo Q F, ad datam aliam distantiam C F. Problema hoc duos habet Casus, quorum alter pendet ab Hyperbola & Circulo, alter ab Ellipsi & Circulo.

Cas. 1. Si fuerit Vis centripeta reciproce ut distantia à Centro. Sit K L M (*Fig.* 1.) Hyperbola quævis rectangula centro C & Asymptoto C B descripta, quæ occurrat normalibus B K, A M super ipsam B C ad puncta B, A erectis, in K & M; ordinatæ vero cuilibet intermediæ F L ad punctum F erectæ, in L. Fiat C D ad C G ut $\sqrt{AFLM}$ ad $\sqrt{ABKM}$, & sit D H normalis super CG: dein capiatur Sector RCB ad Aream HDCB ut data Area Hyperbolica A B K M ad datum Rectangulum $CA \times AM$. Tum recta R C occurret circulo F Q in puncto Q, quod quidem est ad Curvam celerrimi descensus E Q A.

Hab-

Habebitur autem punctum E, à quo inciperet Corporis casus, capiendo Sectorem B C E ad Aream Quadrantis B C G, in eadem ratione Areæ Hyperbolicæ A B K M ad rectangulum sub C A & A M contentum.

Coroll. Hinc si recta R C, circa centrum C revoluta, faciat Sectores R C B proportionales Areis H D C B, in quibus quadrata Basium C D sumuntur in progressione Arithmeticâ: tum rectæ C R intersecabunt Curvam E Q A ad distantias à centro C Q, quæ decrescant in progressione Geometricâ.

Cas. 2. Si vero Vis centripeta fuerit reciproce ut alia quævis Potestas distantiæ à centro; sit $n+1$ Index istius Potestatis (ubi n potest esse Numerus quilibet integer vel fractus, affirmativus vel negativus) sitque H = C B altitudo maxima Curvæ quæsitæ E Q A, h = C A altitudo minima ejusdem, & A = C F altitudo alia quævis intermedia. *Fig.* 2.

In recta C G capiatur C D ad C B ut $\sqrt{h^n}$ ad $\sqrt{H^n}$, atque etiam C H ad C D ut $\sqrt{A^n - h^n}$ ad $\sqrt{H^n - h^n}$. Centro C, semiaxibus C D, C B, describatur Ellipsis B L D, cui occurrat ordinatim applicata H L in puncto L; & ducatur recta L K, quæ Ellipsin tangat in L, & Axi minori C D producto conveniat in K: dein Tangenti K L parallela ducatur N M, circulum B E M G tangens in M & ipsi C D occurrens in N. Denique capiatur Sector R C B, qui sit ad Aream N M B L K N, inter Circulum & Ellipsin & utriusque Tangentes rectamque N K comprehensam, in ratione Numeri binarii ad Numerum n. Tum recta R C intersecabit Circulum F Q in puncto Q, quod erit ad Curvam celerrimi Descensus E Q A.

Quod si fiat Sector B C E ad aream B D G, inter Ellipseos & Circuli Quadrantes interceptam, in ratione dictâ Binarii ad Numerum n, coeuntibus scilicet punctis L, D & M, G; (ob $A^n = H^n$) erit punctum E unde inchoa-

choaretur Casus Corporis brevissimo tempore descendentis ad A, descensuque suo Curvam E Q A describentis, quam tangit recta C E in E, quamque ad angulos rectos secat C B in A.

Harum Constructionum Demonstrationes è Celeberrimi D. *Newtoni Quadraturis*, ejusdemque *Philos. Nat. Principiis* (Prop. XXXIX. & sequentibus aliquibus) petitæ, aliâ datâ occasione ostendentur. Problema autem est alterius generis, Describere Curvas per quas Corpora, de puncto summo E, seu principio casus, demissa, celerrimo descensu ad inferiora data puncta Q, urgente qualibet Vi centripeta, ferrentur; cujus quidem solutio in potestate est. In præsentia sufficiat generalem hujusmodi Curvarum tradidisse Ideam, earumque ad Circuli & Hyperbolæ Quadraturas relationes indicasse, absque quibus easdem Geometrice construere haud adeo proclive est.

II. De

II. *De Potentiâ Cordis.*

Dissertatio Authore JAC. JURIN, *M.D.*

Reg. Soc. Sodale.

Viro Eruditissimo

RICHARDO MEAD, *M.D.*

S. P. D.

Jacobus Jurin.

DIsquisitionem istam, Vir Clarissime, utut rudem & imperfectam, acri tamen ac perspicaci tuo Judicio multis nominibus non illibenter permitto. Quem enim mihi potero aut Judicem æquiorem præoptare, aut Cognitorem deligere magis idoneum, quam cujus Viri candorem animi singularem, morumque humanitatem, non minus atque Mentis dotes præcellentes illas, & optimo quoque Literarum genere perpolitas, omnes suspicimus; cujusque tum acumine Ingenii, tum Judicii subtilitate, Theoriam Medicam videmus clarissimâ luce perfusam & illustratam, Usum vero medendi confirmatum pariter tenemus & expeditum? Nec sane quisquam est Mortalium, cujus calculo cogitata ista nostra comprobari magis studeamus, aut cujus auctoritate, si tibi forte fortuna minus displicuerint, ea contra Hominum quorundam perversorum iniquitatem tutiora sint futura. Ex quibus alii præjudicio ducti & famâ magnorum Nomi-

Tttttt num,

num; quorum ſententias in ſequentibus paſſim redarguimus, noſtra forſitan ne examine quidem aut perlectu digna cenſuri ſunt. Alii vero, ut ſive labore diſcendi, ſive imperitiæ pudore ſe expediant, omnia ſcilicet, quæcunque ipſi non intelligunt, videri volunt alto ſupercilio contemnere. Quibus uti non gravate concedimus doctos Viros & olim exſtitiſſe, & hodie reperiri non paucos, qui nullâ inſtructi diſciplinâ Mathematicâ medendi Artem tamen feliciter & cum laude exerceant; ita viciſſim ipſos fateri æquum eſt eam doctrinam in Praxi expediendâ non inutilem, ad naturam vero & cauſas Morborum explorandas plane eſſe neceſſariam. Corpora enim Animalium, quod tu profecto, ſi quis alius, optime intelligis, cum partim ſolidis canalibus, partim fluido conſtent per eoſdem jugiter propulſo, Machinas eſſe patet, ac proinde opus eſſe, ad eorum Fabricam, Vires, Actiones, & agendi Impedimenta ſive Morbos rite perſpiciendos, rei Mechanicæ peritiam.

De quibus tamen multa traduntur etiam à Mathematicis Scriptoribus adeo parum accurata, ſecumque invicem & cum ratione pugnantia, ut nobiliſſimæ ſcientiæ non modo commendationem non addant & dignitatem, ſed etiam contemptui & hominum indoctorum ludibriis eandem objiciant. Quis enim, non ipſe doctrinâ Mathematicâ imbutus, cum videat, Exempli gratiâ, Cordis Humani vires jam ponderi 3,000 librarum pares, jam 180,000 pondo ſuperantes, jam vero ad uncias 5 vel 8 deductas; Aerem quoque ex Pulmone inter exſpirandum propulſum modo 100, modo 50,000 librarum vi; Quis inquam, qui iſtas concluſiones legerit diſcrimine tam immani à ſe invicem remotas, & tamen omnes demonſtrationibus ſuis munitas, ſi forte ſe à riſu temperet, non tamen inutilem plane & ineptam pronuntiaverit ad explorandas Corporis facultates ſcientiam Mechanicam? Sed meminerint oportet æqui rerum Judices neutiquam

mirandum

mirandum esse, si quandoque in difficili Problemate vel summa ingenia allucinentur, neque errores, siqui forte inciderint, Arti ipsi, sed Artifici imputandos. Quod ut Exemplo manifestius declaretur, libet celeberrimi Problematis de Cordis viribus indagandis solutionem novam proponere. Utque facilius mihi temeritatis opinionem detraham, qui ejusmodi inceptum post *Alphonsum Borellum* aggredi ausim, utque viam simul Lectori expediam ad æquam certamque sententiam in tantâ scriptorum dissensione ferendam, primo loco ostensurus sum, quæ in *Borelli* demonstratione reprehendi debeant, deinde Virorum Doctissimorum, *Morlandi*, & *Kellii* solutiones, cum eâdem philosophandi libertate ad examen revocabo.

Primum nobis, & quidem longe præcipuum videtur *Borelliana* solutionis vitium, quod Cordis Potentiam per pondus iners & quiescens exposuerit. Cor enim cum & ipsum inter contrahendum movetur, & corpora opposita, Sanguinem nempe & Arteriarum tunicas, in motum impellit, patet ejus Potentiam non aliâ ratione sciri posse quanta sit, quam ut motus hujus quantitatem cognitam teneamus. Motus autem quilibet cum pondere quiescente comparari non magis potest, quam Linea cum Rectangulo.

Secundum, quod in ipso Experimento à Circulatore instituto, neutiquam constet pondus illud suspensum fuisse à solâ Musculorum vi contractrice; quum etiam vis illa, quâ tum Musculi adhibiti, tum genæ quoque, & ipsa forsitan ligamenta divulsioni sui ipsorum & fibrarum ruptioni obstiterint, quâque Musculi etiam ex cadavere exsecti pondera satis magna sustinent, venire in subsidium potuerit.

3. Quod vires Musculorum pondere æqualium a *Borello* pares statuantur: quod profecto dubium admodum videtur, præsertim ubi Musculi sunt figurâ dissimiles.

4. Quod integram Cordis Potentiam, quanta maxima exeri potest cum summâ fibrarum contentione & molimine, ad singulas Systoles adhiberi posuerit. Quum ipse Circulator, si pondus suspensum vel continenter, vel alternis vicibus brevissimâ quiete interpositâ, sublevare contenderet, non ita longo tempore plane succubiturus labori fuisset.

5. Quod Sanguinis & Arteriarum resistentiam sexagecuplam statuerit totius Potentiæ Cordis, loco ejus Potentiæ, quæ ad systolem peragendam à Corde impenditur, quæque forte totius Potentiæ minima pars est.

6. Quod in eâ ratione sexagecuplâ definiendâ errorem insignem admiserit. Nam in *Prop.* 60. loco rationis, quam obtinet Summa Potentiarum *P* & *Q* ad Summam *R* & *S*, adhibuit rationem, quæ est inter Rectangulum ex Potentiis *P*, *Q* confectum, & Rectangulum ex *R*, *S*. Quod errati si per Propositiones subsequentes corrigatur, habebitur in *Prop.* 73, resistentia longe major quam ab ipso *Borello* definita est, nempe pondus librarum 1,076,000, loco librarum 180,000, idque secundum positiones ab ipso Viro Clarissimo usurpatas.

7. Denique quod pondus illud librarum 180,000, quum à Cordis Potentiâ libris 3,000 æquali superetur, miraculi cujusdam aut monstri loco Lectoribus obtrudat; & Vim Percussionis, quasi quendam Θεὸν ἀπὸ μηχανῆς in auxilium advocet. Reipsâ enim nihilo plus hic inest prodigii, quam ubi pondus 3,000 librarum pondus aliud 180,000 librarum, ad subsexagecuplam distantiam à centro Libræ inæqualium radiorum appensum, in æquilibrio sustinet.

Minora aliquot Sphalmata, & Hypotheses plures tum prorsus arbitrarias, tum alias aliis contrarias, non illibenter omittimus. Et quidem delicta supra reprehensa, aut saltem majorem eorundem partem, non tam ipsi Vi-

ro Doctissimo imputandam censemus, quam Operi Posthumo condonandam.

Proximus sequitur Vir Doctissimus *Josephus Morlandus*, qui in Disquisitionibus de Cordis vi Sermone Anglicano editis, Methodum peringeniosam exposuit Potentiam Cordis ad Experimentum revocandi. Hic autem, præter delictum supra in *Borello* reprehensum, quod Cordis vires cum pondere quiescente contulerit, nobis videtur eo quoque nomine notandus, quod integram Cordis actionem in tunicas Arteriarum distendendas impendi posuerit. Cor enim non solum Arterias tendit, sed Sanguinem quoque certâ velocitate per totum Arteriarum & Venarum tractum propellit.

Superest, ut Viri Acutissimi *Jacobi Keilii* solutionem, in Tentaminibus Medico-Physicis ad Oeconomiam Animalem pertinentibus, non ita pridem cum Publico communicatam, expendamus. Qui primus omnium ausus est Potentiam Cordis à *Borello* definitam, ac magno Scriptorum consensu exceptam & laudatam, non solum rejicere, sed aliam eidem infinito prope discrimine minorem numeris disertis expressim substituere.

Hunc autem censemus, præterquam quod primum illud *Borellianæ* solutionis vitium imitatus sit, in sequentibus etiam à vero aberrasse.

Quod Corollarium *Newtonianum*, quo utitur ad Cordis vires definiendas, aut male intellexerit, aut certe non satis apte usurpaverit. Pondus enim illud ab *Archimede Britannico* determinatum, quo Motus aquæ ex vase effluentis generari potest, nequaquam generat Motum aquæ; quippe quæ gravitatis vi cadendo ipsa Motum suum acquirat. Sed hoc pondus per datum tempus cadendo, Motum concipit Motui aquæ eodem dato tempore effluentis æqualem.

Præterea ponit Vir Clarissimus velocitatem Sanguinis ex Corde effluentis perpetuo æqualem per totam Systoles

les durationem, quam nos insigniter inæqualem fieri in sequentibus ostendemus.

In Methodo illa simpliciore, quam postea adhibet Vir Doctissimus, præter delicta hactenus reprehensa alia etiam bina admittit.

Adsumit enim Vires Cordis in diversis Animalibus eam inter se rationem obtinere, quæ est inter pondera eorundem; quod infra falsum esse demonstrabimus. Tum ponit velocitatem Sanguinis ex sectâ Iliacâ Arteriâ profluentis, eandem esse quâ ex Corde in Aortam emittitur. Atqui cum omnis fere sanguis ex Corde expulsus per Iliacam alteram resectam emittitur, patet ejus velocitatem tanto esse majorem in Iliacâ quam in Aortâ, quanto sectio Iliacæ circularis à sectione Aortæ superatur. Præterquam quod velocitas æquabilis, quâ Sanguis per Aortam fluit, longe distet ab eâ velocitate, quâcum exit ex ipso Corde.

Similiter fere redargui potest & illa Methodus, quâ usus est Vir Cl. ad rationem definiendam inter velocitates diversas Sanguinis, resistentiâ nunc oppositâ, nunc sublatâ, per Aortam profluentis. Sed cum isto Experimento non altera solum, sed utraque velocitas major æquo reperiatur, unde ratio, quæ est inter ipsas, non magnopere perturbetur, poterit satis tuto proportio ab ipso exposita, tanquam veræ propinqua, usurpari.

Cursu hactenus expedito, scopulisque detectis, in quos impegerunt Viri egregii supra laudati, erit modo nobis ipsis, ut in viâ difficili & erroribus plenâ, summâ adhibitâ cautione progrediendum. Et primo quidem loco ad ambiguitatem præcidendam necesse est, ut id, quod quæritur, quale sit, accuratius paulo declaretur.

Cordis Virium, sive Potentiæ, nomine significamus vel ipsum Cordis Motum, dum in contractionem agitur, vel Motum ponderis cujuslibet, quod Sanguini objectum

ex

ex Corde proruenti. & velocitate idoneâ delatum in partes contrarias, Sanguinis effluxum, adeoque ipſam Cordis contractionem, æquali vi librare valet & ſiſtere.

Potentiam iſtam, cum à priori vix ſperandum ſit ut definire poſſimus, quod neque fabricam Cordis interiorem, neque cauſæ contrahentis naturam, aut vires ſatis habeamus exploratas, relinquitur, ut eandem per effecta, ſive à poſteriori, æſtimemus.

Cordis actio in Ventriculorum ſuorum contractione omnis conſiſtit. Ventriculi autem inter contrahendum in ſanguinem impingunt, eique Motus ſui partem communicando, eundem magna vi, qua datur porta, urgent, & expellunt. Sanguis hoc modo in Arterias, Aortam & Pulmonalem, protruſus, impetu in omnes partes facto, partim in tunicas Arteriarum ex Syſtole ſua prægreſſâ collapſas & flaccidas, partim in Sanguinem priorem tardius fluentem impingit. Unde gradatim extrorſum truduntur Arteriarum tunicæ, & Sanguis antecedens curſu celeratur. Quod ſi animo concipiantur Arteriæ ſectionibus tranſverſis minimis diſtinctæ, primâ Sanguinis portiunculâ ex Corde in primam ſectionem irruente, partim diſtenditur iſta ſectio, partim Sanguis eâdem antea contentus in ſectionem proximam detruditur, eamque diſtendit, atque iſta actio per ſuccedentes Arteriarum ſectiones continuatur. Deinde ſecunda, & tertia ſanguinis portiuncula, & cæteræ deinceps, in primam Arteriæ ſectionem incidunt, eamque paulo magis dilatant, & ſanguinem eâdem contentum in proximas ſectiones ſucceſſive propellunt; idque fieri pergit, donec omnis ſanguis ex Ventriculis fuerit ejectus. Cæterum id utique obſervandum eſt Arterias, quo magis contractæ & flaccidæ fuerint, eo minus dilatationi obſiſtere, quanto autem magis fuerint dilatatæ, tanto fortius ulteriori diſtractioni reniti; atque idcirco Vim Sanguinis ex Corde prorumpentis primo magis impendi in diſtentionem Arte-

 riarum,

riarum, quam in Sanguinis præcedentis protrusionem, sub finem vero magis propelli Sanguinem antecedentem quam distendi Arterias, quippe quæ jam rigidæ factæ majorem dilatationem vix admittant.

Sanguis autem ex Corde prosiliens, cum, uti dictum est, Motus sui partem Arteriarum tunicis, partem Sanguini præcedenti communicat, ipse necessario de pristinâ celeritate remittit; adeoque dum Ventriculorum contractionem moratur, novum ab iis impulsum excipit, ejusque partem, eâdem ratione atque antea, tunicis Arteriarum & præcedenti Sanguini impendit, unde iterum retardatur, & alium Ventriculorum ictum suscipit, & sic deinceps, donec omnis ex Ventriculis fuerit expulsus.

Præter causam supra expositam, superest alia, quâ Sanguis ex Corde effluens gradatim retardatur, adeoque novos successive impetus excipit ex Ventriculis sese contrahentibus. Nam Sanguis in Arteriam Aortam influens, etiamsi nulli omnino resistentiæ occurrere ponatur, adeoque nullam pati Motus sui imminutionem, tamen, cum ex lato in angustum fertur, longitudine perpetim crescit, donec totus in Aortam pervenerit; cumque sectio Aortæ non minuatur, necessario minuitur Sanguinis velocitas. Motus enim Sanguinis est in ratione compositâ, ex ratione Sectionis Aortæ, velocitate in eâdem, & longitudine Columnæ Sanguineæ, per Theorema nostrum III. De Motu Aquarum fluentium. Cum vero ea Sanguinis portio, quæ jam pervenerit in Aortam, gradatim retardetur, retardabitur inde Sanguis iste qui adhuc Ventriculo continetur, & hinc retardabitur ipsius Ventriculi contractio. Unde Ventriculi perpetuo aliam atque aliam Motus sui partem Sanguini contiguo, his de causis perpetim retardato, communicabunt. Patet vero isthinc, ut id obiter notemus, alium esse Motum Sanguinis ex Corde erumpentis, alium ejusdem jam ex Corde expulsi, & intra Arterias

fluentis.

fluentis. Item ictum sive impulsum Ventriculorum in Sanguinem impressum, qui alioqui unicus esset futurus, & puncto temporis transigeretur, tamen causarum supra dictarum vi, quibus Sanguis perpetim retardatur, per totam Cordis Systolen continuari.

Ventriculum itaque alterutrum Cordis Sanguinem impellentem licebit spectare, ut datum corpus cum datâ celeritate impingens in aliud corpus quiescens, cui Motus sui parte communicatâ ambo corpora communi velocitate deferuntur. Æquatur autem Potentia ejusdem, vel Facto ex pondere Ventriculi & velocitate ejus initiali, priusquam in Sanguinem impingat; vel Summæ Motuum ipsius Ventriculi ac Sanguinis ex eodem profluentis, & Motûs qui tunicis Arteriarum & Sanguini præcedenti communicatus est; vel etiam, si abesse ponatur omnis Arteriarum & Sanguinis præcedentis resistentia, Summæ Motuum ipsius Ventriculi & Sanguinis effluentis.

Theorema I.

Motus, quo Machina cava inæqualiter contractilis in contractionem agitur, æqualis est Summæ Factorum ex singulis Machinæ particulis ductis in velocitates respectivas.

Patet ex Mechanicâ.

Corol. 1. Machinæ Motus minor est Facto ex pondere Machinæ ducto in velocitatem earum Machinæ partium, quæ omnium celerrime moventur inter contrahendum.

2. Motus Machinæ æquatur Facto ex pondere ejusdem, ducto in velocitatem aliquam mediam inter velocitates earum Machinæ partium quæ omnium celerrime, & earum quæ omnium tardissime, moventur.

3. Si Machinæ plures similes similiter sese contrahant, velocitate mediâ, vel æquabili vel inæquabili, similiter tamen auctâ vel imminutâ in omnibus Machi-

nis; Motus, quo Machina quæque in contractionem agitur, rationem obtinet compositam ex ratione quadruplicatâ Diametri homologæ ipsius Machinæ, & ratione inversâ temporis, quo Machinæ contractio perficitur; vel rationem compositam ex ratione ponderis Machinæ, ratione ejusdem ponderis subtriplicatâ, & ratione temporis inversâ. *Theoremata reliqua huc spectantia in* Transactione *proximè edenda exhibebuntur.*

III. *A Brief Account of the Contagious Disease which raged among the Milch Cowes near* London, *in the Year* 1714. *And of the Methods that were taken for suppressing it. Communicated to the* Royal Society *by* Thomas Bates *Esq;* *Surgeon to His Majesties Houshold, and* R. S. S.

ABout the middle of *July* the Distemper appeared at *Islington*, and thereupon their Excellencies the Lords Justices having notice of it, were pleased to Command that I should examine into the truth of the Report of its being Contagious; and order'd the Lord *Harcourt*, then Lord *High Chancellor*, to grant such Authority as wou'd be proper to make the Discovery. Accordingly Mr. *Milner*, Mr. *Offley*, Mr. *Richardson*, and Mr. *Ward*, four Justices of the Peace for the County of *Middlesex*, were appointed to make the necessary Examinations.

Pursuant to those Orders we went to *Islington*, where Mr. *Ratcliff* had lost 120 out of 200; Mr. *Rufford* 62 out of 72; and Mr. *Pullen* 38 out of 87. They were very unwilling to own it, because so soon as it should be known, none wou'd buy their Milk; but Mr. *Ratcliff*, a Man of good Judgment in Cattle, after much perswasion, gave us the following account, *viz.* That they first

first refused their Food; the next Day had Huskish Coughs, and voided Excrements like Clay; their Heads swelled, and sometimes their Bodies. In a Day or two more there was a great discharge of a Mucous Matter by the Nose, and their Breaths smelled offensively. Lastly, a severe Purging (sometimes Bloody) which terminated in Death. That some died in three Days, and others in five or six, but the Bulls lived eight or ten. That during their whole illness, they refused all manner of Food, and were very hot.

We then advised with several of the *Cow-leeches*, or Doctors, who all agreed that it was a Murrain, or rather a Plague; and that the Methods they had tryed for a Cure, had proved unsuccessful. This Disease was so surprising, that some of those Men who used to look after them, were afraid to go near them.

We then ordered some of the sick Cows to be Housed, and several sorts of Cattle to be kept with them, to see whether the Contagion would affect any other Species.

The next Day I made a Verbal Report to their Excellencies, of all the several Opinions and Discourses which I have had about it, and left them debating what Method to take; at last I was called in, and Ordered to consider of it again the next Day, and to deliver to them in Writing what would be proper to be done. Accordingly I drew up, and gave them the following Proposals.

I. That all such Cows as are now in the possession of Mr. *Ratcliff*, *Rufford*, and *Pullen*, be Bought, Kill'd, and Burnt: or, at least, that the Sick be Burnt; and the Well kept and secured on the Grounds where they now are, that such of them as Sicken or Dye of this Distemper may be Burnt.

 II. That

II. That the Houſes in which thoſe Sick Cows have ſtood be Waſhed very clean, and then ſmoaked by the burning of Pitch, Tarr, and Wormwood, and be kept three Months at leaſt before any other Cows are put therein.

III. That the Fields where thoſe Sick Cows have Grazed, be kept Two Months before any other Cows are ſuffered to ſtand or Graze thereon.

IV. That the Perſons looking after ſuch as are Ill, ſhou'd have no Communication with thoſe that are well.

V. That the ſame Methods be Obſerved if any other of the Cow-keepers ſhou'd get this Diſtemper among them; and that they be all Summoned and told, that as ſoon as they perceive any of their Cows to refuſe their Meat, or have any other Symptoms of this Diſtemper, that they immediately ſeparate them from their others, and give notice to ſuch Perſons as your Excellencies ſhall appoint, that they may be Burnt; and the places where they have ſtood or Grazed to be ordered as before.

VI. That the Cow-keepers be required to divide their Cows into ſmall Parcels, not more than ten or twelve in a Field together; and that they be allowed ſuch ſatisfaction for complying with theſe Propoſals, as your Excellencies ſhall think fit; all which is moſt humbly ſubmitted, *&c.*

The next day their Excellencies conſulted the four Gentlemen before-named, and gave them Orders to comply with the preceeding Propoſals, and to allow *Forty Shillings* for every Sick Cow which they Burnt, that belonged to Mr. *Ratcliff*, *Rufford*, and *Pullen*; but the free intercourſe which both Maſters and Servants had had with each others Cows (before we were appointed) had

had ſpread the Contagion; and the Diſeaſe began ſoon to appear in ſeveral other Neighbouring places.

The Gentlemen then ſummoned all the Cow-keepers in the County, and acquainted them with the above-named Propoſals (to moſt of which they readily Complyed, as being viſibly their intereſt) and offered them *Forty Shillings* for every Cow which they Burnt, that had not been Sick above twenty-four Hours; but for ſuch as had been longer Ill, or were Dead, they wou'd allow them only the value of their Skins and Horns.

Some of the Cow-keepers appeared not content with this Regulation, and believing that the Diſeaſe wou'd become general, deſign'd to have ſold their Cows at ſome diſtant Market; which the Gentlemen having notice off, appointed ſeveral Butchers to Watch near their Grounds, and count their Numbers every Morning, with Orders to follow ſuch as they ſent to any Market, and prevent their being ſold, by telling the people what they were.

Another great Obſtacle at the firſt was the Cow-keepers not owning the Diſeaſe, till they had loſt ſeveral of their Cows; for ſo ſoon as it was known that any Man had but one Sick, none wou'd buy his Milk; and to thoſe who kept many Cows, that loſs was conſiderable.

Nor was there ever wanting one or other who gave them hopes of a Cure.

To obviate theſe three difficulties, the Gentlemen encouraged them to hope for a Brief, but aſſured them that ſuch only as complyed with theſe Directions, ſhou'd have any benefit by it. Accordingly they ordered a daily account to be taken of the Conduct of each Cow-keeper, and allowed or diſallowed their pretenſions to this Brief, as well as to the *Forty Shillings per* Cow, as they complyed or diſregarded theſe Directions.

This

This had a pretty good effect; but here in *England*, where every Man is at liberty to dispose of his Cattle as he pleases, nothing but making them sensible that it was each Mans particular interest to comply with these Methods cou'd do; this, tho' true in fact, yet the Reader will readily judge to be very difficult among such a Number; but the Gentlemen spared no labour to accomplish it; for that purpose they summoned them once or twice every Week, urged all that cou'd be said to induce their Complyance, and omitted no warrantable means to frustrate their Folly.

I had Orders from the beginning to assist those Gentlemen with my Advice, which I did at most of their Meetings; as also to make a stricter enquiry into the Disease by Dissections, *&c.*

Accordingly I discoursed the *Cow-leeches* about the Customs and Diseases that Cows were subject to, and consulted such Books as treated of them; but concerning this Disease, I cou'd gain but small assistance from either.

I then made Dissections of sixteen Cows, in different degrees of Infection; and found the Putrefaction of their *Viscera* to encrease, in proportion to the time of their Illness.

The first five that I opened, had hearded with those that were Ill, and the Symptoms of this Distemper were just become visible; in these, the Gall-bladders were larger than usual, and filled with Bile of a natural Taste and Smell, but of a greener Colour. Their *Pancreas*'s were shrivelled, some of the Glands obstructed and tumified. Many of the Glands in their *Mesenterys* were twice or thrice their natural bigness. Their Lungs were a little inflamed, and their Flesh felt hot. All other parts of their *Viscera* appeared as in a healthful State.

The

The next six that I opened, had been ill about two Days; in them the Livers were blacker than usual, and in two of them, there was several Cysts filled with a Petrified Substance like Chalk, about the bigness of a Pea. Their Gall-bladders were twice their usual bigness, and filled with Bile of a natural Taste and Smell, but of a greener Colour than the first. Their *Pancreas*'s were shrivelled, some of their Glands very large and hard, and of a blackish Colour. The Glands in their *Mesenterys* were many of them five times their natural Bigness, and of a blackish Colour. Their Lungs were inflamed, with several small Cysts forming. Their Intestins were full of red and black Spots Their Flesh was very hot, tho' not altered in Colour.

The five last that I opened, were very near dying; in them I found the Liver to be Blackish, much Shrivelled and Contracted, and in three of them, there was several Cysts as big as Nuts or Nutmegs, filled with a Petrified Substance like Chalk. Their Gall-bladders were about three times their usual bigness, and filled with Bile of a natural Taste and Smell, but of a deep Green Colour. Their *Pancreas*'s were Shrivelled and Contracted, many of their Glands very large and hard, and of a black Colour. The Glands in their *Mesenterys* were many of them distended to eight or ten times their natural bigness, were very Black, and in the *Pelvis* of most of those Glands in two Cows, there was a yellow Petrefaction, of the consistence of a sandy Stone. Their Intestines were the Colour of a Snake, their inner Coat excoriated by Purging. Their Lungs were much Inflamed, with several Cysts containing a yellow Purulent Matter, many of them as big as a Nutmeg. Their Flesh was extream hot, tho' very little altered in Colour.

I have

I have here only given you a general account of my Diſſections, in the three different Stages of the Diſeaſe; for as the difference was but ſmall, and the Diſeaſe incurable, it could neither be uſeful nor pleaſant to the Reader, to have each particular Diſſection at large, tho' I have now the Minutes by me. But the following Caſes being very extraordinary, I cou'd not omit the mention of them, *viz.* In one of them the Bile was Petrified in its Veſſels, and reſembled a Tree of Corral, but of a dark yellow Colour, and brittle Subſtance.

In another there were ſeveral Inflammations on the Liver, ſome as large as a half Crown, cracked round the Edges, and appeared ſeparating from the ſound part, like a Peſtilential Carbuncle.

In a Third, the Liquor contained in the *Pericardium* (for Lubricating the Heart in its Motion) appeared like the ſubſidings of *Aqua Calcis*; and had excoriated, and given as yellow a Colour to the whole Surface of the Heart and *Pericardium*, as *Aqua Calcis* cou'd poſſibly have done.

In giving my Opinion of this Diſtemper, I muſt beg leave to premiſe, that all Cows have naturally a Purgation by the *Anus* for five or ſix Weeks in the Spring, from (as the Cow-keepers term it) the frimneſs of the Graſs; during which time they are briſk and lively, their Milk becomes thinner, and of a blewiſh Colour, ſweeter to the Taſte; and in greater Plenty: but the Spring preceeding this Diſtemper, was all over *Europe* ſo dry, that the like has not been known in the Memory of any one living; the conſequence of which was little Graſs, and that ſo dry and void of that frimneſs which it has in other Years, that I could not hear of one Cow keeper, who had obſerved his Cows to have that Purgation in the ſame degree as uſual; and very few who had obſerved any at all. They all agreed

that

that their Cows had not given above half ſo much Milk that Summer as they did in others; that ſome of them were almoſt dry; that the Milk they did give was much thicker, and yellower than in other Years. It was obſerved by the whole Town, that very little of the Milk then ſold wou'd Boyl without turning; and it is a known Truth, that the weakeſt of the common Purges you can give a Cow entirely takes away her Milk; from all which Circumſtances, I think it evident, that the want of that natural Purgation was the ſole cauſe of this Diſeaſe; by producing thoſe Obſtructions, which terminated in a Putrifaction and made this Diſtemper Contagious.

During my daily Converſation at that time with Cow-keepers, *&c.* there occurred many other Circumſtances of leſs Moment, to confirm me in this Opinion: but as there was no one reaſon to give me the leaſt notion of any other Cauſe, I ſhall not trouble the Reader with a uſeleſs detail of them.

Cows are likewiſe ſubject to a Purgation (tho' in a leſs degree) from the ſame quality in the Graſs, about the latter end of *September*; which is called the latter Spring; and which I believe contributed not a little, to the preventing the encreaſe of this Diſtemper; for this Purgation coming ſo ſoon after the Diſeaſe appeared, it is not unreaſonable to ſuppoſe, that it freed ſuch Cows as were not much injured, from the ill effects of thoſe Obſtructions, occaſioned by the want of their Vernal Evacuations.

Several Phyſitians attempted the Cure, and made many Eſſays for that purpoſe; but the Diſſections convinced me of the improbability of their ſucceeding, with which I acquainted their Excellencies. However they having received the following *Recipe* and Directions from ſome in *Holland*, ſaid to have been uſed

there with good succefs, gave me Orders to make tryal of it: But the effect was anſwerable to my expectation, for in very many Inſtances, I was not ſenſible of the leaſt Benefit.

Herb. *Ariſtoloch. Rotundæ,*
Veronicæ, ãa M.viij
Pulmonariæ,
Hyſſopi,
Scordij, ãa M. 4
Rad. *Gentianæ,*
Angelicæ,
Petaſitidis,
Tormentillæ,
Carlinæ, ãa ℔ ſs.
Bacc. *Lauri,*
Juniperi, ãa ℥xij *Miſce fiat Pulv.*

See *Phil. Tranſact.* N°. 338. *in fine.*

This Powder is to be given in Water, one Ounce at a time, three or four Mornings ſucceſſively; then reſt four Days, and if the Diſeaſe continues, repeat the Powders in warm Water, as before.

I think there is no one Method in Practice, but what was tryed on this Occaſion, tho' I cannot ſay that any of them was attended with an appearance of Succeſs; except that of Bleeding plentifully, and giving great quantities of Cooling and Diluting Liquids. But by this Method, the inſtances of Succeſs were ſo few, that they do not deſerve any further mention.

Their Excellencies being informed that the feeding Cows with Diſtillers Grains was a new Cuſtom, and was the cauſe of this Diſeaſe, gave me Orders to examine into the Truth of it; but upon enquiry, I found it to have been the Practice of ſeveral of the Cow-keepers above twenty Years, without the leaſt appearance of

any inconvenience; and that ſome of thoſe Perſons who had ſuffered moſt, had never given any. Nor is there any difference between thoſe of Brewers and Diſtillers, only that the latter are the dryer.

It was likewiſe ſaid, that the want of Water was the cauſe of this Diſeaſe, for that the Springs and places where People uſed to Water their Cowes, were almoſt every where dry; and that many were obliged to ſend them ſeveral Miles for Water. This might produce ſome Diſeaſes, but ſuch only as they got by the fatigue of being driven ſo far; for Mr. *Ratcliff*, Mr. *Rufford* and Mr. *Pullen*, the three Perſons where this Diſeaſe firſt appeared, had the New River Water running thro' the very Grounds where their Cows conſtantly Grazed, and cou'd drink at their Pleaſure, and ſo had moſt of the Cow-keepers at *Iſlington*.

There were at that time ſeveral other reports of the cauſe of this Diſeaſe, but none that had a ſhew of Reaſon.

About the latter end of *September*, the Diſeaſe increaſed, and the Numbers brought to be burnt were ſo great, that it cou'd not be well executed; therefore it was judged proper only to bury them fifteen or twenty Foot deep: but firſt to make large Inciſions in their moſt Fleſhy parts, and to cover them with quicklime.

At the ſame time, having notice that it was a Cuſtom with the Cow-keepers, to ſend their Calves when a Week old to *Rumford, &c.* to be Sold; and apprehending by this means that the Contagion might be carried into the Country, I required all ſuch as had Sick Cows, to bring their Calves to be buried; to which they readily conſented, and were allowed from *Five* to *Ten Shillings per* Calf.

In the beginning of *October*, being informed that ſome of the Cows in *Norfolk*, *Suffolk*, and *Hertfordſhire*, had

 got

got this Disease, and apprehending that it wou'd become general; I gave in the following Report to a Committee of Council.

The Distemper among the Cattle encreasing, and beginning to appear in several other Counties, I thought it my Duty to acquaint your Lordships, with the hazard that may attend their not being duely buried. It is the Opinion of all Authors in Physick that treat of Contagious Diseases, as well as of several of the Physitians in Town, that a Putrifaction of so many Cows as there is reason to fear will dye of this Distemper, may produce some Contagious Disease among Men; unless they are buried so deep that the Infectious *Effluvia* cannot injure the Air, which I am certain has very seldom been complyed with, except in the Counties of *Middlesex*, *Essex*, and *Surry*, the Gentlemen employed being capable of acting in those Counties only. It is affirmed by several now living, that there was a Mortality among the Cattle, a little before the last great Plague in the Year 1665, which was imputed to the want of a due Care in burying them. And your Lordships may know of what importance it was judged by the King of *Prussia*, the States of *Holland*, and several other Princes and States, by the Care they took to publish Decrees and Placarts, commanding them to be buried upon pain of Death, or other severe penalties; and I humbly conceive it wou'd be necessary, not only to bury those which shall Dye, but that such as are already Dead may have the same Care; as also that they be buried nine or ten Foot deep at least. All which is most humbly submitted, *&c.*

Their Lordships thought fit to defer all proceeding upon this Report, till the Distemper becoming more general shou'd make it Necessary; but I thank God that Necessity never happened, for within three Weeks

or

or a Month after the giving in of that Report, the following particulars concurred to put an end to the Disease.

The Cows began their latter Purging, which contributed much to prevent the Disease from appearing in fresh Places; and the Cow-keepers were convinced that the Disease was incurable.

The knowledge of the Disease was spread all over *England,* so that none wou'd buy a Cow in the Country; and the Gentlemen prevented their being kill'd in Town, by having the Markets examined daily; and such Meat condemned as appeared Suspicious.

They now divided their Cows into small Parcels, by which they lost only that in which the Disease happened; whereas before that Method, when one Cow got this Disease, if she had herded with One, Two, or Three Hundred (the Contagion was such) scarce one did escape.

Those who had no Sick Cows avoided all Communication with such as had.

They likewise found that the keeping their Cows so long when Ill, had been the chief Cause of their Loss; they therefore now brought them to be Buried on the first appearance of the Disease, before the Contagion cou'd possibly have got to any great height.

These were the effects of the Cow-keepers dear bought Experience; but it was the indefatigable Care and Diligence of those four Gentlemen, who gave a daily Attendance, both early and late, that secured *Great Britain* from that terrible Ravage, which was made by this Distemper in several parts of *Europe*.

The severity of this Disease in *England* did not last above three Months; tho it was not entirely suppressed till about *Christmas*: But in several other Countries it continued two or three Years; and I am credibly assured,

ſured, that in *Holland* it now rages with as much violence as ever; and that they have loſt in Cows, Oxen and Bulls, above Three Hundred Thouſand.

The Providence of God has ſo diſpoſed the matter of Animal Bodies, as to render Contagious Diſeaſes very ſeldom infectious to different Species; but Experience demonſtrates, that Contagions may be communicated to the ſame Species, by touching the Woolen, Linnen, *&c.* to which the Infectious *Effluvia* of the Diſeaſed had adhered, tho' the two Bodies ſhould be at a very great diſtance; and I verily believe that more Hundreds died from the Infection, which was carried by the Intercourſe that the Cow-keepers had with each other, than ſingle ones by the original Putrifaction.

The Nature of Contagious Diſeaſes are but little underſtood, and it would neither be agreeable to my Deſign, nor uſeful to the Publick, to ſay more of this than what was evident: But I have been particularly careful, not to omit any thing Material, either for deſcribing the Diſeaſe, or manifeſting the Methods that were taken for ſuppreſſing it; becauſe it is more than probable that the ſame Care wou'd be equally ſucceſsful in any other Species of Cattle.

The number of Bulls and Cows loſt by this Diſeaſe, in the Counties of *Middleſex*, *Eſſex* and *Surry*, were Five Thouſand Four Hundred and Eighteen; and of Calves, Four Hundred and Thirty Nine; and the Money iſſued for them, at *Forty* or *Ten Shillings per* Cow, *&c.* was the Royal Bounty of his Majeſty, from his own Civil Liſt: and tho' neither the four Gentlemen, nor I, made any demand for a Reward, or for Expences, yet it amounted to 6774 *l.* 1 *s.* 1 *d.* But the entire loſs to the Cow-keepers, as delivered in upon Oath, was 24500 *l.* (excluſive of the 6774 *l.* 1 *s.* 1 *d.*) tho' computed but at *Six Pounds per* Cow; which at a Medium, was

was not more than their Prime Coſt; the dearneſs of keeping them near *London* neceſſitating the Cow keepers to buy the very beſt.

His Majeſty was further pleaſed, on the Sollicitation of the four Gentlemen, to grant a Brief for the 24500 *l.* but the many falſe Reports that were then induſtriouſly propagated, to leſſen the value of thoſe poor Mens loſſes, ſo fruſtrated that Charity, that the entire Sum Collected (the charges of Collecting being firſt paid) was but 6278 *l.* 2 *s.* 6 *d.* which on a Dividend, amounted to *Five Shillings* and *Three Half Pence* in the Pound, computing their Loſs as above, at *Six Pounds per* Cow; tho' if we conſider their Contracts with Brewers for Grains, their Rent of Grounds which lay uſeleſs, Servants wages, *&c.* their real Loſs may (by a modeſt Computation) be allowed to be *Ten Pounds* for every Cow that died.

IV. *A Deſcription of the* Organ *of* Hearing *in the* Elephant, *with the Figures and Situation of the* Oſſicles, Labyrinth *and* Cochlea *in the Ear of that large Animal. Communicated to the* Royal Society, *by Dr.* Patrick Blair, *R. S. S.*

IN the Deſcription I formerly wrote to the Honour'd Sir *Hans Sloane*, Bart. of the *Elephant* I Diſſected in *Dundee, Anno* 1706. which he was pleaſed to Communicate to the *Royal Society*, as you have it in *Philoſ. Tranſ.* N°. 226. 227. I treated of the Bony part of the Ear of that prodigious Animal a little too ſuperficially; becauſe I was unwilling at that time to break up the *Os Petroſum* of the right Ear, which had accidentally

been

been separated on dividing of the Scull, by which the account I then gave of the *Linea Semilunares*, or *Labyrinth* and *Cochlea* was but Lame. But I have chosen since rather to destroy that Bone (however seldom such Bones are to be met with) than that the Publick should be depriv'd of an exact Description of that curious Organe, and that I may give a clear Idea of all its Bony parts, I shall repeat what I formerly advanc'd upon that Subject, and add what Improvements I have made upon it since.

Before I proceed, 'tis fit I observe that the *Auris Externus* of this big Creature lyes flat, and not Protuberent as in other Quadrupeds, whose Cartilaginous Substance is capable of divers Motions perform'd by several Muscles, whereby the inner Ear is preserv'd from the great violence of the External Air, which upon some occasions might perhaps injure or break the thin and delicate Membrane of the *Tympanum*. It is also for this reason that the *Meatus* is further guarded, by the Contorsions and oblique Position of the Cartilage at the Orifice of the *Meatus*, which only admits of a determinate quantity of Air, sufficient for the vibration of the *Membrana Tympani*, by which a distinct sound is convey'd to the *Sensorium commune*; whereas did the Air admitted exceed its due proportion, nothing but the confus'd Idea of a Sound would follow, such as resemble the rushing of Waters, &c. or that noise often observ'd when, by a supervenient Cold or the like, obstructions are generated within the Ear it self. And in Man, because the *Auris externus* is also flat, not only are these turnings and windings observable in the Cartilage at the entry, but the *Meatus* it self is likewise obliquely Situated, to prevent the aforesaid Inconveniencies. But there is no need for such a contrivance in the *Elephant*, whose external Orifice of the

the *Meatus* is patulent, open (ſcarſe being guarded by the Cartilage) and ſtreight, whoſe length (it reaching from the external to the internal Table of the Scull) is ſufficient to prevent the acceſſion of too great a quantity of Air to the *Tympanum*; for in its progreſs moſt of the *Columnæ Aeris* beat againſt one or other of the ſides of the *Meatus*, in ſo much that their force is inhibited, and only ſo many as ſuffice to convey the Sound, can reach the *Tympanum* it ſelf.

Meatus Auditorius.

The *Meatus Auditorius* then is a long ſtreight Tube or *Canule* ſituated Horizontally, and reaching from the outer to the inner Table of the Scull, in Figure not unlike the Barrel of a Piſtol, but ſomewhat Oval, the ſides of whoſe Cavity are hard and ſolid, about the thickneſs of a Halfpenny, from whoſe outer Part ſeveral of the *Laminæ* betwixt the two Tables of the Scull do ariſe, (*Fig.* I.) Its Cavity is an Inch or $\frac{3}{4}$ of an Inch Diameter, and length 9 $\frac{1}{2}$ Inches; being ſomewhat enlarg'd as it arrives at the *Crena* for the *Membrana Tympani.* (*Fig.* II.)

Crena and Cavitas Tympani.

This *Crena* is two Inches in Circumference, within which is the *Cavitas Tympani*, conſiſting of two different Surfaces; the one much deeper and Cellulous, the other more ſuperficial and Smooth. The firſt runs perpendicularly down $\frac{1}{4}$ Inch from the *Crena Tympani.* Its bottom is variouſly divided into ſeveral *Celluls*, not unlike a Hony Comb, but irregularly diſpos'd. Its Bony *Laminæ*, by which theſe Cellules are diſtinguiſh'd from each other, are thicker at the Top than at the Bottom; they being one Line, two Lines, or 1 $\frac{1}{2}$ Line diſtant from each other, and about $\frac{1}{4}$ Inch deep. Could I have got it ſo well cleans'd as I wiſh'd for, doubtleſs I might have obſerv'd their Communication with each other, by means of certain Orifices which ſerve to convey what ſuper-

fluous Moisture is contain'd in them. for we may reasonably suppose, as in all other Cavities of the Body, there are certain Glands for separating proper Liquors convenient for the uses design'd; so here there seems to be a necessity for separating a certain quantity of Moisture, fit to lubrifie the Muscles of the Officles, and facilitate their Motion; as also to preserve the *Membrana Tympani* from becoming too dry. This dryness of the *Membrana Tympani,* and the thickness of the Liquor separated by these Glands, is often the cause of a Deafness in Human Subjects; especially those that are advanc'd in Age. This cellulous Structure of the *Cavitas Tympani,* seems to be very proper for receiving of the superfluous Humidity; and these Communications are requisite for conveying it from one Cellule to another, till it is emptyed into the *Receptaculum Commune* the *Aqueduct,* whereof hereafter.

This first or cellulous Cavity is two Inches broad, and reaches from the *Crena Tympani* to the *foramen Ovale,* or entry into the *Vestibulum,* which is shut by the *Stapes.* The second Part of this Cavity is more superficial (*Fig.* II. (*e*), in form not unlike a Pear, from a narrow beginning becoming broader and more superficial, terminating Semicircularly, smooth in the Bottom, and having several incurvated Lines running across it; it reaches much farther than the *Vestibulum,* being one Inch five Lines from before to behind, and one Inch transversly where broadest. What superfluous Moisture it contains is discharg'd into the fore-named Aqueduct.

Beside the above-mentioned uses for these two Cavities, *viz.* to receive and discharge the superfluous Moisture; they are also most beneficial and assisting to the Hearing: for, no sooner is the external Air modulated, and the *Membrana Tympani* mov'd thereby, than

than the Sound is conveyed by the *Ossicles* to the *Nervus Auditorius*, and the Undulation continued, first by the *Anfractuosities* of the first Cavity, and then by the *Gyres* and incurvated Lines of the second, so that we may easily account for the acute Sensation of Hearing, wherewith *Elephants* are said to be endow'd: For as the Tame ones are most exact in obeying their Masters commands; so the wild Ones are soon aware of what Traps or Snares are laid to catch them, by the tremulous Motion convey'd to their Ear from the Cavous parts of the Earth, where the Pit into which it is expected they should fall, is digg'd. It is easy therefore to explain whence the acuteness of the Sensation of this Animal may proceed; for as the *Nervus Olfactorius* has a large Space and Bounds wherein to be dispers'd, *viz.* the two Cavities of the *Proboscis*, which are both long and large, so that scarce any *Columna aeris* can enter them, but some one or another of the Filaments of the *Nervus Olfactorius* dispers'd in these Cavities must be toucht, whereby the Idea of smelling must be conveyed to the *Sensorium commune* in a more intense Degree, and the Animal soon become sensible of whatever approaches that is noxious or nauseous to it, and thereby is taught how to avoid it; so this Structure, for a quick conveyance and long continuance of the Sound, is a great means both to make the *Elephant* soon receive the Sound and have a deep impression of it.

The Aqueduct.

The *Aqueduct* is a flat Tube or Pipe, whose Orifice is so situated betwixt the two forementioned Cavities, that if there be any superfluous Humidity contain'd in them, it must needs be discharg'd (at least in this Animal) into the Mouth; for as it is situated where the first Cavity terminates, so the second, from a broader and more superficial be-

ginning muſt needs diſcharge its Moiſture, by its more narrow and deeper termination, into this receptacle; alſo it deſcends directly towards the Mouth, paſſing through the Scull below the hole for the Jugular Vein (*mm*) betwixt the hole for the *Carotid* Artery, (*pp*) and that for the *Arteria duræ matris* (*q q*) whence deſcending (*nn*) it is joyn'd with its Fleſhy part, which diſcharges it ſelf into the Mouth on each ſide, behind the back part of the inner Teeth of the upper Jaw. This ſituation of the Aqueduct makes it plainly appear, that its Uſe is to receive the ſuperfluous Moiſture from the *Cavitas Tympani*; for beſide the Glands above-mentioned, fit for ſeparating ſuch a quantity of Humidity as may lubrifie the Muſcles, and facilitate both their Motion and that of the *Oſſicles*; the very Vapours that ariſe in ſuch a Cavity as that of the *Tympanum* in this Animal, muſt at laſt be converted into a Liquor, and that muſt either again be receiv'd into the Blood Veſſels, or otherwiſe diſcharg'd by ſuch a Receptacle as this. Further if there be a neceſſity for Glands in the *Meatus Auditorius* without the *Tympanum*, to ſeparate a certain Liquor, by which the acrimonious Particles of the Air are obtunded, and hindred from being offenſive to the Nervous Membrane of the *Tympanum*, (which muſt be of a moſt acute Senſation) and for moiſtning it, by which it the more eaſily receives the Vibration of the Air; ſo ſuch Glands as theſe ſeem to be moſt requiſite in the *Cavitas Tympani* for the Uſes above nam'd. And ſince what ſuperabounds of this Moiſture, cannot be diſcharg'd outwardly as that of the *Meatus*, this Aqueduct ſeems to be moſt convenient for that purpoſe. Some are of opinion that this Aqueduct is alſo aſſiſting to the Hearing, eſpecially in Men; becauſe it is generally obſerv'd that

Oſteographia Elephantina, or *Philoſ. Tranſact.* No. 327. Tab. 3. Fig. 3.

that they who are Deaf, open their Mouths wide, when they are desirous to hear more distinctly: But I see not how that can be, for tho' the Cavity of the Bony part of the Aqueduct, in most of Animals, is proportionally large enough; yet its carnous or fleshy Part lyes for the most part so flat, and its two sides are so collaps'd together, that scarce any Air can be admitted, at least so far as to be subservient to the Hearing.

The four Bones of the Ear.

The *Officles* in this as in other Animals are three or rather four in number; for though I did not procure the *Os quadrangulare* of *Du Verney*, yet I have good reason to believe it was there; because there is a conspicuous *Sinus* in the extremity both of the *Incus* and *Stapes*, where they are articulated, so big as to contain the Head of an ordinary Pin; and when I consider the Angle which must have been form'd by the articulation of these two Bones, I look upon this small Bone to serve for the same purposes as the *Patella* in the Knee, and *Sesamoide* Bones in the Fingers and Toes.

The Malleolus.

The *Malleolus* is an irregular Bone, and doubtless has been endow'd with pretty large Muscles, because of the rugosities, protuberances and *Sinus*'s observable in it. It has a protuberant Head (*Fig.* IV. (1) four Lines broad, next to that a *Crena* or semicircular *Sinus*, (2) after which the Bone is rais'd, affording a protuberant Margin to an oblong *Sinus* (3) for receiving the head of the *Incus*, four Lines broad The opposite part of this *Sinus*, or back part of the Bone, is convex of an unequal rugous Surface, with a great many protuberances and depressions, for the Origins and insertions of the Muscles, for the space of five Lines; where it forms an Angle, from whence it becomes Flat and Smooth, being three Lines broad and reaching four

Lines

Lines to another Angle, (5.) where the *Manubrium Malleoli* begins, and where it becomes more round; from whence it gradually Tapers to the Point, being six Lines in length.

The head of the *Incus* is four Lines broad, (*Fig.* VI. (1) below which is the Neck or an oblique *Sinus*; (2) next to that are two *Apophyses*, one on each side. These descending obliquely outwards, and becoming flat, meet in a Point, (*Fig.* VII. (5.) whence ascending obliquely inward, this Production is join'd to another small round one, like the *Manubrium Malleoli* 4 ½ Lines long (6.) This has the fore-mentioned small excavation or half round *Sinus*, (7.) which with the extremity of the *Stapes*, I suppose to have contain'd the *Os quadrangulare*, or rather *Orbiculare*, according to the Figure of the *Sinus*.

The *Stapes* differs much in Figure from the Human one. From its Concave extremity 'tis enlarg'd on each side by two small slender Productions, not unlike the Processes of the *Vertebræ* of some Fishes (*Fig.* VI. (2 2) to which is join'd the *Basis*, (3.) so thin almost as the Scale of a Fish. This was accidentally separated from its two Sides, and remain'd in the *Foramen Ovale*, from whence I pull'd it with a Pin; 'Tis Concave towards the *Stapes*, and Convex toward the *Vestibulum*.

The *Foramen Ovale* lyes so hid and obliquely in the side of the *Cavitas Tympani*, that it could not be delineated in its true Dimensions. Near to it is another hole Oblong and Sharp at both ends, both which give an entry into the *Vestibulum*.

The *Vestibulum* is of an irregular Figure, (*Fig.* X. (*a*) 'tis for the most part three Lines from the one side to the other, and perforated by eight Orifices, *viz.* five for the Canals of the *Labyrinth*, (*Fig* IX. X (*a*) one

one for the *Cochlea*, (*Fig.* X. (*h*) and two for the *Fenestræ* (*b*,*c*.)

The *Cochlea* is a long Cavity consisting of three *Gyres* or *Meanders*; (*Fig.* XI. (*d e f*) Its Orifice where it proceeds from the *Vestibulum* is but small; but it afterwards widens, so that the first course of this Cavity is a third part larger than the second (*e*), and proportionally the third is less than the other two (*f*), till it terminates in an Orifice (*g*) situated in the Top, for receiving a branch of the soft portion of the *Nervus Auditorius*, which accompanies and passes along all its *Gyres*.

The hardness and solidity of the Bone (for which it may be justly called *Os Petrosum* in this Subject) was such that I could not so exactly trace the three Canals or Ducts of the *Labyrinth*, so as to give a true Idea of the manner of their several Turnings. But *Valsalva*'s Figures of the Humane Ear directed me so exactly, that I easily found out the several Orifices, and opened them so far as to find out their situation and true Dimensions, by introducing a Hogs bristle, then cutting it off and stretching it out to the Scale. Thus after laying open the two *Foramina* which gave an inlet to the *Vestibulum*, I soon perceiv'd the several Orifices which in so large a Subject were pretty conspicuous. I first turn'd to the one hand and discovered the *Duct* of the *Cochlea*; this I pursued all along the Protuberance, (*Fig.* III. (*d*) in doing of which I laid wholly open the *Lesser Duct* of the *Labyrinth*. (*Fig.* IX. X. (*d*) Then turning up the other side of the Bone, I trac'd the soft Portion of the *Nervus Auditorius* divided into two Branches, one whereof was distributed into the *Cochlea*, and the other to the *Labyrinth*. In filing the Bone a little further, I opened a small part of the *Middle Duct*, and in a short time I discovered the *Ductus*

Major;

Major; after which I meaſured their ſeveral lengths as is ſaid.

Labyrinth. The *Labyrinth* then conſiſts of three *Lineæ Semilunares* or incurvated *Ducts*, whereof the *Major* lyes in that part of the *Proceſſus Petroſus* which regards the ſeat of the Brain (*b*) This is twenty Lines or one Inch eight Lines long. The *Medius Ductus*, one part whereof regards the Orifice of the *Cochlea*, and the other is common with the *Major* for the ſpace of three Lines; (*e*) this is fifteen Lines or one Inch three Lines long: And the *Minor* which regards the *Cavitas Tympani*, has one Orifice which is near to the *Medius*, were it approaches the *Cochlea*; and the other near to the Orifice of the *Major*. This is one Inch long.

The ſeventh pair of Nerves called in general the *Nervus Auditorius*, enters the *Proceſſus Petroſus*, and is divided into the hard and ſoft Portions, as in other Animals. In this Subject I find one Canule entring the Bone from the ſides of the Orifice for the *Carotide* Artery, about three Lines diameter, (*e*) (*h*) from thence running forward for the ſpace of one Inch four Lines, then bending downwards one Inch, till it meets with the Orifice at the Sides of the *Meatus Auditorius*, by which it pierces the Scull, and paſſes outward. This Canule, after it is entred the *Proceſſus Petroſus* for the ſpace of eight Lines, communicates with the Orifice which uſually enters the foreſaid Proceſs from the Baſe of the Scull; and both theſe Orifices, after they have accompany'd one another about five Lines, are ſeparated, and the ſoft Portion penetrates the Bone at two places, as is ſaid.

Oſteog. Eleph. Tab. 3. Fig. III. Fig. IV.

I have now endeavoured to give ſuch a Deſcription of the *Oſſeous* or Bony part of the Ear of this ſtupendious Animal, as I am in hopes may be uſeful for the

* clearing

clearing up of ſome *Phænomena* in leſſer Subjects. At leaſt we may hereby obſerve, what a variety of Mechaniſm the great Author of Nature has thought fit to employ, in the ſeveral parts of different *Species* of Animals. Thus both the external Ear of Man, and of the *Elephant* lye flat, as being moſt convenient: for if they had been Protuberant as in moſt Quadrupeds, how unſuitable would it have been in Man, who is the moſt perfect of all Creatures, not upon the account of his Reaſon alone, but alſo as he is a Pattern for Beauty and the Symmetry of his Parts; and how unſeemly would it have been in the *Elephant*, if his external Ear had ſtuck out, and been proportional to his other Parts; conſidering what an extraordinary aſpect he makes already by his Trunk and Tusks? But the Ears in theſe two Subjects differ by the tortuoſity of the Cartilage, and oblique *Meatus*, to prevent the injury of the Air, by its immediate acceſs into the inner Ear in Man: whereas in the *Elephant* the external Orifice is fully expos'd to the Air; but then the length of the *Meatus* hinders any more Air than is convenient from arriving at the *Tympanum*. We likewiſe ſee in the *Seal* and *Otter*, that thoſe two Amphibious Quadrupeds have no external Ear further protuberant than the other Parts of their Head; for had it been otherwiſe, their ſwimming and diving would have been much hindred: But its two ſides are ſo collaps'd, that no Water can enter in when in the Deep, though it can receive ſufficient Air when aſhoar. The cellulous Cavity of the *Tympanum* in the *Elephant*, may well be compar'd to the *Apophyſis Maſtoides* in Man; and the ſecond Cavity of a plain Surface ſeems to be Analogous to the Cavous *Maſtoides* in *Sheep, Cats, Dogs, &c.* So that we ſee that whereas other Animals have but one Cavity for aſſiſting the Vibration of the Air, and continuation of the

Sound in the *Tympanum*; this Animal has two, or a large one with two different Surfaces. The Aqueduct both by its Figure and Position in this Animal doth plainly shew us the Use of it in other Animals, which is to receive the superfluous Humours in the *Tympanum*, and convey them to be discharg'd in the Mouth.

Explication of the FIGURES.

Figure I.

R*Epresents the Bony part of the* Meatus Auditorius *of the Right Ear.*

a *The external Orifice of the* Meatus Auditorius.

b *The* Processus Petrosus.

c *The Orifice where the* Nervus Auditorius *enters.*

d *The* Meatus Auditorius.

e *A part of the* Laminæ *which proceed from it on each side, by which the* Cellules *betwixt the two Tables of the Scull are form'd; those situated above the* Meatus *being remov'd.*

f *Part of the inner Table of the Scull.*

Fig. II.

Represents part of the Meatus Auditorius *opened, with other parts of the inner Ear.*

a *The ragged part of the Bone from whence the* Os Petrosum *was separated.*

b *The* Processus Petrosus *opened.*

c *The* Crena *for the* Membrana Tympani.

d *The Hony comb Cavity of the* Tympanum.

e *Its inner Cavity of a smooth Surface.*

f *Its Semicircular or undulated Lines.*

g *The Orifice of the Aqueduct.*

h *The Orifice of the hard Portion of the* Nerve.

Fig. III.

Represents the lower Surface of

of the Os Petrosum, *as it was separated from above the* Tympanum *and other parts of the inner* Ear.

aa *The ragged Margine of the Bone.*

bb *The upper part of the* Cavitas Tympani.

c *The* Foramen Ovale.

d *The Protuberance in which the* Labyrinth *and* Cochlea *are lodg'd.*

e *The Orifice of the hard portion of the* Nervus Auditorius.

Fig. IV.

Represents the Malleolus *alone in its true dimensions.*

1 *The Protuberant Head.*

2 *The Semicircular* Sinus *betwixt it and the Margin.*

3 *The* Sinus *which receives the head of the* Incus.

4 *The angle below the* Sinus *for the head of the* Incus.

5 *The angle where the* Manubium Malleoli *begins.*

6 *The* Manubrium Malleoli.

Fig. V.

Represents the Incus.

1 *The head of the* Incus.

2 *The* Sinus *or neck of the* Incus.

3 *Two* Apophyses.

4 *A long protuberance with the* Sinus *for the* Os quadrangulare *at its extremity.*

Fig VI.

Represents the Stapes.

1 *The small part of the* Stapes, *where it is articulated with the* Incus, *with a* Sinus *at its extremity, being the other half of the Cavity for the* Os quadrangulare.

2 2 *Two small portions of the* Stapes, *where it is articulated with the* Basis.

3 *The* Basis *of the* Stapes *separated.*

4 *The whole* Stapes.

Fig. VII.

The Malleolus *and* Incus *join'd together, with their lower side turn'd up.*

1 *The* Malleolus.

2 *Its articulation with the* Incus.

3 *The* Incus.

4 *The* Manubrium Malleoli.

5 *A point of the* Incus, *fram'd by the other two Productions.*

6 *The*

6 *The long protuberance of the* Incus.

7 *The* Sinus *in the extremity of its long Production.*

Fig. VIII.

The Malleolus, Incus *and* Stapes *articulated together.*

1 *The* Incus.

2 *The* Malleolus.

3 *The* Stapes *where it shuts the* Foramen Ovale.

Fig. IX.

Represents the upper part of the Lineæ Semilunares, *or that side which is towards the passage of the* Nervus Auditorius.

a *The five extremities cut off.*

b *The* Linea Semilunaris Major.

c *The* Semilunaris Media

d *The* Minor.

e *The common Canule between the* Major *and* Media.

Fig. X.

Represents the Cochlea *and* Labyrinth *together.*

a *The* Vestibulum.

b *The* Foramen Ovale.

c *The* Foramen Oblongum.

d *The* Linea Semilunaris Minor, *which is towards the* Cavitas Tympani.

e *The common Canule to the* Major *and* Media.

f *The* Major.

g *The* Media.

h *The* Cochlea.

Fig. XI.

Represents the Cochlea.

a *The* Vestibulum.

b *The third* Gyre *or turning.*

c *The Orifice.*

d *The first* Gyre *or turning opened.*

e *The second turning.*

g *The Orifice at the top of the* Cochlea.

F I N I S.

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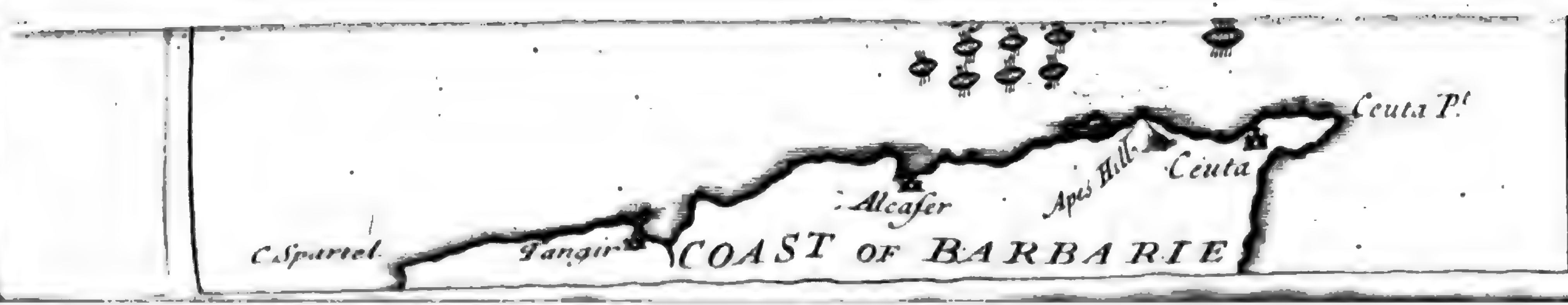

Ceuta P.t
Ceuta
Apes Hill
Alcaser
Tangir
C. Spartel
COAST OF BARBARIE

PHILOSOPHICAL TRANSACTIONS.

For the Months of *January* and *February*, 1718.

The CONTENTS.

I. *Curious Observations of the Transit of the Body and Shade of* Jupiters *Fourth Satellite over the Disque of the Planet. Communicated by the Reverend Mr.* James Pound, R. S. S.

FInding by the Tables of *Jupiter*'s Satellites that the Fourth Satellite was to pass over the Disque of *Jupiter* the 16*th* of this present *February*, at Night; we were very desirous to observe the same with the *Hugenian* Telescope, having never before, since I have had the Use of it, been able, by reason of the foulness and inconstancy of the Weather, to make any tolerable Observation of this kind.

At $6^{h}\frac{1}{4}$ thro' a short Tube, we saw all the 4 Satellites, the 3 outermost on the *East* side of *Jupiter*, and the innermost near the *Western* Limb approaching to an Eclipse. The Fourth at that time was about half a Semidiameter of *Jupiter* from the *Eastern* Limb. Then it proved Cloudy till about 8^{h}, at which time (thro' the long Glass) we could see only the second and third Satellites, the first being behind *Jupiter* in the Shadow, and the fourth entred upon the Disque. We saw at this time a dark Spot, a little *Northward* of the great *Northern* Zone, and near the *Eastern* Limb, where the Satellite was to enter on the Disque; which Spot we took for the Shade of the Satellite. The Clouds then again intercepted our View, till $8^{h}.\ 53'$. *Æq. T.* at which time the first Satellite was lately emerged out of the Shadow, and the Spot advanced so far, that we perceived it would arrive at the middle of *Jupiter*, near two Hours sooner than the

the Shade ought to have done by our Computation; but not imagining that this dark Spot could be any thing elſe but the Shade, we concluded there had been ſome Error in the Calculation, which we thought to re-examine afterwards. On this preſumption we left off obſerving till $9^h. 35'$. at which time we were ſurprized to ſee a Notch in the Limb of *Jupiter*, near the place where the former Spot entred. This laſt appearance agreeing well with the time that the Shade of the Satellite ought to have entred the Diſque, ſoon made us alter our former Opinion, and conjecture that this and not the other Spot was the ſaid Shade. At $9^h. 39'\frac{1}{2}$ *Æq T.* the Notch vaniſhing, a round black Spot appeared within the Limb, but in contact with it. At $9^h. 45'$. we judged the firſt Spot, and at $11^h. 45$. the ſecond, to be in the middle of *Jupiter*.

At $11^h. 50'$. the firſt Spot touched the Limb, being within the Diſque; ſoon after which the Limb in that place ſeem'd a little protuberant. At $12^h. 5'$. appeared the fourth Satellite juſt come out of the Diſque, and touching the Limb in the place where the Protuberancy was. At $12^h. 7'$, we could perceive the Satellite ſeparated from the Limb At $13^h. 56'$. the ſecond black Spot, ſtill within the Diſque, juſt touched the *Weſtern* Limb; ſoon after which there appeared a Notch in this part of the Limb, as it did on the other at the coming on of this Spot. At $14^h. 6'$. the Spot was all gone off, and the Limb appeared clear and entire The firſt Spot, when in the middle of *Jupiter*, was almoſt as black as the ſecond when near the Limb, but ſomewhat leſs and a little more *Northerly*.

At the time that the firſt Spot was in the middle of the Diſque, the three innermoſt Satellites appeared

to

to the *East* of *Jupiter*; the first (as aforesaid) having lately emerged out of the Shadow; the second being almost at its greatest distance; and the third having passed the Axis of the Shade about twelve Hours before, and appearing at this time about three Diameters of *Jupiter* from his Limb. The times that these Spots arrived at the middle of the Disque are agreeable to the times found by Calculation, in which the fourth Satellite and its Shade ought to have appeared there. From all which 'tis very plain, that the first of these Spots was the fourth Satellite itself, and the second its Shadow.

We have seen the first and second Satellites appearing not as dark Spots, but as bright ones (somewhat different from the light of *Jupiter*) for some little time after they entred his Disque, but as they approached nearer the Middle we lost sight of them. And we have frequently observed that the same Satellites appear brighter at some times than at others; and that when one of them hath shined with its utmost Splendour, the Light of another hath been considerably diminished. From whence 'tis very probable at least, not only that the Satellites revolve upon their proper Axes, but also that some parts of their Surfaces do very faintly (if at all) reflect the Solar Rays to us.

All which hath for some time since been observed and taken notice of by Mess. *Cassini* and *Miraldi*, as may be seen in the *Memoirs* of the *Academie Royale*, for the Years 1707 and 1714

II. *A*

II. *A Discourse tending to shew the Situation of the ancient* Carteia, *and some other* Roman *Towns near it. By* John Conduitt *Esq;* Fellow *of the* Royal Society.

ABout four *English* Miles *N.W.* from *Gibraltar*, at the end of the Bay, there are considerable Ruins. The place is called at present *Rocadillo*, and consists of a few Huts, and a Modern Square Tower, which appears to have been raised on the Foundation of a much greater Pile. The Walls of the old City are very easy to be traced. They seem to have been about two *English* Miles in Circumference, and were built upon the Brow of a rising Ground. The space within is covered with Ruins, among which are a great many pieces of very fine Marble well wrought; and innumerable fragments of Vessels of that kind of red Earthen Ware, which *Ambrosio Morales* in the first Chapter of his *Discurso de las antiguedades de las ciudades de Espanna*, lays down for a certain mark of a *Roman* City, and takes to have been a Composition of the Clay of *Saguntum*, often mentioned among the *Romans*.

Ficta Saguntino pocula malo luto. Mar. Lib. VIII. Ep. 6.

Sume Saguntino pocula ficta luto. Lib. XIV. Ep. 108.

There are remains of a rude Semicircular Building, raised on Arches, which descends gradually into an Area, and seems to have been a kind of Theatre. I brought away with me a Marble Pedestal of a Statue, dug up near to the Square Tower. The Marks where

the Feet and the extremities of the Drapery were fastened to it, are ſtill to be ſeen, and the following Letters finely cut VARIA MARCE. It was given me by the owner of the Ground, who ſaid he had read upon it formerly three other Letters L L A ſince broken off There are other Inſcriptions, but ſo Defaced and ill Cut, that they do not deſerve a particular mention. I have a conſiderable number of Medals, that were found among theſe Ruins; moſt of them have a *Caput turritum* with CARTEIA in very legible Characters. The Reverſe is generally a *Fiſh* a *Neptune*, or a *Rudder*. Towards the *Weſt* there is an eaſy Deſcent to the River *Guadarranque*, which takes its Source at *Caſtellar*, about four Leagues in the Country, and is very deep at *Rocadillo* There is a Bar where the River falls into the Bay; but it does not hinder the entrance of Veſſels of 15 Tun, to load Charcoal and other neceſſaries, that are Shipt off from thence for *Ceuta*. Along the ſide of the River there is ſtill a great deal of Stone Work and viſible remains of an Ancient Key. At a ſmall diſtance to the Eaſt, upon an Eminence, there are conſiderable ruins of a Square Caſtle, which appears to have been an ancient Building of very great Strength The Country People now call it *Callillon*. but the *Corrigidor* of that Diſtrict told me he remember'd it called *Torre Cartagena*. The Situation agrees exactly with the Tower of that Name, mentioned in the 2[illegible]th and 316th Chapters of the Chronicle of *Alphonſo* XI of *Caſtile*. A Book of great Authority among the *Spaniards*, who are generally of Opinion that it was formed upon the Memoirs of *Fernando Nunnez de Valladolid*, a Favourite and Miniſter of that King, tho' it goes under the name of another Perſon.

All

All the *Spaniards* who live about the Ruins I have been describing, say they are the remains of a City of the *Gentiles* called *Cartago*. The corruption of *Carteia* into a name so much more talked of, might easily happen in an Oral Tradition of so many Years; and I cannot help thinking that, where other Circumstances concur, an account deliver'd down from Father to Son is an evidence not to be slighted, in matters of so much obscurity.

Frequent mention is made of *Carteia* by the Ancient Geographers and Historians. I build so much on two Passages of *Livy*, that I am obliged to insert them at length. The first is Lib. XXVIII. C. 30. (*Livy* does not mention from what Port *Lælius* sailed for *Carteia*, but by what goes before, it seems to have been from *Cartagena*, at that time *Scipio*'s Head Quarters) Lælius *interim, freto in Oceanum evectus, ad* Carteiam *classe accessit. (Urbs ea in orâ Oceani sita est, ubi primum è faucibus angustis panditur mare)* Gades *sine certamine proditione recipiendi (ultrò, qui eam rem pollicerentur, in castra* Romana *pervenientibus) spes, sicut antea dictum est, fuerat. Sed patefacta immatura proditio est, comprehensosque omnes* Mago Adherbali *Prætori* Carthaginem *devehendos tradit.* Adherbal, *Conjuratis in quinqueremem impositis, præmissaque eâ, quia tardior quam triremis erat, ipse cum octo triremibus modico intervallo sequitur. Jam fretum intrabat quinqueremis, cum* Lælius *&* *ipse in quinquereme, è portu* Carteiæ, *sequentibus septem triremibus, evectus, in* Adherbalem *ac triremes invehitur; quinqueremem satis credens deprehensam rapido in freto, in adversum æstum reciprocari non posse.* Pœnus, *in re subitâ parumper incertus, trepidavit utrum quinqueremem sequeretur, an in hostes rostra converteret. Ipsa cunctatio facultatem detrectandæ pugnæ ademit: jam enim sub ictu teli erant, & undique instabant hostes. Æstus quoque arbitrium moderan-*

di naves ademerat; neque erat navali pugnæ similis, quippe ubi nihil voluntarium, nihil artis aut consilii esset. Una natura freti æstusque totius certaminis potens, suis, alienis navibus, nequicquam remigio in contrarium tendentes invehebat; ut fugientem videres retro vortice intortam victricibus illatam, & sequentem, si in contrarium tractum incidisset maris, fugientis modo sese avertentem. Jam in ipsâ pugnâ hæc, cum infesto rostro peteret hostium navem, obliqua ipsa ictum alterius rostri accipiebat: illa, cum transversa objiceretur hosti, repente intorta in proram circumagebatur.

The other Passage is Lib. XLIII. C. 3. *Et alia novi generis hominum legatio ex* Hispaniâ *venit: Ex militibus* Romanis *& ex* Hispanis *mulieribus, cum quibus connubium non esset, natos se memorantes, supra quatuor millia hominum, orabant ut sibi oppidum in quo habitarent daretur. Senatus decrevit, uti nomina sua apud L.* Canuleium *profiterentur, eorumque siquos manumisisset, eos* Carteiam *ad Oceanum deduci placere Qui* Carteiensium *domi manere vellent, potestatem fore uti numero colonorum essent, agro assignato.* Latinam *eam coloniam fuisse, Libertinorumque appellari.*

The best *Spanish* Authors, and *Ortelius* and *Cellarius* trusting to them, take this *Carteia* of *Livy* to be different from that which was the next to *Calpe*, and place it generally about *Conil. Rodrigo Caro* in his *Convento Juridico de Sevilla* C. 24. applies the *Carteia* in the XLIII Book of *Livy* to *Rocadillo*, and in Cap. 74. to *Cartaia* near *Lepe.* It is surprizing he takes no notice of the Passage in the XXVIII Book, For the particular mention of *ad Oceanum*, and *Urbs ea in orâ Oceani sita est*, implies they both relate to the same place; perhaps it was because he could not reconcile it with his *Cartaia* near *Lepe. Cellarius* makes *Bæsippo* this *Carteia* of *Livy*. L. 2. c. 1. *Bæsippo, quæ videtur* Carteia Livii *esse, extra fretum*

fretum & columnas posita. Aliam pro Livio Carteiam *non invenio*; tho' in all the ancient Geographers *Bæsippo* is mentioned by it self as a distant Town. I am so far from seeing any necessity of erecting a new *Carteia* in the Ocean for these Passages in *Livy*, that I take that in Lib. XXVIII. to be rather a proof that the City there mentioned, stood at *Rocadillo*. It certainly agrees much better with that Situation, than with *Conil* or *Cartaia* near *Lepe*. It is not to be reconciled with the latter, b cause that lies *North West* of *Cadiz*, from whence *Adherbal* set out for *Carthage*, and is a good way up the Country, on the side of a River, and not *in orâ Oceani*. Neither can *Conil* be said properly to be Situated *Ubi primum è faucibus angustis panditur mare*; for the Sea widens considerably before it reaches the Capes *Spartel* and *Trafalgar*, and becomes an Ocean where that Town stands. It is observable that *Mela* applies words of the same import with those of *Livy* to the Sea between *Calpe* and *Abila*. *Barbesul*; *Aperit deinde angustissimum pelagus*. There is no Harbour at *Conil*, or any other place between Cape *Trafalgar* and *Cadiz*. If the *Carthaginian Quinqueremis* had only *been going into* (*intrabat*) the Mouth of the Streights between Capes *Spartel* and *Trafalgar*, *Lælius* could not have believed it *satis deprehensam rapido in freto, in adversum æstum reciprocari non posse*, for there is no such strong Current there; and the action between him and *Adherbal*'s *Triremes*, which were at some distance behind the *Quinqueremis*, must have happened *Westward* of those Capes; which is inconsistent with the description *Livy* gives of it; because in that part of the Ocean there are none of those Eddies, that appear to have had so particular an effect on both the Fleets, during the Engagement, and are peculiar to the Middle of the Gut.

This

This general miſtake ſeems to have been occaſioned by giving too eaſily into the opinion, that *Livy* underſtood by the *Fretum* all the Sea between the Capes *Spartel* and *Trafalgar*, and the Rock of *Gibraltar* and *Apes-Hill*; when it is more probable that he termed ſtrictly ſo only the narroweſt Part, which was generally reckoned to be between the two latter: *Mela. Proxima Africæ & Europæ littora montes efficiunt Calpe & Abila. Pliny* takes *Mellaria* to be neareſt to *Africk*, and therefore places there the *Fretum ex Atlantico mari Lib.* 3. which is an argument his *Fretum* was not the ſame with our Streights, and that he carried the Atlantick Ocean much farther *Eaſt* than the Capes *Spartel* and *Trefalgar*.

Other Authors ſeem to make the Pillars of *Hercules* the Boundary of the Mediterranean and the Ocean. *Marcianus Heracleotes.* Ἐνταῦθα πέρας ἔχει τῆς Βαιτικῆς Ἱσπανίας τὸ μέρος τὸ παρῆκον παρ' ἑκατέρας τὰς θαλάσσας τὰς περὶ τὸν Ἡράκλειον πορθμὸν τυγχανούσας, τήν τε καθ' ἡμᾶς καὶ τὴν ἔξω, τοῦτ' ἔστι τὸν Ὠκεανόν. *Hic finem habet Hiſpaniæ Bæticæ pars contingens utraque maria quæ circa fretum Herculeum. tam mare noſtrum quam mare exterius, h. e. Oceanum.* Τῆς μὲν Βαιτικῆς τὸ πλεῖστον παρὰ τῆς καθ' ἡμᾶς κεῖται θαλάσσης, τῶν Ἡρακλείων ἐντὸς στηλῶν, μέρος δέ τι παρὰ τὸν δυτικὸν Ὠκεανόν. *Bætica quidem pars maxima prætenditur noſtro mari, Herculeas intra columnas, pars vero quædam occidentali Oceano.*

Polybius L. III. Καλεῖται δὲ τὸ μὲν παρὰ τὴν καθ' ἡμᾶς παρῆκον ἕως Ἡρακλείων στηλῶν, Ἰβηρία· τὸ δὲ παρὰ τὴν ἔξω καὶ μεγάλην προσαγορευομένην κοινὴν μὲν ὀνομασίαν οὐκ ἔχει, διὰ τὸ προσφάτως κατοπτεύεσθαι. *Quæ porrigitur ſecundum mare noſtrum portio ad columnas uſque Herculis Iberia nominatur; quæ ſecundum mare externum quod & magnum appellatur, communem appellationem nondum invenit, quia non diu eſt cum fuit explorata.*

Appian

Appian L. II. Ἐμφυλ. περάσαντι τὰς στήλας τὰς Ἡρακλέους τὸν ὠκεανὸν ἐπέραν. *Trajecto ad Columnas Herculis Oceano.*

Florus Lib. IV. C. 2. *In ipso ostio Oceani* Varus D[illegible]e que *legati conflixere; sed acrius fuit cum ipso m*[illegible] *inter se navibus bellum: Siquidem velut furore* [illegible] *castigaret Oceanus, utramque classem naufragio* [illegible]*it. Quinam ille horror, cum eodem tempore fluctus, procellæ, viri, naves, armamenta confligerent? Adde situs ipsius formidinem, vergentia in unum hinc* Hispaniæ *inde* Mauritaniæ *littora; Mare & intestinum & externum, imminentesque* Herculis *speculas; cum omnia undique simul prælio & tempestate sævirent.* Here the Pillars of *Hercules* are made the very Mouth of the Ocean. If you understand the *Fretum* of *Livy* in this Sense, and reckon it to signify only the Sea between *Calpe* and *Abila*, and the Ocean to begin from thence *Westward*, the Passage in the 28*th* Book is an accurate description of *Rocadillo. Lælius interim freto in Oceanum evectus ad Carteiam classe accessit. Urbs ea in orâ Oceani sita est, ubi primum è faucibus angustis panditur mare.* And allowing *Lælius* to set out against *Adherbal* from thence, every circumstance mentioned by *Livy* is so easy to be accounted for, that it is needless to make Application. A Passage in *Dio Cassius* Lib. XLIII. induces me to believe the Vessels anchored in the *Guadarranque*, and that that River, and not the Bay, was properly *Portus Carteiæ.* Οὔαρος δὲ ὑπὸ τοῦ Διδίου περὶ Καρτηΐαν ἐναυμαχηκὼς ἡττήθη, καὶ εἰ γε μὴ προκαταφυγὼν ἐς τὴν γῆν, ἀγκύρας ἐς τὸ στόμα τοῦ λιμένος ἄλλας πρὸς ἄλλαις προσεβεβλήκει, καὶ περὶ αὐτὰς οἱ πρῶτοι τῶν διωκόντων σφᾶς, ὥσπερ περὶ ἔρμα ἐπταίκεισαν, πᾶν ἂν τὸ ναυτικὸν ἀπολωλέκει. Varus *vero à* Didio *apud* Crantiam *navali prælio superatus in terram evasit, conjectisque in introitum portûs anchoris, ita ut una ab aliâ teneretur, cum ad eas, tanquam ad septum quoddam, primæ insequentium naves*

naves offendissent, periculum totius classis amittendæ declinavit. This cannot be understood of the Bay, because that is three Leagues over at the narrowest part, and much too deep for a work of such a Nature, which might easily have been effected upon the Bar of the River *Guadarranque*.

There is no room to doubt of the emendation *Luis Nunnes*, in his *Hispanica*, has made here of Καρτηΐα for Κεραντια; for no ancient Author mentions any other Town or Harbour thereabouts of a name like that; and *Carteia* was the place which held out the longest for the younger *Pompey*, and where he kept his Fleets.

Florus in the Passage I have already quoted, relating the same Action between *Didius* and *Varus*, represents in very lively Colours, the very Scene near *Rocadillo*. *Adde situs ipsius formidinem, vergentia in unum hinc Hispaniæ inde Mauritaniæ littora; mare intestinum & externum, imminentesque Herculis speculas* &c.

Hirtius, in the latter part of his Book *de bello Hispanico*, says *Cn Pompeius ad navale præsidium parte alterâ contendit* Carteiam, *quod Oppidum abest à* Corduba *millia passuum* CLXX. which distance, as well as the circumstance of *navale præsidium*, agrees with the Situation of *Rocadillo*. The ancient Geographers place *Carteia* next to *Calpe* Westward. *Pomponius Mela*, after having given us a perfect Picture of *Calpe*, and described those lasting Marks, in which so many Centuries have made no alteration, says—*Sinus ultra est, in eoque Carteia*. *Strabo* L. III. Ἐνταῦθα δὴ ὄρος ἐστὶ τῶν Ἰβήρων ἡ Κάλπη, &c. καὶ πρὸς αὐτὸ Κάλπη πόλις ἐν τετταράκοντα σταδίοις, ἀξιόλογος καὶ παλαιά, ναύσταθμός ποτε γενομένη τῶν Ἰβήρων. Ἔνιοι δὲ καὶ Ἡρακλέους κτίσμα λέγουσιν αὐτήν, ὧν ἐστι καὶ Τιμοσθένης, ὅς φησι καὶ Ἡρακλείαν ὀνομάζεσθαι τὸ παλαιόν, δείκνυσθαί τε μέγαν περίβολον καὶ νεωσοίκους. *Ibi mons* Hispanorum *est* Calpe, &c. *& ad* XL *inde Stadia Urbs* Calpe

vetusta

vetusta & memorabilis, olim Statio navibus Hispanorum. *Hanc ab* Hercule *quidam conditam aiunt, inter quos est* Timosthenes, *qui eam antiquitùs* Heracleam *fuisse appellatam refert, ostendique adhuc magnum murorum circuitum & Navalia.* *Casaubon*, in his Notes on this Passage is of Opinion it should be Καρπηΐα πόλις. *Legendum censeo* Καρπηΐα πόλις, *nam eam urbem hic intelligi res ipsa docet; & ex eo colligi potest, quod toties eam infra nominans nihil tamen de ejus situ alibi dixisse reperitur. At* Calpen *Urbem nemini Veterum ne nominatam quidem reperio.*

Marcianus Heracleotes makes *Carteia* 50 Stadia from *Calpe*. Either of these Distances agrees with *Rocadillo*, according to the part of the Rock from which they reckon; for it is above six Miles from *Europa* Point to *Rocadillo*.

Bochart in his *Geographia Sacra* Lib. I. Cap. 34. strengthens *Casaubons* Opinion *Nec frustrà* Heraclea Carteiæ *fuisse vetus nomen, tanquam ab* Hercule *conditore.* Herculem *enim suum* Phœnices Μέλκαρθον *appellabant.* Philo Biblius *ex* Sanchoniathone *apud* Eusebium L. I. *Præparat.* τῶ δὲ Δημαρῶντι γίνεται Μέλκαρθος ὁ καὶ Ἡρακλῆς. *Ex* Demarunte *autem natus est* Melcarthus *qui &* Hercules. Μέλκαρθος *autem est* מלך קרתא Melech Kartha. *Rex Urbis. i.e.* Tyri. *Idem* Græcis Melicertes *sive* Palæmon *Maris Deus, quem* Cadmi *nepotem esse fingunt. Hinc* Hesychius *rursus* Μάλικα τὸν Ἡρακλέα Ἀμαθούσιοι. *Omnino igitur ex* Melcartho, *vel* מלך קרתא Melech Kartha. *Urbs quam ad* Calpen *condidit* Hercules Phœnicius, primo Melcartheia *vocata est*, Melech Karthiia, *quasi* Ἡρακλεῖαν *dixeris; deinde per Aphæresin* Cartheia *vel* Carteia. *Apud* Hebræos *frequens est hæc Aphæresis in nominibus locorum compositis. Tale* Sittim *pro* Abel Sittim, Salem *pro* Jerusalem, &c.

I have some Medals that were dug up at *Rocadillo*, with the Head and Club of *Hercules* upon them;

which seem in some measure, to support that great Man's Assertion. Upon the Reverse are Tunny Fishes, which according to *Strabo* and *Pliny* abounded formerly near *Carteia*, and are still taken in great quantities near the Shoar of the East Sea, at a small distance from *Rocadillo*.

Bernardo Aldrete an Author of such Weight, that *Bochart* does not disdain to copy him on several occasions, in the second Book and second Chapter of his *Antiguedades de Espanna*, accounts for the Addition of *eia* to *Cartha*; which in the *Syriack* and *Chaldean* signifies *Pulcher*, *Formosus*, and was affixed to the Name of this City to distinguish it from the *Cartha* in *Syria*, mentioned in the 21*st* Chapter and 34*th* Verse of *Joshua*.

By all accounts, the *Phœnicians* founded most of the Cities on this Coast, and probably *Carteia* was one of their earliest Settlements; for it lies very near *Africk*, in a most inviting Situation, having on one side a Bay, and on the other a River, which waters a rich Country. Its height gave it strength and a very beautiful Prospect; circumstances which seem to justifie *Aldretes* interpretation of the latter part of it's name.

In the Itinerary of *Antoninus*, it is *Calpe-Carteiam*, not *tanquam duæ urbes diversæ*, as *Casaubon* intimates in his Notes on the third Book of *Strabo*, for then it would be *Calpen Carteiam*; nor, according to *Surita's* Comment on that part of the Itinerary, *ut significet non rectâ iter ex* Suel Carteiam *deduci, sed paululum ad* Calpen *deflecti*; because *Calpe* stands at the end of a narrow neck of Land, which projects to the *Southward* a great way from the rest of the Continent; and consequently is quite out of the Road from *Suel* to any other place *Westward* of it; probably *Calpe-Carteia* is for *Carteia ad Calpen*, to distinguish it from the other *Carteia* in *Celtiberia*, mentioned in the 21*st* Book and

5*th*

5*th* Chapter of *Livy*: for, as *Caro* observes, there is no necessity for the alteration *Sigonius* has made in that passage of *Althæa* for *Carteia*, from the Text of *Polybius*; because *Livy* never mentions the other *Carteia* without adding *ad* Oceanum, *Urbs ea in ora Oceani sita est*; which distinction were needless, had there been only one City of that Name. *Strabo* in his third Book mentions a City called Καρπαλίας, and places it near *Saguntum*, which is agreable to the Situation given this *Carteia* by *Livy*.

I am very much surprized that *Mariana*, and several others, should take the present *Gibraltar* to have been the ancient *Heraclea*; when neither *Pliny*, who resided so long in those Parts, *Mela* who was born there, nor any ancient Geographer or Historian that I have met with, makes the least mention of such a City thereabouts, except *Strabo*; and he places it 40 Stadia from *Calpe*, at the Foot of which *Gibraltar* is situated. The *Spanish* Historians give good ground to believe there was no Town upon that Mountain till the *Moors* invaded *Spain* under *Tariff*, who gave it the name it has retained ever since. I shall not enter into the detail of the reasons of those Authors who place *Carteia* at *Tarifa* or *Algezeira*: the true one seems to have been their not knowing any other place which agreed better with the old accounts of *Carteia*, or where the ruins of a City, which made so great a Figure, could be buried; the common practice of Authors who describe places they have not seen. This appears to have been the case of most of those, especially *Mariana*; who, had he been in these Parts, would not have been guilty of the oversight he has committed Lib. XVI. C. 9. where he places two Bays in the *Streights*, one at *Gibraltar*, and the other at *Tarifa*; which error he was probably led into (as it often happens) by another.

For, giving into the Opinion that *Tarifa* was the antient *Carteia*, and finding that City placed in a Bay by *Mela*, he concluded there must be one at *Tarifa*, which is an open Road, and so much exposed, that in the least bad Weather, the smallest Vessels must be haul'd ashore. Which Circumstance alone is a sufficient proof of its not being *Carteia*, by all accounts, a famous Harbour.

Tho' there are very great Ruins at *Algezeira*, they are not such as give any room to believe they are the remains of a *Roman* City. For neither pieces of Marble, nor Inscriptions are found there, nor any *Roman Coins*. The Circumstance of *Varus* his shutting up the Mouth of the Harbour of *Carteia*, and the distance of 40 or 50 *Stadia* from *Calpe*, are not applicable, either to *Tarifa* or *Algezeira*; and if one of those Towns was *Carteia*, to what City belong those Ruins I have been describing? since all the ancient Geographers make *Carteia* not only the nearest Town to *Calpe*, but the only one in that Bay. There is better ground to believe *Tarifa* stands on the Ruins of an other Town, as I shall endeavour to shew presently.

But before I proceed to a Description of the Coast, it may not be improper to mention some Ruins I saw at *Ximena*; an inland Town, about five Leagues *North* from *Gibraltar*, situated on a Rocky Hill, at the bottom of which to the *Eastward* is a very plentiful Country, washed by the *Josgarganta*, a small Branch of the River *Guadiaro*. On the top of the Hill is the old Town, which by the Arches and Vaults, appears to have been built by the *Moors*. On the right-hand Corner of the second Gate of it, there is a course Stone with Mouldings on the Edges, which has the following Inscription.

L. H E-

L. HERENNIO HE
RENNIANO
L. CORNELIVS HEREN
NIVS RVSTICVS
NEPOS EX TESTA
MENTO POSVIT
NONIS MARTIIS
SEX. QVINTILIO CON
DIANO SEX. QVIN
TILIO MAXIMO COSS.

Rodrigo Caro in his *Convento Juridico de Sevilla* C. 13. says he saw the beginning of this Inscription in *Bejer de la miel*; but when I was in that Town, I was informed by a very intelligent Person, that there is no *Roman* Inscription in any part of it. The Author of *Cadiz el Emporio del Orbe*, when he inserts this inscription, makes it SEXOVINTILIO CONDIMIO; But the Dash of the Q is very plain, and the other word seems rather CONDIANO. The *Latin Fasti* in A. U. C. 903. place Consuls

SEX. QUINTILIUS GORDIANVS
SEX. QUINTILIUS MAXIMUS.

But the very learned Dr. *Bently* has observed to me that the *Greek Fasti* and *Dio* call him Κονδιανὸς, which reading is confirmed by this Inscription.

I have brought with me from this Town a piece of Marble with the following Words upon it.

AVCTINVS CLEMEN
TIS SIBI
ET SVIS BRITTÆ
MATER AN LX
H.S.E. SIT T.T. LEVIS.

I saw

I saw another on the Wall of the great Church which seems to have been the Base of a Statue; the Inscription is as follows.

RESPVBLICA OBEN
SISE..LO DATO
DEDI...VIT CVRAT
LIBE..OR H..REN
NIO RVSTICO H. M.
SINILO RESTITVTO
II VIR.

The manner in which the *Moors* have placed these Inscriptions plainly shews the little value they set upon them, and there is so great a plenty of Stone on the Rock where *Ximena* stands, that it is not to be thought they would fetch them for such an use, from any distant place; which induces me to believe a *Roman* Town formerly stood there called OBA.

I do not find any Town of that name in the ancient Authors. *Strabo* L. III mentions Σάνοβα, Μαίνοβα καὶ ἄλλας πόλεις, which may possibly comprehend *Oba*. The *Geographia Nubiensis*, in the fourth *Clima*, makes a Town called *Rothan*, the first Station from *Algezcira* to *Seville*, which perhaps may have been this *Oba*; for it is about a Days journey from *Algezeira*, and in the direct Road from thence to *Seville*.

Mariana places Lib. III. C. 2. the Cave where *Crassus* hid himself, near *Ximena*; the Marks of it, given by *Plutarch*, are common to most others. I went three Leagues in search of it; but the Country People having a notion that there is a Treasure in it, and not being to be persuaded that I would take so much Pains out of pure Curiosity, would not shew me the Way, tho' they acknowledged there were several such Caves thereabouts. I cannot help taking notice of one very odd

odd tho' trifling circumstance. The name of the Person who owns the Land where those Caves are, is *Pacheco*, which is very near the same with that of the *Spaniard*, who is said by *Plutarch* to have entertained *Crassus* so courteously, Πακιακός. *Hirtius* in the beginning of his Book *de bello Hispanico* mentions a *Spaniard* of Note, *in provinciâ Bæticâ*, called *Patiecus*. *Quibus præfecit hominem ejus provinciæ notum & non parum scientem, L. Julium Patiecum*, which was probably the *Roman* Name; and therefore I am surprized the *Latin* Translator of *Plutarch* makes it *Pacianus*.

Most of the antient Geographers describe the Coast Westward of *Carteia* in the following manner. *Julia Traducta, Mellaria, Bælo fluvius & oppidum, Portus Basippo, Promontorium Junonis*, &c. The Itinerary of *Antoninus*, makes no mention of *Julia Traducta*, and *Pliny* places it on the *African* Coast, which *Hardouin* endeavours to account for Pag. 227. in his *Nummi Illustrati*. *Strabo* calls it *Juliam Jozam*, which as *Bochart* observes Lib. I. C 24. signifies the same in the *Phenician* Language as *Traductam* in the Latin. *Ptolomy* calls it *Transducta*. He places *Barbesula* between that and *Carteia*. But all the other old Geographers put both the Town and River of that Name Eastward of *Calpe*. I saw some Ruins on the *East* side of the River *Guadiaro*, four Leagues *East* of *Gibraltar*; which I take to be the remains of the antient *Barbesula*. For I find in the *Cadiz Emporio del Orbe*, mention made of two pieces of Marble, brought from thence to *Gibraltar*; on one of which was M M BARBESVLANI. I was credibly informed they were used for the Fountain on the Parade. The Letters probably were either sawed off, or turned inwards; for they do not appear. This *Barbesula* is probably the *Barbariana* placed in the Itinerary X. M. P. *East* from *Carteia*.

Pompa-

Pomponius Mela, who was born in those Parts, and therefore is most to be depended on, gives the following account of the Coast, according to the Edition of *Gronovius*. *Sinus ultra est, in eoque* Carteia, *ut quidam putant, aliquando* Tartessus, *& quam transvecti ex* Africâ Phœnices *habitant*; *atque unde nos sumus*, Tingentera. *Tum* Mellaria *&* Bælo, *&* Bæsippo *usque ad* Junonis promontorium *oram freti occupat*. The Text of *Mela* in this place has occasioned great disputes among the Learned. *Casaubon* in his Notes upon *Strabo*, says, *lego autem—atque unde nos sumus* Tingi *contraria Mellaria, aut Tingi è regione sita Mellaria. Nam* Tingis *factam hic à* Melâ *mentionem mihi est persuasissimum*; *primùm quidem veterem lectionem spectanti, quæ est, ut dixi*, Cingenteratum; *aut etiam ut in suis libris doctissimus* Elias Vinetus *reperit* Tingentera; *ut jam de eo dubitari non possit. Deinde autem video morem* Melæ *hunc esse, ut locorum in alterâ orâ oppositorum mentionem faciat. Sic alibi:* Majorem Sabæi tenent partem, ostio proximam, & Carmanis contrariam Macæ. *Nec moveri quisquam debet quod alii* Tingin Bæloni *non* Mellariæ *faciunt contrariam. Nam* Bælo *&* Mellaria *ita sunt vicinæ, ut mirari hoc nemo debeat*. *Salmasius*, whose opinion is approved by *Bochart*, makes it *Tingis altera, tum Mellaria*, &c. and takes the preceding *transvecti* to denote *Julia Traducta*. *Casaubon* seems to have been once of the same Opinion. *Sed a* Strabone *stare* Ptolemæus *videtur, qui in hâc* Hispaniæ *orâ oppidum quoddam memorat cui nomen* Transducta, *in quod scilicet collocati fuerint isti, de quibus nunc loquitur* Strabo; *& de quibus dubitavi aliquando, an hæc* Melæ *verba essent accipienda*, In eoque Carteia, ut quidam putant, aliquando Tartessus, & quam transvecti ex Africâ Phænices habitant. *Nam videbatur satis apertè* Transductam Ptolomæi παραφράζειν. *Nunc iis assentior qui ad* Carteiam *ea referunt*. The opinion of

* *Salma-*

Salmasius seems to be the most probable; for *Bælo* and not *Julia Traducta* is said to be over against *Tingis*. *Marcianus Heracleotes* makes the two former about 250 Stadia distant from one another, and *Mellaria* is generally placed between them; therefore they could not be so near one another as *Casaubon* insinuates. Tho' *Carteia* was originally founded by the *Phœnicians*, it had been erected into a *Roman* Colony long before *Mela's* time, and therefore he could not very properly say *Carteia, quam Phœnices habitant*; and had he intended to take notice of the Founders of that City, it is probable that one whose Stile is so pure and accurate, would have made use of another word, or at least another Tense. Besides, if *Julia Traducta*, according to *Casaubon*, is not meant by that passage, it must have been entirely omitted by *Mela*; which is very unlikely, considering he was Born in or near it; and that it is mentioned by *Strabo*, who lived before him, and *Ptolomy* and several others who were after him; and appears to have been remaining at the time the *Vandals* were in possession of *Spain*; for *Greg. Turon.* Lib. II. says *Prosequentibus* Alamannis *usque ad* Traductam, *transito mari*, Vandali *per totam* Africam *ac* Mauritaniam *sunt dispersi*. The Letters of *Tingi altera* come nearer the *Tingentera* of *Elias Vinetus*, and the *Tinge Hiera* of *Gronovius*, than *Casaubons Tingi contraria* or *Tingi è regione sita*. The & and the *atque*, by making the stop at *Tartessus* instead of *Habitant*, may very well relate to the same place; and it is not improbable that *Mela* was desirous to illustrate the obscure place of his Birth by a Periphrasis, and a name of some *Eclat*; tho' it has happened, the method he took to do Honour to it, has been the occasion, that we are in doubt even of its Name.

I met with two Medals of *Julia Traducta* among the Brass *Spanish* Coins; but as I cannot ascertain where they were found, I will not pretend to form from thence any judgment of the situation of the Town to which they belong. But I presume in matters so dark, a conjecture may be offered. It does not seem very improbable, that *Julia Traducta* stood where *Tarifa* is at present. The *Spanish* Authors reckon that Town to have been built by *Tarif* at his second coming to *Spain*. I cannot see what could invite him to settle on a Spot which has neither the convenience of a River, nor a Harbour, and is commanded by a rising Ground; unless he found some Tenements standing, or Ruins to serve for Materials to Build. I have several *Roman* Coins that were found there after great Rains, in the Common Sewer; which is some slight inducement to believe it was formerly a *Roman* Town.

About a League and half to the *West* of *Tarifa*, is a place which goes now by the name of *Val de Vaca*. The Country People have a Tradition among them, that it was once a considerable Town, since swallowed up by the Sea. There is a small Brook called *el Arroyo de Juan Francisco*, which serves to turn some Mills, that a Priest of that Name was encouraged to build there, by finding an antient Stone Channel for the Water. I saw some other small Ruins, and was credibly assured there are visible remains of an old Town a good way under Water. There is a Shoal almost off this place, that runs pretty far in the Sea, on which a *Hamburgher* was lost some Years ago. Perhaps *Mellaria* stood hereabouts.

Wherever it was, the Ruins of it must be a considerable way in the Sea, if credit is to be given to *Pliny*, who upon the Testimony of one Born there, reckons only five Miles from thence to *Afric*. Lib. III. whereas

whereas it is at present five Leagues over at the narrow Part. *Casaubon* is mistaken in that Note on *Strabo* L. II. where he says, *At Maris Mediterranei ostium vix LXX Stadia latum est* ᾗ στενώτατον.

I cannot help observing that the best Hony in all *Spain* is made in these Parts, and that the same cause to which the ancient *Mellaria* ow'd its Name, still subsists, and has given a modern Appellation to several places hereabouts, as *Playa de Orimel, Rio de la Miel, Bejer de la Miel.* The latter of these is generally reckoned by the *Spaniards* to be the old *Mellaria*, for no other reason, that I can see, but the Name. For it is at least two Leagues from the Coast of the Streights, and, by what I could judge when I was on the Spot, as near the Ocean, and therefore may as well be ascribed to the one as the other. Whereas *Mellaria*, according to all the old Geographers, was situated on the Sea-side in the Streights, and is reckoned by *Pliny* the nearest Town to *Afric*; a plain proof that it was not what is now *Bejer de la Miel.*

About a League and half further *West*, in a small Bay, there are very great Ruins, which appear evidently to be the remains of a *Roman* Town. A League *Eastward* from that place, upon an Eminence, are to be seen the Quarries from which the Stone was fetched for building it; and all the way from thence are large remains of an Aqueduct, of which in some places there are entire Arches still standing. Among the Ruins of the old Town, I saw the Body of a *Roman* Statue of fine Alabaster, something bigger than the Life. Our Guide said his Father had seen it entire; but as it was an Idol of the *Gentiles*, they, like good Catholicks, had broken it to pieces. He likewise told us that Urns of old Coins had been found there;

but not being Current in *Spain*, they had thrown them away. The place is called *Balonia*. It is over against *Tangier*, and frequently infested by the *Moors* from thence; on which account it is uninhabited. A small River, called *Alpariate*, runs by it: all which circumstances correspond with the ancient accounts of *Bælo*. I have a Medal that was given me at *Tarifa*, with the following Letters upon it BAILO, which probably belonged to this City, called by *Ptolomy* Βαίλων. *Martianus Capella* Lib. VI. mentions it under the name of *Velonensis Bæticæ Civitas*. The Itenerary of *Antoninus* places *Bælo* VI.M.P. *West* of *Mellaria*, which is about the distance of these Ruins from *Val de Vaca*.

About five Leagues farther is the Cape of *Trafalgar*; the sight of which immediately brought to my mind *Mela*'s description of it. *Illud jam in Occidentem & Oceanum obliquo jugo excurrens, atque ei, quod Ampelusium esse dixeramus, adversum*, &c. Near the Capes Point are the Ruins often mentioned by the *Spanish* Authors, under the name of *Aguas de Mecca*. I was not there, but was assured at *Bejer de la Miel*, that there were still some Ruins on the Shore, and more in the Sea, that run all along under the Cape; particularly remains of a Mole, which must have made it a tolerable Harbour. These Ruins seem to be the remains of old *Bæsippo*. *Plin*. L. III. *Portus Bæsippo*. Mela *Bæsippo usque ad Junonis promontorium, oram freti occupat*.

The placing of Watch-Towers along the Coast of *Spain* to Alarm the Country, upon any Descent, seems to have been a practice of a long standing. *Livy* Lib. XXII. C. 19. *Multas & locis altis positas turres* Hispania *habet, quibus & speculis & propugnaculis contra latrones utuntur: inde primo, conspectis hostium navibus, datum signum* Asdrubali *est*, &c.

III. A

III. *A Letter of M.* l' Abbé Conti, R. S. S. *to the late M.* Leibnitz, *concerning the dispute about the Invention of the* Method *of* Fluxions, *or* Differential Method; *with M.* Leibnitz *his Answer.*

J'Ai differé jusqu' à cette heure de repondre à votre Lettre, parce que j'ai voulu accompagner ma Réponse de celle que M. *Newton* (*a*) vient de fair à l'Apostille que vous y avez ajoutée. Je n' entrerai dans aucun detail à l'égard de la dispute que vous avez avec M. *Keill*, ou plutôt avec M. *Newton*. Je ne puis dire qu' historiquement ce que j'ai vû, & ce que j'ai lû, & ce qu'il me manque encore de voire & de lire, pour en juger comme il faut.

J'ai lû avec beaucoup d' attention, & sans la moindre prevention, le *Commercium Epistolicum*, & le petit Livre (*b*) qui en contient *l' Extrait*. J'ai vû à la *Societé Royale* les Papiers Originaux des Letters du *Commercium*; une petite (*c*) Lettre écrite de votre main à M. *Newton*; & l' ancien Manuscript (*d*) que M. *Newton* envoya au Docteur *Barrow*, & que M. *Jones* à publié depuis peu.

(*a*) In his Letter dated *Feb.* 26. 1715-16. *st. vet.* and Printed at the end of *Raphson*'s History of Fluxions.

(*b*) Printed in the *Philos. Trans.* N. 342. and in Tome VII. *du Journal Literaire.*

(*c*) Dated 17 *March*, 1693. and Printed at the end of *Raphson*'s History of Fluxions.

(*d*) Entituled *Analysis per Series numero terminorum infinitas.*

De

De tout cela j'en infere, que si on ôte à la dispute toutes les digressions étrangeres, il ne s'agit que de chercher si M. *Newton* avoit le Calcul des Fluxions ou infinitésimal, avant vous, ou si vous l'avez eu avant lui. Vous l'avez publié le premier, il est vrai; mais vous avez avoué aussi que M. *Newton* en avoit laissé entrevoir beaucoup dans les Letters qu'il a écrites à Mr. *Oldenbourg* & aux autres. On prouve cela fort à long dans le *Commercium*, & dans son *Extrait*. Quelles son vos Réponses? Voila ce qui manque encore au Public, pour juger exactement de l'affaire.

Vos amis attendent votre réponse avec beaucoup d'impatience, & il leur semble que vous ne sauriez vous dispenser de répondre, si non à M. *Keill*, du moins à M. *Newton* lui même, qui vous fait un deffi en termes exprès, comme vous verrez dans sa Lettre.

Je voudrois vous voir en bonne intelligence. Le public ne profite guere des disputes, & il perd sans ressource, pour bien de siecles, toutes les lumieres que ces mêmes disputes lui dérobent.

Sa Majesté a voulu que je l'informasse de tout ce qui s'est passé entre M. *Newton* & vous. Je l'ai fait de mon mieux, & je voudrois que ce fut avec succès pour l'un & pour l'autre.

Votre Probleme a été resolu fort aisément en peu de tems. Plusieurs Geometres à *Londres* & à *Oxford* en ont donné la Solution. Elle est générale, car elle s'étend à toutes sortes de Courbes, soit Geometriques, soit Méchaniques. Le Probleme est un peu équivoquement proposé: mais je croi que M. *de Moivre* ne se trompe pas, en disant, qu'il faudroit fixer l'ideé d'une suite de Courbes. Par exemple supposer qu'elles ayent la même Soûtangeante pour la même Abscisse; ce qui conviendra non seulement aux Sections Coniques, mais à une infinité d'autres tant Geometriques que

* Me-

Mechaniques. On pourroit encore faire d'autres suppositions pour fixer cette idée.

Je vous parlerai une autre fois de la Philosopie de M. *Newton*. Il faut convenir auparavant de la Methode de Philosopher, & distinguer avec beaucoup de soin la Philosophie de M. *Newton*, des consequences que plusieurs en tirent fort mal á propos. On attribuë à ce grand homme bien de choses qu'il n'admet pas; comme il l'a fait voir à ces Messieurs Francois qui vinrent à Londres, à l'occasion de la grand Eclipse.

Je suis avec tout le respect possible

A *Londres* le Monsieur, vôtre *&c.*
de *Mars* 1716.

N. B. *Mr.* l'Abbé Conti *spent some Hours also in looking over the old Letters and Letter-Books kept in the Archives of the* Royal Society, *to see if he could find any thing which made either for Mr.* Leibnitz, *or against Mr.* Newton, *and had been omitted in the* Commercium Epistolicum Collinii & aliorum: *but could find nothing of that kind.*

A Letter of M. Leibnitz *to M.* L' Abbe Conti, *in Answer to the former.*

Monsieur, Hanover *ce* 14. *d'*Avril, 1716.

POur ne vous faire attendre, je vous dirai par advance que j'ai répondu d'abord à l'honneur de votre Lettre, & en même tems à celle que Mr. *Newton* vous a écrite; & j'ai envoyé le tout à Mr. *Remond* à *Paris*, qui ne manquera pas de vous le faire tenir. Je me suis servi de cette voie, pour avoir des temoins neutres & intelligens de notre Dispute: & M. *Remond* en fera encore part à d'autres. Je lui ai envoyé en même

tems

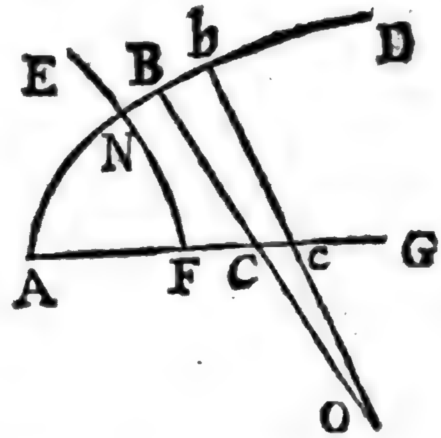
tems une copie de votre Lettre & celle de Mr. *Newton.* Après cela vous pourrez juger, si la mauvaise chicane de quelques uns de vos nouveaux Amis m'embarrasse beaucoup.

Quant au Probleme dont quelques-uns parmi eux ont voulu resoudre des cas particuliers, pour en fixer, disent-ils, les idées; il y a de l'apparence qu'ils se seront jettez sur des cas faciles: car il y en a dans les Courbes Transcendantes, aussi-bien que dans les ordinaires; mais il s'agit d'une solution generale. Ce Probleme n'est point nouveau. M. *Jean Bernouilli* l'a deja proposé dans le mois de *May* des Actes de *Leipsic* 1697, *p.*211. Et comme M. *Fatio* méprisoit ce que nous avions fait; on en repeta la proposition pour lui & pour ses semblables, dans les Actes de *May* 1700. *p.* 204: Il peut servir encore aujourd'hui à fair connoitre à quelques-uns, s'ils sont allez aussi avant que nous en Methodes: & en attendant qu'ils trouvent le moyen de parvenir à la solution génerale, ils pourront essayer ce qu'ils peuvent, en fixant les idées sur un cas particulier, qu'on leur propose dans le papier cy-joint. Sa solution vient encore du même M. *Bernoulli.* Ainsi vous aurez la bonté de ne pas vous rendre trop tôt aux insinuations de ceux qui nous sont contraires; comme lorsqu'ils vous font a croire que notre Probleme leur étoit aisé. Je suis avec zele, Monsieur Votre *&c.*

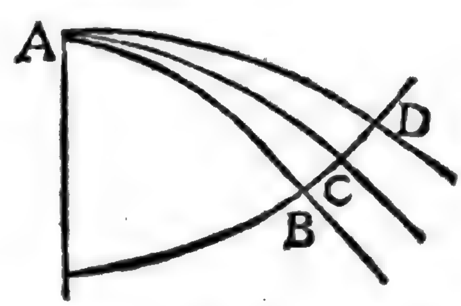
Problema continens casum specialem Problematis generalis de invenienda Serie Curvarum, quarum qualibet sit ad aliam Seriem Curvarum perpendicularis.

Super

Super recta AG *tanquam axe, ex puncto* A *constructis Curvis quotcunque qualis est* ABD, *ejus naturæ ut radius osculi ex singulis singularum Curvarum punctis* B *eductus* BO *secetur ab axe* AG *in* C *in Data semper constanti ratione: ut nempe sit* BO *ad* BC *ut* M *ad* N. *Construendæ jam sunt Trajectoriæ qualis est* ENF, *priores Curvas* ABD *secantes ad angulos rectos.*



Thus far this Letter. Mr. *Leibnitz* first proposed the general Probleme to M. *l'Abbé Conti* in these words; *Trouver une ligne* BCD, *qui coupe à angles droits toutes les courbes d' une suite determinée d'une même gendre; par exemple, toutes les Hyperboles* AB, AC, AD, *qui ont le même sommet & le même centre; & cela par une voye generale.* And in the *Acta Eruditorum* for *October*, 1698. *p.* 470, 471. he calls the Curves in this determinate Series, *Curvas ordinatim datas, & positione datas, & positione ordinatim datas.* And by all this, the Series of Curves to be cut is given, and nothing more is to be found, than the other Series which is to cut it at right Angles. But Mr. *Leibnitz* being told that his Probleme was solved, he changed it into a new one, of finding both the Series to be cut and the other Series which is to cut it. And the particu-

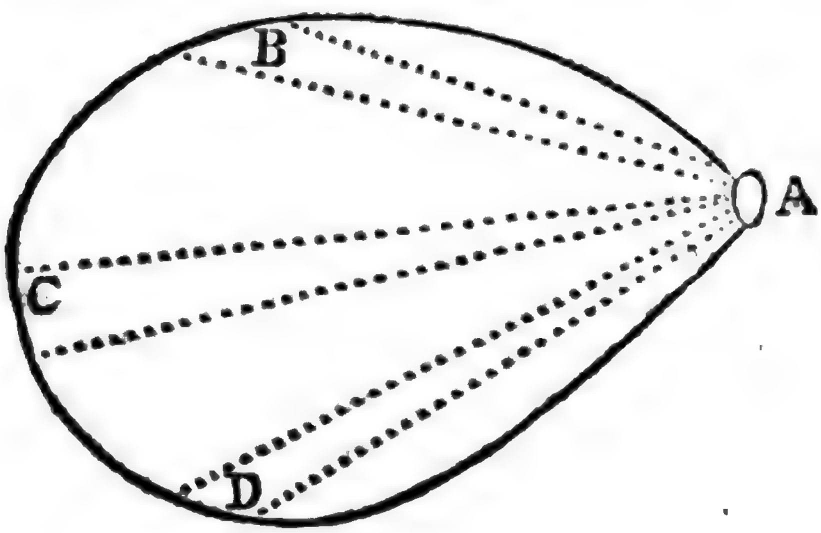


lar Probleme proposed in this Letter is a special Case, not of the general Probleme first proposed, as it ought to have been, but of this new double Probleme. And the first part of this double Probleme (*viz.* by any given property of a Series of Curves to find the Curves) is a Probleme harder than the former, and of which a general Solution is not yet given. Mr. *Leibnitz* in a Letter to Mr. *John Bernoulli*, dated 16 *December*, 1694. and published in the *Acta Eruditorum* for *October* 1698. *p.* 471, set down his Solution of the Probleme, when the given Series of Curves is defined by a finite Equation, expressing the relation between the Abscis and Ordinate. The same Solution holds when the Equation is a converging Series, or when the property of the Curve to be cut can be reduced to such an Equation, by the *Analysis per Series numero terminorum infinitas.* But Mr. *Leibnitz* was for solving the Probleme without converging Series.

IV. *Pars reliqua Dissertationis* De Potentiâ Cordis. *Authore* Jacobo Jurin, *M. D.* & R. Societatis Soc.

Theorema II.

S*I ex Machinâ cavâ inæqualiter contractili*, A B C D, *aqua per Machinæ contractionem exprimatur, Motus aquæ ex orificio* A *prosilientis æquatur Summæ Factorum ex*



Sectionibus quibusvis transversis omnium aquæ filamentorum A B, A C, A D; *singulis ductis in longitudines & velocitates respectivas.*

Demonstratio. Loco filamentorum aquæ, concipiatur Machina tubis minimis, inæqualiter amplis, A B, A C, A D, in orificium A desinentibus, tota constare.

Est aquæ Motus in quovis tubo æqualis sectioni cuivis ipsius tubi, ductæ in velocitatem aquæ per sectionem istam fluentis, & longitudinem tubi, per *Theor.* 3.

 De

De Motu Aquar. fluent. Proinde Summa Motuum aquæ in omnibus tubis simul sumptis, sive Motus aquæ ex Machinæ orificio prorumpentis, æqualis est Summæ Factorum ex omnium tuborum sive filamentorum aquæ sectionibus, ductis in longitudines, & velocitates, respectivas. *Q. E. D.*

Corol. 1. Motus aquæ effluentis minor est Facto ex orificio A, velocitate aquæ exeuntis, & longitudine filamenti aquæ omnium longissimi. Est enim Factum ex orificio & velocitate aquæ effluentis, æquale Summæ Factorum ex sectionibus filamentorum singulis ductis in velocitates respectivas; & Summa horum Factorum, ducta in longitudinem filamenti omnium longissimi, major est quam Summa eorundem ductorum cujusque in suam longitudinem.

2. Motus Aquæ æquatur Facto ex orificio A & velocitate aquæ exeuntis, ducto in longitudinem aliquam mediam inter longitudines filamentorum longissimorum & brevissimorum: vel æquatur Facto ex quantitate aquæ dato tempore effluentis, & longitudine mediâ prædictâ, applicato ad tempus illud datum.

3. Si Machinæ plures similes aquâ plenæ similiter contrahantur, sive æquabili velocitate mediâ, sive inæquabili, similiter tamen in omnibus Machinis auctâ, vel imminutâ; Motus, quo aqua ex Machinæ cujusque orificio prorumpit, rationem habet compositam ex ratione quadruplicatâ Diametri cujusvis homologæ ipsius Machinæ, & reciprocâ temporis ratione, quo peragitur Machinæ contractio: vel rationem compositam, ex ratione ponderis Machinæ, vel molis aquæ, sive Machinâ contentæ, sive ex eâdem expulsæ, ratione ejusdem ponderis, vel molis, subtriplicatâ, & ratione temporis reciprocâ.

Problema.

PROBLEMA.

Invenire Potentiam Cordis.

Sit p = Pondus Ventriculi sinistri, sive quantitas Sanguinis eidem ponderi æqualis.

S = Superficies interna ejusdem.

l = Longitudo media filamentorum Sanguinis ex eodem prodeuntium.

s = Sectio Aortæ.

q = Quantitas Sanguinis Ventriculo sinistro contenti.

t = Tempus, quo Sanguis ex Corde expelleretur, sublatâ Arteriarum & Sanguinis præcedentis resistentiâ.

v = Velocitas variabilis, quâ Sanguis ex Corde prosiliens per Aortam flueret, sublatâ resistentiâ.

x = Longitudo variabilis Aortæ à Sanguine ex Corde effluente percursa.

z = Tempus, quo longitudo x percurritur.

Inde velocitas media variabilis Sanguinis Ventriculo contigui, sive media velocitas ipsius Ventriculi $= \frac{sv}{S}$.

Motus Ventriculi (per *Theor.* 1. *Cor* 2.) $= p \times \frac{sv}{S}$.

Motus Sanguinis effluentis (per *Theor*. 2. *Cor*. 2) $= sv \times \overline{l + x}$.

Horum Summa, sive Potentia Ventriculi $= sv \times \overline{\frac{p}{s} + l + x}$. Est autem $v = \frac{x}{z}$. Unde per Methodum *Newtonianam* inversam, elicitur Potentia Ventriculi $= \frac{sx}{z} \times \overline{\frac{p}{S} + \frac{x}{2} + l}$. Sed cum $z = t$, erit $sx = q$.

Hinc Potentia Ventriculi $= \frac{q}{t} \times \overline{\frac{p}{S} + \frac{q}{2s} + l}$.

Simili

Simili ratione invenitur Potentia dextri Ventriculi

$$=\frac{q}{t}\times\overline{\frac{\pi}{\Sigma}+\frac{q}{2\sigma}+\lambda}.$$

Literis autem Græcis eadem significantur in dextro Ventriculo, quæ Latinis in sinistro.

Hinc tota Cordis Potentia

$$=\frac{q}{t}\times\overline{\frac{p}{S}+\frac{\pi}{\Sigma}+\frac{q}{2S}+\frac{q}{2\sigma}+l+\lambda}.\quad Q.\ E.\ I.$$

Si ponatur

$p = 8$ unc. Avoird $= 13.128$ unc. cub.
$\pi = 4 \quad\quad = 6.564$
$S = 10$ unc. quadrat.
$\Sigma = 10$
$l = 2$ unc.
$\lambda = 1\frac{1}{2}$.
$q = 2$ unc. Avoird. $= 3.282$ unc. cub.
$s = 0.4185$ unc. quadrat. } Ex *Keillianis* Experi-
$\sigma = 0.583$ } mentis.
$t = 0.1''$

Erit Potentia Ventriculorum æqualis motui ponderum subscriptorum, nempe,

	lib.	unc.
Ventriculi sinistri —— ——	9	1
Ventriculi dextri —— ——	6	3
Cordis totius —— ——	15	4

Quorum ponderum ea est velocitas, quâ percurratur longitudo uncialis singulis minutis secundis.

Cor. 1. Quoties Pulsus fit celerior; aut minuitur resistentia, aut Potentia Cordis augetur, aut minor solito Sanguinis copia singulis contractionibus ex Corde expellitur.

2. Si Pulsus solito tardior fiat; necesse est, vel augeatur resistentia, vel Cordis Potentia minuatur, vel major Sanguinis moles ex Corde ejiciatur.

3. Aucta

3. Auctâ resistentiâ, necessario vel Pulsus retardabitur, vel augebitur Cordis Potentia, vel Sanguinis quantitas solito minor ex Corde exprimetur.

4. Imminutâ resistentiâ, vel Pulsus acceleratur, vel major Sanguinis copia quâque Systole ejicitur, vel Cordis vires minuuntur.

5. Auctis Cordis viribus, necessario vel augebitur resistentia, vel Pulsus accelerabitur, vel plus Sanguinis ex Corde ejicietur.

6. Viribus Cordis imminutis, vel minuatur necesse est resistentia, vel Pulsus tardior fiat, vel minus Sanguinis ex Corde exprimatur.

7. Cum minor Sanguinis moles ex Corde projicitur; vel acceleratur Pulsus, vel Cordis vires minuuntur, vel augetur resistentia.

8. Cum plus Sanguinis ex Corde exprimitur; vel Pulsus tardior fiet, vel augebitur Cordis Potentia, vel resistentia minuetur.

Schol. 1. Ventriculorum superficies internas, cum factu difficillimum videatur, ut accurate determinentur, aut etiam ratio habeatur imminutionis, quam inter contrahendum patiuntur, contenti fuimus præterpropter æstimare: cum sive easdem 12, sive 8 unciis quadratis singulas æquales statueris, perparva reperiatur Potentiarum facta mutatio. Quod etiam observari poterit de longitudine mediâ filamentorum Sanguinis. Præterea differentias, quâ Arteriæ ambæ, earumque rami proximi à Corde progredientes, sectione augentur, ut æstimatu perdifficiles & pene insensiles, negligimus. Alioqui esset Cordis Potentia tantillo minor statuenda, quam quæ supra definita est.

2. Determinavit Vir Celeberrimus, *Jacobus Keillius*, velocitatem Sanguinis, resistentiâ submotâ, ex Corde effluentis, eam circiter, quâ percurrantur pedes $6\frac{1}{3}$ singulis minutis secundis. Ponit vero ille celeritatem Sanguinis

guinis per totam Syſtolem æquabilem, quam nos inſigniter inæqualem fieri, & perpetim à Syſtoles initio retardari ſupra oſtendimus. Hanc ſi cui definire libuerit, ſubſtituenda eſt, in quartâ Æquatione ſupra poſitâ, Potentia Ventriculi proxime inventa, & ipſi x valor quivis tribuendus, ut eliciatur v, ſive velocitas eidem reſpondens. Ita, cum initio Syſtoles ſit $x = 0$, ſub finem vero $x = \frac{q}{s}$, determinatur inde ea Sanguinis velocitas initio Syſtoles, quâ pedes $14\frac{1}{4}$; in fine autem quâ $4\frac{1}{4}$, minuti ſecundi ſpatio percurrantur. Pariter in dextro Ventriculo: velocitas Sanguinis initialis pedes circiter $10\frac{5}{6}$, ultima vero 3 pedes eodem temporis ſpatio conficiet.

Adhibuimus hactenus eam Hypotheſin, quâ Muſculi Cordis Ventriculos conſtituentes Motum omnem, quo adiguntur in contractionem, Momento temporis concipiunt. Quod ſi ponamus Motum iis communicari non unico quidem Momento, ſed tantillo tamen temporis ſpatio, quod cum totâ Syſtoles duratione comparatum rationem obtineat admodum exiguam; erit Cordis Potentia paululo major ſtatuenda, quam quæ ſupra determinata eſt. Si vero ſtatuatur iſte Motus, procedente Syſtole, in ratione temporis augeri; erit totus Motus in fine Syſtoles acquiſitus duplo major quam ſupra poſuimus, ubi nulla reſiſtentia Sanguini ex Corde profluenti objicitur: Ubi autem ſolita adeſt reſiſtentia, erit idem quintuplo major; quod inſtituto calculo facile patebit. Pari ratione poterit calculus noſter ad aliam quamlibet Hypotheſin, quâ Ventriculorum Motus in duplicatâ vel ſuperiori quâvis ratione temporis augeatur, accommodari. Potentia vero in fine acquiſita ſuprapoſitâ elicietur longe major, nempe ex ratione duplicatâ Potentia tripla, ex triplicatâ quadrupla, ex quadruplicatâ quintupla, & ſic in infinitum.

Nobis

Nobis autem videtur secunda Hypothesis, quâ Ventriculi parvo admodum temporis spatio Motum omnem concipiunt, cæteris longe verisimilior. Quum necesse sit, ut aliquid temporis impendatur ad Motum quemlibet generandum; neque videatur adeo tarde increscere Ventriculorum Motus, ut non celerius augeatur, quam secundum temporis rationem. Motus enim Musculorum impetu solo Fluidorum quorumcumque, quæ ex Sanguine proveniunt, perfici nequit; quum Brachio alterutro Motum exerere possimus Motu Sanguinis per vasa Corporis universa profluentis longe majorem. Relinquitur ergo, ut Musculorum fibræ Ventriculos Cordis constituentium, rarescentiâ quâdam liquorum in easdem influentium, in Motum impellantur. Hæc autem, quoties vim magnam concipit, plerumque subita est, & fere instantanea. Adde quod Ventriculorum Motus secundùm hanc Hypothesin longe minor efficitur, quam in tertiâ. Non solet autem sapientissimus Artifex, Rerum Conditor, in operibus suis plus Virium adhibere, quam quantum sufficit ad finem propositum consequendum.

Cæterum sive admittatur ista Hypothesis, sive alia quæcunque ex supra dictis verior censeatur, poterunt omnia Corollaria nostra eodem jure ex Problemate deduci. Quæ utrum aliquid adjumenti afferant ad Morborum Historiam explicandam Medico sagaci considerandum permittimus. Facile autem ex Morbi cujusque Naturâ sciri poterit, utrum aucta sit vel imminuta resistentia. Augeri vero credibile est vel imminui Cordis vires auctis vel imminutis Musculorum reliquorum viribus; quamvis aliter statuisse video Virum Celeberrimum, *Laurentium Bellinum*.

Ffffff *Theorema*

Theorema III.

Totus Motus resistentiæ, quæ Sanguini ex Corde erumpenti durante Systole objicitur, sive totus Motus, qui Sanguini præcedenti & Arteriarum tunicis communicatur, toti Cordis Potentiæ quamproxime æqualis est.

Dem. Peractâ Cordis Systole, quæ pars Aortæ & Arteriæ Pulmonalis Cordi proxima est, perstat plena Sanguine per totam Systolem Arteriarum. Nec enim patitur earum fabrica & nexus, quo Cordi conjunctæ sunt, ut tunicis in sese penitus collabentibus totæ occludantur, neque potest earum cavum Sanguine vacare. Alioqui enim, contrahentibus sese reliquis Arteriarum partibus, Sanguis iisdem contentus retro in vacuum impelleretur motu, & inutili & motui Sanguinis naturali contrario. Tum etiam Valvulæ Semilunares non tenderentur versus Ventriculos, adeoque Sanguis ex Auriculis in Ventriculos expressus, etiam in Diastole Cordis, in Arterias protruderetur.

Hinc patet Sanguinem proxime ex Corde expulsum Systole peractâ immotum in Arteriis persistere, adeoque tum omnem Ventriculorum Motum excepisse, tum eundem totum partim Sanguini antecedenti, partim tunicis Arteriarum communicasse. *Q. E. D.*

Theorema IV.

Motus, qui in Systole Cordis communicatur Sanguini præcedenti, est ad Motum tunicis Arteriarum communicatum, ut tempus Systoles Cordis ad tempus Diastoles quam proxime.

Dem. Quum Sanguis per vasa Corporis universa, si partes Arteriarum Cordi propiores exceperis, æquabili cursu deferatur; necesse est, ut tum Motus affrictu Sanguinis ad vasorum latera deperditus, tum Motus Sanguini redditus à Systole sive Cordis sive Arteriarum

rum, æqualibus temporibus æqualis ſit. Qui autem Motus à Syſtole Arteriarum Sanguini communicatur, idem eſt præciſe, qui prius à Cordis Syſtole Arteriarum tunicis fuerat impreſſus, cum Arteriæ eodem impetu quo diſtractæ fuerint etiam reſtituantur; & Syſtole Arteriarum cum Cordis Diaſtole duratione conveniat. Unde patet Propoſitum. *Q. E. D.*

Cor. Si ponamus cum Viro Doctiſſimo *Jacobo Keillio*, Syſtolen Cordis peragi tertiâ parte temporis inter Pulſus binos intercepti; erit Motus Sanguini præcedenti communicatus totius Potentiæ Cordis pars tertia: Motus vero Arteriis communicatus prioris duplus, ſive duæ partes tertiæ totius Cordis Potentiæ.

Theorema V.

In diverſis Animalibus Potentia Cordis rationem obtinet compoſitam, ex ratione quadruplicatâ Diametri cujuſvis homologæ Animalis, & ratione inversâ temporis, quo Cor contrahitur: vel rationem compoſitam, ex ratione ponderis vel ipſius Cordis vel integri Animalis, ratione ponderis ejuſdem ſubtriplicatâ, & ratione temporis reciprocâ.

Facile demonſtratur vel ex *Corol.* 3. *Theor.* 1 & 2, vel ex Potentiâ Cordis Problemate præcedente definitâ.

Cor. 1. Si ponatur Cordis Potentiam rationem obtinere ponderis vel ipſius Cordis, vel integri Animalis, vel Sanguinis copiæ in toto Animali; erit Animalis longitudo in ratione temporis, quo Cordis Syſtole perficitur, ſive in ratione inversâ frequentiæ Pulſuum.

2. Si ratio longitudinis integri Animalis major fuerit ratione inversâ frequentiæ Pulſuum, neceſſe eſt major ſit ratio Potentiæ Cordis ratione ponderis ejuſdem.

Schol. Quum conſtet Experimentis Puerorum Pulſus non eſſe tanto frequentiores Pulſibus Virorum, quanto Pueri Virorum longitudine ſuperantur, concludendum

eſt, vi ſecundi Corollarii, Potentiam Cordis Virilis majorem obtinere rationem ad Potentiam Cordis Pueri, quam eſt ratio ponderum. Et par eſt ratio in cæteris Muſculis. Nam ſi Corporis robur rationem ponderis ſequeretur, poſſent Pueri æqualia itinerum ſpatia eodem tempore cum Viris conficere.

Simili ratione ac Motum Sanguinis ex Ventriculis Cordis erumpentis ope ſecundi *Theorematis* determinavimus, poterit quoque Urinæ Motus ex Urethrâ profluentis determinari. Nempe ſi ponatur Urethræ & Veſicæ longitudo 12 unciis æqualis, & binæ unciæ Urinæ minuti ſecundi ſpatio emittantur, erit Motus Urinæ effluentis æqualis Motui ponderis libræ $1\frac{1}{2}$, quod uncialem longitudinem ſingulis minutis ſecundis percurrat. Quoniam vero Urina non ſolis Veſicæ Urinariæ viribus contractivis, ſed etiam Diaphragmatis & Muſculorum abdominalium ope in ſubſidium vocatâ, expellitur, nequit Veſicæ Potentia ex Motu Urinæ profluentis æſtimari.

Hæc tu, Vir Doctiſſime, æqui bonique conſulas rogo: ipſe autem ut diutiſſime valeas, utque exiſtimationem tuam, & ipſam Artis Æſculapiæ dignitatem uſque ut hactenus feciſti, inſigniter tueri pergas, ac magis indies magiſque extendere, idcirco ex animo voveo, quia publicam ad ſalutem pertinere arbitror.

Calendis *Januarii*,
$171\frac{7}{8}$.

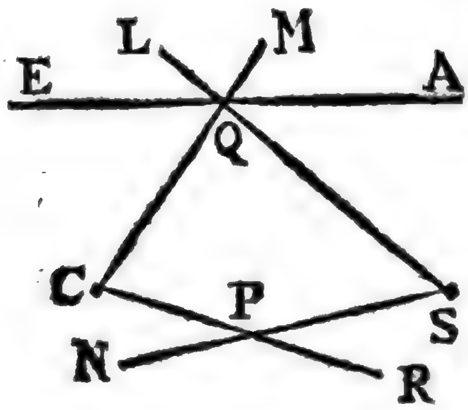
V. *Nova*

V. *Nova Methodus Universalis Curvas Omnes cujuscunque Ordinis Mechanicæ describendi sola datorum Angulorum & Rectarum Ope. Per* Colin Maclaurin *in Collegio Novo* Abredonensi *Matheseos Professore.*

INter innumera sublimiaque Magni *Newtoni* inventa, quibus Geometria amplissime ditata in immensam excrevit luculentissimæ Cognitionis molem, Constructionem exhibuit Curvarum Mechanicam, post Enumerationem Linearum Tertii Ordinis, ad finem *Opticæ* editam, arduo summi Viri ingenio dignam; qua simpliciorem & simul adeo Universalem aliam exhibuit Nemo. Methodum vero suam ad Curvas Tertii Ordinis puncto duplice carentes, aut eas altiores Ordinis puncto multiplice destitutas, non extendit; earumque descriptionem Problematibus Geometriæ difficilioribus annumerandam pronuntiat. Atque hinc in spem venio Methodum sequentem, qua Curvæ Geometricæ cujuscunque Ordinis, licet puncto duplice aut multiplice quovis destitutæ, construuntur, non fore Geometris ingratam.

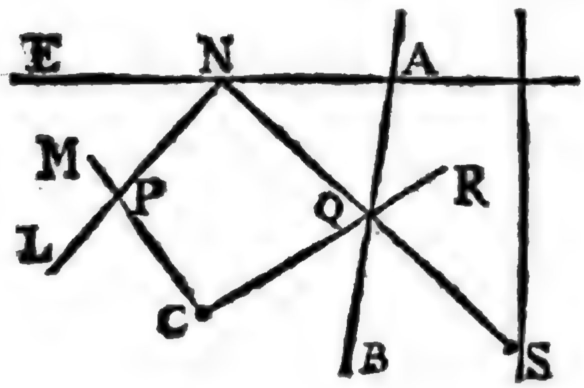
I. Lineæ primi Ordinis ipsæ sunt Rectæ; quæ in uno solo puncto sibi mutuo occurrere possunt. Lineæ secundi Ordinis sunt Sectiones Conicæ, quæ in pluribus punctis quam duobus à rectâ quavis secari non possunt. Eæ vero omnes secundum Lemma 21. Lib. I. *Princip.* D. *Newtoni* sic construi possunt. Circa data duo puncta

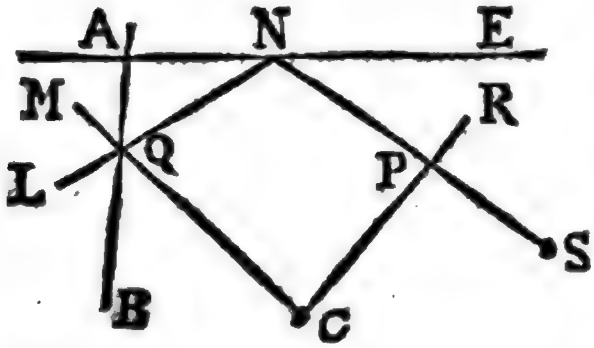
C & S



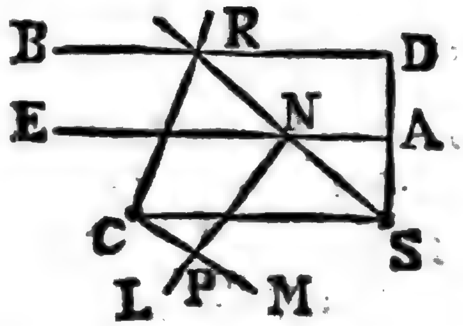
C & S moveantur Anguli dati MCR, LSN; ita ut Crurum CM SL concurſus ſemper ducatur per rectam indefinitam poſitione datam AE; tunc crurum aliorum CR & SN concurſus in P deſcribet Lineam ſecundi Ordinis ſeu Sectionem Conicam.

II. Moveatur ut prius Angulus MCR (*v. Fig.* 2.) circa datum punctum C; Angulus vero datus LNQ ſemper percurrat Angulari ſuo puncto N rectam datam AE, ita ut crus NQ ſemper tranſeat per datum punctum S. 1. Si concurſus crurum CR & SN, tum punctum Q ducatur per rectam infinitam AB, concurſus crurum CM & NL deſcribet Curvam lineam Tertii Ordinis punctum duplex habentem in C. 2. Reliquis manentibus, ſi crurum CM & NL concurſus (*vide Fig.* 3.) ducatur per rectam indefinitam AB: concurſus crurum CR & SN in P deſcribet Curvam Tertii Ordinis punctum duplex habentem in S.

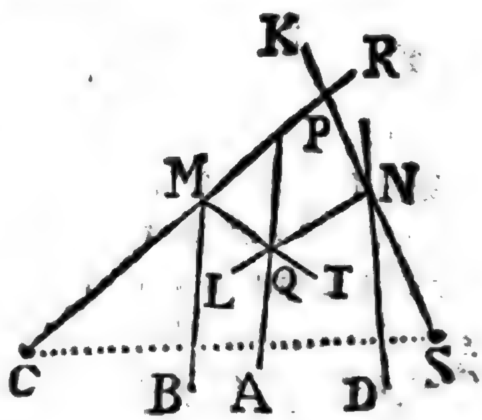




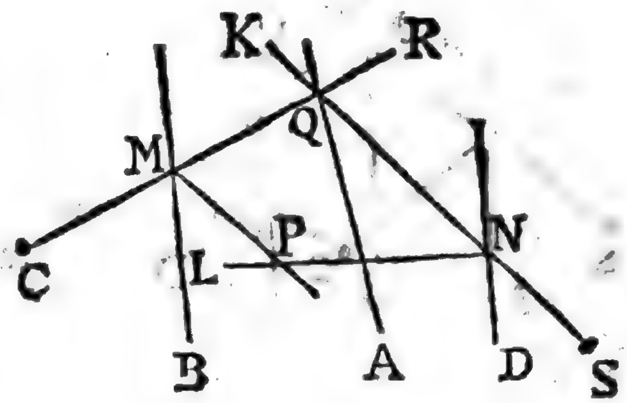
Exem-



Exemplum Casus 1. Sint anguli MCR, LNS recti, (*vide Fig.* 4) & AE, DB, CS parallelæ; sint quoque SA & SD normales respective in rectas AE & DB; sitque SD = 2 SA. Hisce positis, si SD sit minor recta CS, Curva secundum regulam Casus primi descripta, erit Parabola Nodata cum Ovali, Speciei 68*va* Curvarum D. *Neutoni*; Quod si SD = CS, Ovalis evanescit & nodus evadit Cuspis, atque Curva descripta erit Parabola *Neiliana* seu semicubica; Si vero sit SD major quam CS, erit Curva Parabola punctata Campaniformis Speciei 69*na*.



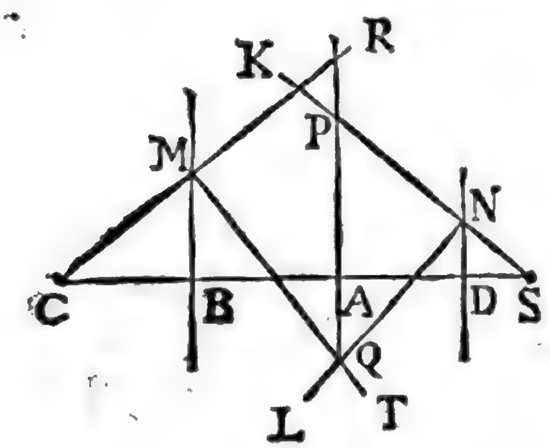
III. Moveantur Anguli dati RMT, KNL, ita ut puncta M & N percurrant rectas indefinitas BM, DN respective; & crura RM, KN semper transeant per data puncta C & S. Si primo Crurum MT & NL concursus Q ducatur per rectam indefinitam AQ; tunc concursus crurum MR & NR in P describet lineam Quarti Ordinis puncta duo duplicia habentem, alterum in C alterum vero in S. Sed secundo si crurum MR & NK (*vide Fig.* 6.) concursus ducatur per rectam indefini-



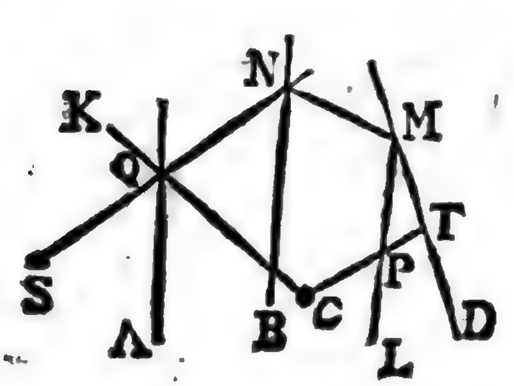
tam

tam A Q; tunc concursus crurum M T & N L describet Lineam Quarti Ordinis puncto duplice carentem.

IV. Quod si in primo casu hujus Constructionis (*v. Fig.* 5.) rectæ C M R, S N K, una coincidant cum C S; tunc puncta C & S evadunt simplicia & Curva erit Tertii Ordinis absque puncto duplice. Exemplum. Sint rectæ B M, A Q, D N, sibi mutuo parallelæ atque omnes perpendiculares in C S. Sint quoque Anguli R M T, K N L recti, & si secundum regulam primi Casus describatur Curva, Crura C M R, S N K una coincident cum C S; & hac constructione describi possunt Curvæ D. *Newtoni* 10, 11, 20, 21, 40, secundum varias positiones punctorum C & S respectu trium rectarum B M, A Q, D N; Omnes vero hæ Species puncto duplice carent.

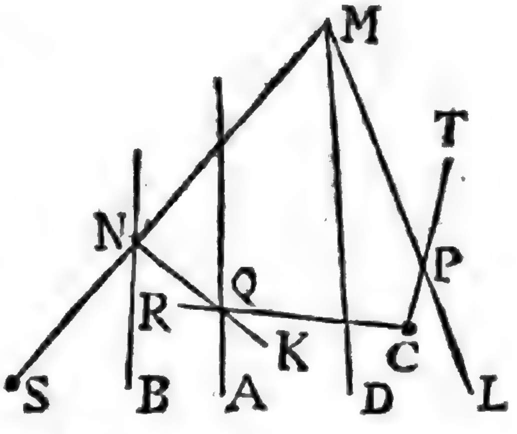


V. Lineæ vero Quarti Ordinis quæ punctum triplex habent sic construi possunt. Sint tres rectæ A Q, B N, D M positione datæ; sint etiam Anguli Q C T, S N M & N M L dati & invariabiles; percurrant puncta N & M rectas B N & D M, ita ut crus N Q semper transeat per datum punctum S: Revolvatur Q C T circa C ita ut concursus crurum C K, S N percurrat tertiam rectam A Q; tunc concursus

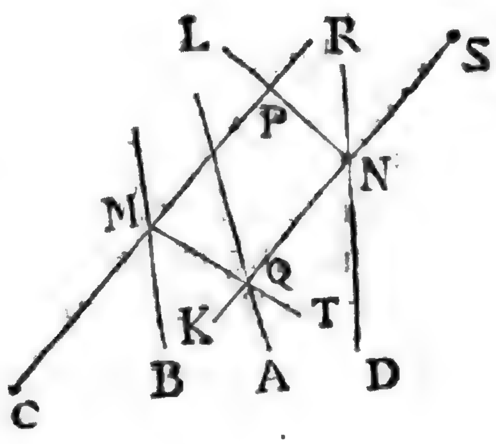


ſus crurum C T, M L deſcribet Lineam Quarti ordinis punctum triplex habentem in C.

VI. Oſtendi quo pacto Lineæ Quarti Ordinis deſcribi poſſunt, quæ punctum triplex habent aut duo duplicia; Aliæ quæ unicum habent punctum duplex ſic commode deſcribuntur.



Sint tres rectæ ut prius poſitione datæ, A Q, B N, D M, dentur etiam Anguli S N K, S M L, R C T; ſint puncta N, M & S ſemper in eadem recta linea; Moveantur puncta N & M ut prius per rectas B N, D M; Si concurſus crurum C R, N K ducatur per rectam indefinitam A Q, tunc concurſus crurum C T, M L deſcribet Lineam Quarti Ordinis habentem punctum duplex unicum in C. Hæ vero duæ ultimæ Propoſitiones novas Methodus ſuppeditant lineas Tertii Ordinis deſcribendi, tum quæ puncta duplicia habent, tum quæ iis deſtituuntur; Eæ vero in brevi hoc Methodus Noſtræ ſpecimine ſunt omittendæ.

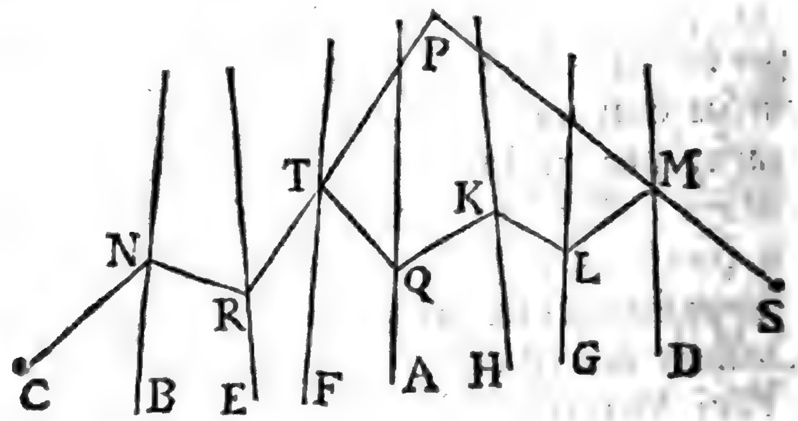


VII. Maneant Anguli atque rectæ ut in *Prop.* III. Concurſus vero nunc rectarum M T, N K ducatur per indefinitam rectam A Q; & Concurſus crurum M R & N L deſcribet Lineam Quinti Ordinis punctum quadruplex habentem in S. Habeo etiam alias Methodus curvas deſcribendi Quinti Ordinis, quæ punctum habent triplex, duplex, aut duo duplicia, vel nulla niſi puncta ſimplicia; ſed

Ggggggg hæc

hæc sufficiant ad simplicitatem & universalitatem Methodus demonstrandam. Notandum vero in specialibus simplicioribus Angulorum & re[illegible]arum circumstantiis, Lineam aliquando migrare in curvam ordinis inferioris quam in *Prop.* explicatur; Imo singulæ Propositiones Methodus suppeditant particulares, curvas aliquas ordinis cujuscunque inferioris describendi.

VIII. *Propositio Generalis.* Sumantur ad libitum Rectæ in eodem plano ubicunque positæ, quarum sit numerus (n) ut BN, ER, FT. Sumantur etiam ad libitum aliæ rectæ ut DM, GL, & HK *&c* quarum sit numerus



(m). Sint Anguli CNR, NRT, RTQ *&c.* atque anguli SML, MLK, LKQ *&c.* invariati, dum puncta angularia N, R, T, M, L, K, percurrant rectas indefinitas BN, ER, FT, DM, GL, HK; Ducatur concursus crurum TQ & KQ per rectam indefinitam AQ; Invenire ordinem curvæ quam concursus cruris SM cum aliqua rectarum CN, NR, RT, TQ *&c.* *ex. gr.* cum RT, perpetuo tanget.

In Serie rectarum CN, NR, RT, TQ *&c.* denotet s numerum rectæ RT, cujus concursu cum SM Curva est describenda, à CN inclusive; qui in hoc casu est ternarius: erit Curva ordinis quem exprimit nume-

rus

rus $sm+s+n+1$: unde in casu quem figura designat, cum $s=m=n=3$ erit Curva ordinis 16^a.

In his descriptionibus Rectas solummodo atque Angulos dari postulavimus; sed facilius sæpe simpliciorum Curvarum ope complexiores describuntur; atque Propositiones his non minus Universales huc pertinentes investigavi: Eas vero cum harum demonstrationibus utpote prolixis impræsentiarum omitto; Easdem postea publici juris facturus, si luce non videantur hæc Geometris indigna.

VI. *Extract of a Letter of the Reverend Mr.* William Rice, *Rector of* Caerleon *upon* Usk, *to* Charles Williams *Esq; giving an account of an ancient* Roman *Inscription lately found there. With some Conjectures thereon, by the Reverend Dr.* John Harris, *S.T.P.* and *R.S.S.*

Sir,

A Person last Week being at Plow in a Close near the Bank of the River *Usk*, which the Ancients called *Isca*, (which glides by us about a quarter of a Mile off and in sight of this Town) came thwart a Stone, and finding Letters thereon, took it up whole; 'tis about a Yard in length, and about three Quarters broad I went to the place, and took a true Copy thereof, which I here make bold to send you. There was underneath it some seeming Oblong Square Sepulcher of Stones, rude in order. A little further in that Close, where that River wears out the Land, there was, some time before, a large Earthen Pot taken out of the Bank by the River-side, which had therein the Scull and Bones of some Person, by some thought to be a Child Murther'd; But I rather conjecture it a *Roman* Urn.

Caerleon, March 21. 1717.

Your humble Servant,
William Rice.

D

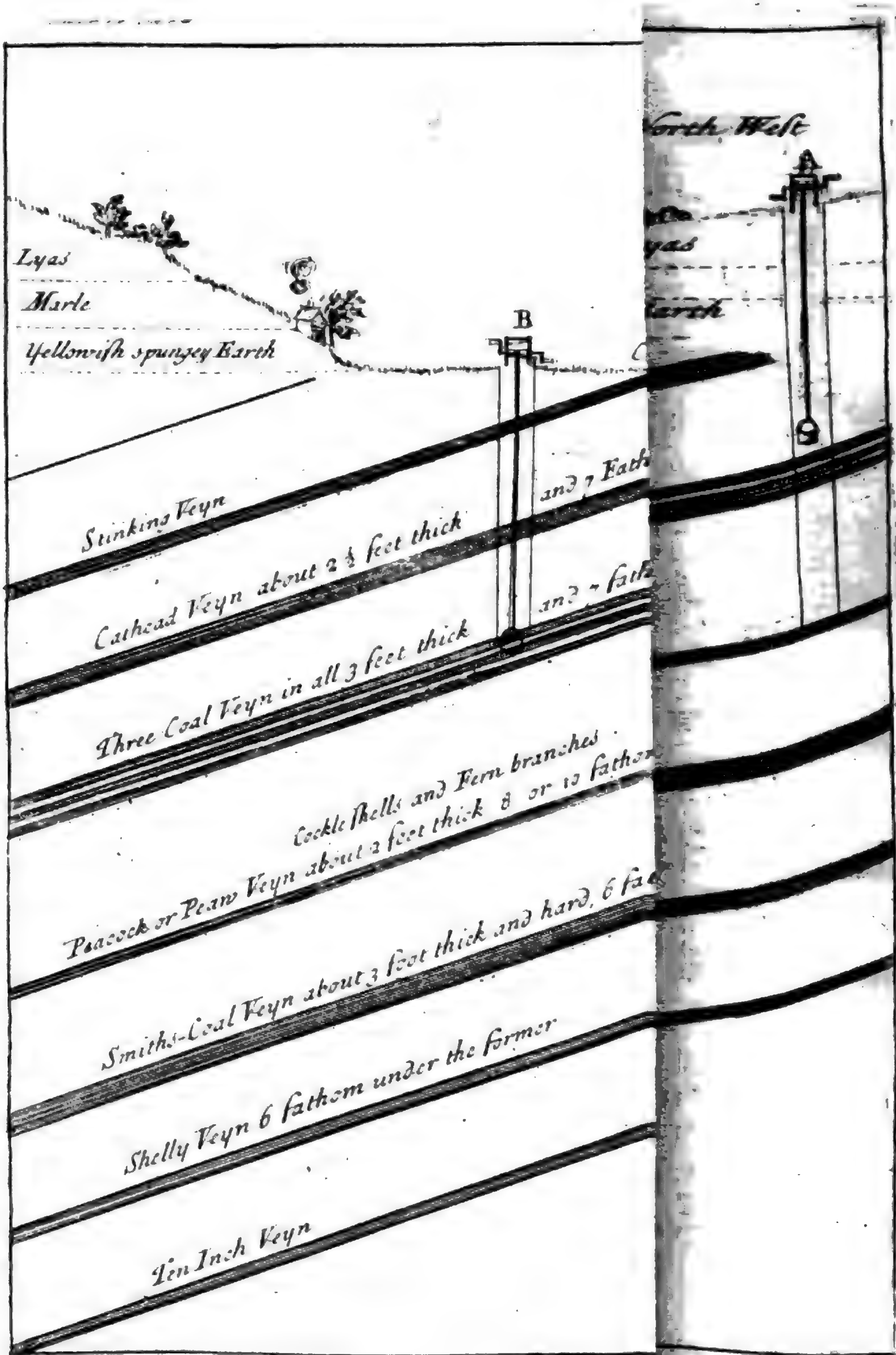
D M
G. VALERIVS. G. F
GALERIA. VICTOR
LVGDVNI. SIG. LEG. II AVG
STIP. XVII. ANNOR XLV. CV
RA. AGINT. AMNIO. PERPITVO. B

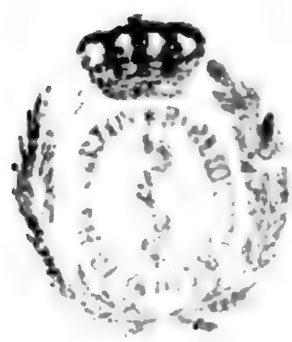
Sir,

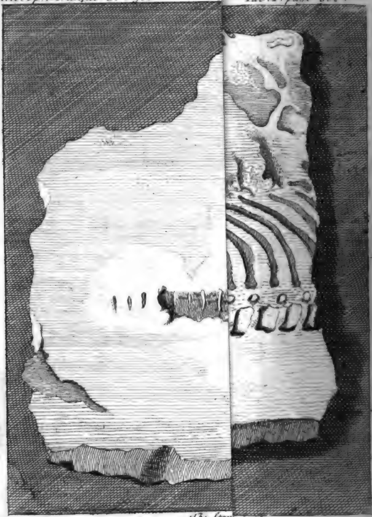
This ancient and fair Inſcription confirms what others have found hereabouts; and what *Cambden* and other Hiſtorians ſhew us, *viz.* That the ſecond *Roman* Legion called *Auguſta,* brought into *Britain* by *Claudius Cæſar* under the Conduct of *Veſpaſian,* was placed here at *Iſca* or *Caer Legion,* by *Julius Frontinus,* in order to awe the *Silures*: And that General obtained ſeveral Victories over them and their Neighbours in ſeveral places hereabouts.

There ſeems to be nothing of Moment or of difficulty in this Inſcription; but *Victor Lugduni*: Which as I think we have no ground from Hiſtory to refer to *Lyons* in *France,* ſo I gueſs that Expreſſion may be thus accounted for. The River *Lugg* is famous in the Neighbouring Parts; and as *Dynas* or *Dyn* hath been ſaid to ſignifie a Town in the Ancient *Britiſh* Language; and that *Dun* doth alſo ſerve to expreſs a Hill or *Down* as we ſtill call it; (which I think is derived from the *Britiſh* alſo) probably *Lugduni* here may expreſs ſome Town or Hill near the River *Lugg*; and ſince there is a Place called to this Day *Luckton,* on the ſide of the River *Lugg* in *Herefordſhire,* perhaps that may bid fair to be the very place where *Valerius* obtained the Victory perpetuated by this Inſcription.

F I N I S.


North West
A
B
Lyas
Marle
Yellowish spungey Earth
Stinking Veyn
and 7 Fath
Cathead Veyn about 2½ feet thick
and 7 fath
Three Coal Veyn in all 3 feet thick
Cockle shells and Fern branches
8 or 10 fathom
Peacock or Peaw Veyn about 2 feet thick
Smiths-Coal Veyn about 3 foot thick and hard, 6 fa
Shelly Veyn 6 fathom under the former
Ten Inch Veyn





The Ston

PHILOSOPHICAL TRANSACTIONS.

For the Months of *March*, *April* and *May* 1719.

The CONTENTS.

I. *De* Maximis & Minimis *quæ in motibus Corporum Cœlestium occurrunt.*

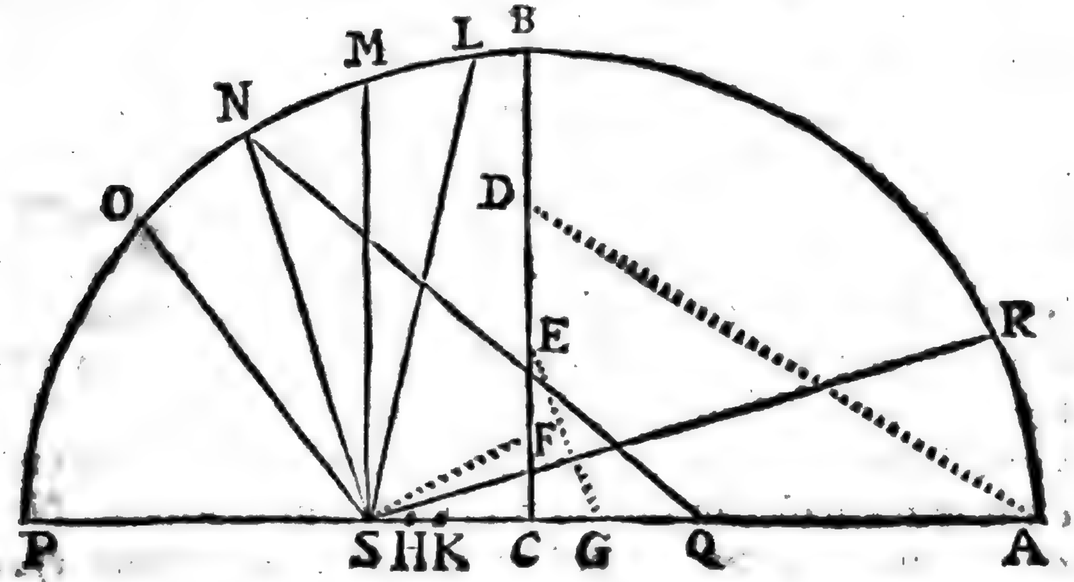
ANTE *Keplerum* Astronomi universi, per tot retro secula, Planetarum motum circularem non ausi sunt in dubium vocare, ex præconceptâ, ut videtur, in figura Circuli nescio qua perfectionis Ideâ. *Keplero* autem Inventori debetur ea qua nunc utimur Theoria, nempe quod Corpora cœlestia Solem ambiunt in communi orbium Ellipticorum Foco situm, ea lege ut Areæ Temporibus proportionales radiis ad Solem ductis describantur. Sublimiorem vero postulat Geometriam, ad ostendendum quam ob causam hoc ita se habeat, quodque aliter esse non possit. Hoc in sempiternam celeberrimi D. *Newtoni* Præsidis nostri gloriam reservatum est.

Hujus vestigiis insistens, Corollaria quædam exhibuit eximius Mathematicus D. *Abr. de Moivre* R. S. S. in *Philos. Transact.* N° 352 edita; Theoremata scil. parata, quibus determinantur Velocitates sive Momenta Motûs tam veri quam apparentis circa Solem, sicut etiam accessûs vel recessûs à Sole, in dato quovis datorum Orbium puncto. Deinde ut Theoriam systematis Planetici penitius excoleret, ope eorundem Theorematum, dictorum Momentorum Momenta perscrutatus est, ostenditque quibus in orbium punctis fiant *Maxime* harum Velocitatum mutationes, idque Solutionibus facilitate & concinnitate præstantibus.

Sit A B P Orbis Planetæ Ellipticus, A P Axis Transversus, C B Semiaxis conjugatus, S Sol, Q Focus alter Ellipseos. Per S ducatur S M ipsi C B parallela: & erit punctum M in quo *Maxima* cum velocitate crescit

ſcit vel decreſcit diſtantia à Sole, & $SM = AC - \frac{SC^2}{AC}$.

Si vero capiatur SL media proportionalis inter Semiaxes AC, CB, erit punctum L in quo *Maxima* fit æquatio Centri, ut vocant; ſive ubi motus angularis fit æqualis medio Motui: Quod ſi Eccentricitas non major ſit quam in pleriſque Planetis, $BL = \frac{1}{2} BM$ quam proximè: Eſt vero $SL = \sqrt{\sqrt{AC^4 - AC^2 SC^2}}$.



Si quæratur punctum N, in quo fit *Maxima* mutatio Velocitatis motûs realis in Curvâ, Problema Solidum eſt. Eſt enim $2NS = 4AC - 2NQ$ ad $3NQ - AC$ ut $AC^2 - CS^2 = CB^2$ ad NQ^2; adeoque ſi ponatur $AC = a$, $CB = c$ & $NQ = y$, habebitur æquatio $y^3 - 2ayy + \frac{3}{2}ccy - \frac{1}{2}acc = 0$. Quâ reſolutâ erit y ſive NQ diſtantia puncti quæſiti N ab altero Ellipſeos foco. In Orbibus autem parum Eccentricis, quales ſunt Planetarum, ſi fiat $CD = SQ$, & junctæ AD æqualis ponatur AK, erit reliqua pars Axis $KP = NS$ diſtantiæ puncti N à Sole quamproxime. Si vero Orbis fuerit Parabolica erit SN ad SP ut 5 ad 4, anguluſque NSP erit 53°. 8′ fere, cujus Sinus eſt $\frac{4}{5}$ Radii.

At Punctum O, in quo motûs apparentis ſive angularis acceleratio Planetæ deſcendentis, vel retardatio

aſcendentis *Maxima* fit, hoc modo obtinebitur. In AC capiatur $CG = \frac{1}{6} AC$, ac fiat angulus CSF 30 *gr.* ductæque SF æqualis ponatur CE, ipſique GE ſit GH æqualis. Dico, ſi diſtantia SO fiat æqualis ipſi PH, quod in puncto O proveniet *Maxima* mutatio motus angularis Planetæ in Orbe Elliptico ABOP gyrantis; eo ſcilicet in Orbis loco ſecundæ differentiæ æquationum centri Planetæ reperientur *Maximæ*. Eſt autem $SO = \frac{7}{6} AC - \sqrt{\frac{1}{36} AC^2 + \frac{1}{3} SQ^2}$. Quod ſi Orbis Parabolica fuerit, ut in Cometis, fiet SO ad SP ut 8 ad 7, angulusque OSP fiet $41^\circ.\ 24'\frac{1}{2}$, ſive cujus Sinus ſit ad Radium ut $\frac{1}{4}\sqrt{7}$ ad 1.

Denique *Minimâ* cum Velocitate mutatur directio Tangentis Orbitæ in puncto R, ſi fiat SR æqualis duabus tertiis Axis majoris AB. Quod ſi Eccentricitas SC minor fuerit quam $\frac{1}{2}$PC, *Minimum* hoc non locum habet, ſed decreſcit ſemper hæc Velocitas quacum revolvitur Tangens, uſque in ipſum Aphelion; quemadmodum ſe res habet in omnium Planetarum motibus. Neque etiam in orbe Parabolico obtinet, ob Axem ejus in infinitum protenſum.

Hæc omnia demonſtrantur, juxta præcepta Doctrinæ de *Maximis* & *Minimis*, ex Theorematis prædictis in N° 352 exhibitis, quæ quidem hac occaſione reviſere Lectorem curioſum non pigebit.

II. *Apologia*

II. *Apologia D.* Brook Taylor, *J V. D.* & *R.S.Soc. contra V. C* J. Bernoullium, *Math. Prof.* Basileæ.

PACIS & concordiæ studioso satius esset injurias vincere ferendo, quàm odiosas contentiones obire ulciscendo. Verum cum patientia nostra pro ignaviâ habetur, silentium pro confessione criminis, & nuperam calumniam jam nova sequitur contumelia, omnino respondendum est, ne nobis ipsis deesse videamur. In *Epistola pro eminente Mathematico D.* J Bernoullio Actis Lipsiensibus An. 1716 insertâ, plagii reus sistor sequentibus verbis: " Hoc nihil novi est in quibusdam Anglis, qui sibi solis licere putant, *aliorum inventa tanquam sua* impunè *usurpare*; quando ipsi Hominesque Deosque invocant, ubi vident, vel saltem videre arbitrantur, Extraneos in suorum Inventa manus inferre. Exempla sunt quorundam, ut Cheynæi, Des Hayes, *Taylori*, aliorumque, *qui passim inventis Bernoullii sunt usi alienisque, vel nullâ prorsus factâ mentione Autoris*, vel ". — Palam est ab ipso Bernoullio promanasse hanc accusationem. Nam in Actis Lipsiensibus An. 1718 [1] per filium suum fatetur se *res ipsas in illa epistola contentas quoad maximam partem amico alicui perscripsisse*. Invidiam equidem prædictæ calumniæ à se amovere sollicitè studet, atque transferre in vicarium illum suum, cum ipse profiteretur, *se non approbare quæ in alios durius dicta censeri possunt* [2]. Sed admodum imperfecta est hæc purgatio. Nam calumniæ sunt quæ duriùs dicta vocat. Ait se *dicta* illa non approbare: Sed improbasse necesse fuit. Testimonium

1 Pag. 261.

2 Pag. 262.

 denique

denique eſt pro ſe teſtantis: Autorem illum anonymum citaſſe oportebat, ut cum ipſo agere liceret: Sed is adhuc latitat. Quam verè autem & ex animo *ſe durius dicta non approbare* videatur, conſtare quodammodo poteſt ex ſequentibus, quæ de me ipſe profert, proprio ſuo nomine, nullâ uſus perſonâ: " Taylorus Geometra " inſignis & acutus, qui ad *profundiora noſtra* feliciter " penetravit, teſte ipſius libro de Methodo incrementorum, probè ſentiens impeditam nimis Analyſeos " fraternæ prolixitatem, eamque in compendium contrahere, ac ſimul generaliorem nonnihil reddere volens, " tantam rei affudit obſcuritatem (quâ in aliis quoque " brevitatem affectans impensè delectari videtur) ut dubitem quenquam fore etiam inter perſpicaciores, qui " ubique & hic imprimis mentem viri aſſequatur, imo " etiamſi aliunde rem cognitam habeat. Ut jam nihil " dicam de ipſo calculo, pro more ejus, conciſo quidem & contracto, ſatis tamen adhuc longo & intricato, ſi quis ſingula ejus capita minutim perſequi " velit; præterquam quòd cum Fratre meo ad tertias " quoque fluxiones excurrat " [3]. Sit ſanè liber ille meus nonnihil obſcurus: Difficile eſt in re ferè nova, & ab uſu communi aliquantulum remota, non eſſe obſcurum. Sed maximè obſcurum oportet eſſe librum, in quem illa omnia verè dicantur. Et ſi verè dicantur, tamen ſine ullâ omnino causâ talia dixiſſe, ab ingenuis moribus prorſus alienum eſt, & mera contumelia.

Sed audio Bernoullium de exordio conquerentem quo nuper uſus ſum, in ſolutione problematis Leibnitiani in Tranſactionibus Philoſophicis editâ. Stylum acriorem reprehendit quàm virum benè moratum deceat, item nimium contemptum Extraneorum. Quæ liberius effatus ſum, hæc ſunt: " Si nondum viderint [fautores

3 Act. Leipſ. An. 1718. M. Jan. p. 18.

Leibnitii]

Leibnitii] quomodo ex illa " [ex anteriori nempe solutione generali] " æquationes sint deducendæ, id pro" fecto illorum imperitiæ tribuendum erit " [4]. Hæc fateor paulò durius sonant; sed si ad causam attendas contumelia vacant. Fautores Leibnitii (non omnes intelligo, sed Bernoullium tantum, & Socios ejus anonymos nobis infensos,) universos Anglos indignè tractarunt. Solutionem illam generalem cum non intelligerent, derisui habebant: In injuriosos & derisores me liberius explicui; contumelia non est. Sed ubi ille contemptus Extraneorum? Neminem ego nominatim citavi: De Fautoribus Leibnitii sum solùm locutus. Sed absit ut omnes designatione illa omnino intelligerem quocunque modo causæ Leibnitii faventes; tanquam ipse causæ Neutonianæ essem tam pertinaciter addictus, ut alios omnes odio habeam. Sed controversia ista Neutonum inter & Leibnitium nihil ad me. Solos intellexi Fautores illos qui in Anglos essent infensi, qui me nominatim calumnia provocarunt; Bernoullium iterum dico quem Principem agnovi causæ istius, sociosque ejus anonymos vel veros vel fictos. Hæc apertius dico, ne alii de nostra in alios contumelia immerito querantur. In immerentes injuria esset, in Bernoullium non est. Sed ad superiora illa redeo.

Plagii accusor, tanquam inventa Bernoullii, aliorum, usurpassem ut mea. Exempla proferat, dabitur responsum. Plura sanè tractavi cum aliis communia; sed inventis alienis sum minime usus ut meis. Propria ubique sum usus Analysi, (si Isoperimetrum excipias, de quo postea dicetur ;) ut nullo modo dici possit me alios fraudasse. At Autores nominasse oportebat, unde artem hauseram. Tanta me quidem tenet reverentia illustrium nominum, Hugenii, Hospitalii, Varignonii, Leib

4 Transf. Phil. N° 354.

nitii, aliorum, ut neſciam an ex hac parte non peccaverim, cum mihi ipſi deeſſe videar, cui tantos viros citaſſe ſemper fuiſſet ornamento. Nimia fortaſſe ignavia erat, quod de rebus cum eſſem maxime ſollicitus, hiſtorias rerum penitus neglexerim. Sperabam tamen me in tantæ fraudis ſuſpicionem incidere non potuiſſe, cum illuſtriſſima tantorum virorum opera eam facilè detegerent. Quæ cum Bernoullio communiter tractavi problemata, ſunt, de Funiculariâ, de Centro Oſcillationis, & de Iſoperimetris. In duobus primis ſum propria omnino uſus analyſi; in Iſoperimetro uſus ſum analyſi Autoris Jacobi Bernoullii, Viri à rebus Mathematicis optimè meriti, cui debitos nunc perſolvo honores. Solutio noſtra problematis de Centro Oſcillationis, cum amicis meis communicata eſt uſque ab initio Anni 1712. ut teſtes poſſum citare epiſtolas autographas Keillii noſtri; Liber item noſter erat penes Societatem Regiam, & cum omnibus fere noſtris Mathematicis communicatus, uſque à menſe Aprilis Anni 1714. quod hic monitu neceſſarium duxi, ne & Solutionem illam ſibi vindicet Bernoullius; cujus Solutiones [5] duæ extant eodem Anno editæ; quarum poſterior cum noſtra, quoad principia, tam mirè conſentit, ut jurares ab eodem homine eſſe utraſque inventas. Materia de Iſoperimetris excogitata primum eſt à Jacobo Bernoullio, ſicut jam innuimus. Ejus extat Solutio cum Analyſi, in Actis Lipſienſibus Anni 1701. Extat Analyſis fratris in Commentariis Regiæ Scientiarum Academiæ Anni 1706. Extat & Solutio in Libro noſtro. De eadem materia Commentarium nuper edidit Bernoullius in Actis Lipſienſibus Anni 1718. proximi [6].

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5 Altera in Act. Lipſ. M. Jun. In Comm. Reg. Sc. Acad. M. Aug. altera.
6 P. 16. & ſeq. Has igitur aliaſque ob rationes, actum agere minimè videbor, &c. p. 18.

Ibi,

Ibi, ne actum agere videatur, non meis ſolummodo, verum etiam fraternis ſolutionibus malevolus detrahere aggreditur; fratri prolixitatem [7], mihi obſcuritatem [8] objiciens. De novis illis inceptis nihil non magnum [9] pollicetur; & *ope cujuſdam principii, ab uniformitatis lege, quam nemo hucuſque obſervavit, petiti,* rem totam pene ſine calculo, nullo labore abſolvet. Sed neſcio quo fato fit, ut in hac materia de Iſoperimetris, Bernoullius Deos omnes ſemper offendat iratos. Nam primo, priſtina illa Analyſis ejus à capite ad calcem quaſi unum aliquod vitium maximum conſtituit: Secundo, quod tantum jactitat Principium, à lege uniformitatis, quam nemo hucuſque obſervavit (ſic enim ſtrenuus affirmat) petitum, à me olim obſervatum eſt: Denique quam hic tanquam novam exhibet Analyſin, tota mera fraterna eſt. Analyſin enim conſtituunt Præcepta, juxta quæ deinde inſtituitur calculus; qui non Analy-

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7 Nullos hic offendet Lector *ſcopulos*, quos objicit *operoſa* Fratris analyſis, hique differentiarum tertiarum *tricas ac ſpinas*, quibus *undique obſeptam* ibi ſentit viam, in noſtra methodo nullas percipiet.——Nec fratris calculi prolixitatem, nec Taylori obſcuritatem æque ingratam ac moleſtam ſibi metuendam habeat, *p.* 18——quam Frater per *operoſiſſimam* ſuam analyſin elicuit, *p.* 23.——non tantum ea, quæ à fratre meo quondam propoſita *magna pompa, nec minori conatu & labore* ſoluta fuere, ego ex ſola lege uniformitatis ſolvi citra calculum analyticum, &c.

8 *Vide Not. præced.* —— *item quæ ex* p. 18. *jam ſunt deſcripta.*

9 —— [illegible] confidam, *publicum et gratiam habituram*, quod occaſio mihi extiterit, talia nunc divulgandi, quæ fortè cum multis aliis in ſchedis meis perpetuò manſiſſent ſepulta, quamvis *reconditæ Geometriæ fines non parum prolatura*, p. 14. quod ibi ex incuria prætervifum reparabo hic *novo ſolvendi modo* qui ſingulari facilitate expedit problemata, non tantùm omnia quæ de Iſoperimetris propoſuerat Frater, ſed & innumera alia illis affinia, *ib.*—— ope cujuſdam principii ab uniformitatis lege, *quam nemo hucuſque obſervavit*, petiti, ex ſola Figuræ inſpectione, ac ſine ullo pene calculo æquationes pro curvis quæſitis ſponte velut ſe offerentes ſtatim eliciam, &c. ut in *Not.* 7. Actum agere minimè videbor, ſi in hoc argumento per ſe difficili viam monſtrem & rationem *brevem, planam, claram, & facilem*, qua quiſque mediocri quoque ingenio præditus ad veritates illas abſtruſiores (non fide aliorum, ſed) propriis oculis ſpectandas pervenire poſſit, ita nempe, ut, &c. *ut in Not.* 7.

ſis.

sis est, sed instrumentum Analyseos. Præceptis semel positis, quivis facilè calculum instituit, more quisque suo, hic prolixius, ille magis concinne, prout unicuique faverit Minerva. Negandum non est, Bernoullium calculum tandem concinnasse, & reddidisse elegantiorem; sed tamen in Analysi fraterna fecit, non sua: Nec dubitandum quin frater, adhuc si vixisset, rem reddidisset non minus illustrem Analysin diximus in præceptis contineri; præcepta verò sunt omnia fraterna. Nam quod curvæ quæsitæ arculum minimum, tanquam ex tribus lineolis elementaribus compositum contemplatur, vel ipso fatente [10] à fratre est: quod ex data longitudine arculi istius minimi quærit rationem differentiarum Ordinatarum in Lemmatis suis, à fratre est: quod rationem eandem denuo quærit, faciendo ut sit areola nascens, ex Functionibus (ut vocat) composita, vel maxima vel minima à fratre est: quod denique ex duplici illa expressione ejusdem istius rationis æquationem colligit qua curvæ quæsitæ natura definiatur, à fratre est. Sed hæc Solutionem constituunt. Ergo Solutio mera fraterna est. Dixi me olim usum esse Principio illo, quod tanta cum ostentatione sibi arrogat Bernoullius: Ex eadem una pagina, en duo exempla. In pagina 113. Libri mei hæc sunt "$\frac{\underset{\cdot}{m}}{\underset{\cdot}{R}}=\frac{m}{R}$.. Sed "est $\frac{\underset{\cdot}{m}}{\underset{\cdot}{R}}$ novus valor ipsius $\frac{m}{R}$, unde est $\frac{m}{R}$ quantitas "data". Luce clarius est me hoc loci ex observata

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10 Utar pro hoc, ut ipse fecit in sua Analysi, contemplatione arculi minimi, &c. *p.* 18.

uniformi-

uniformitate inter formulas, $\frac{\underset{,}{m}}{\underset{,}{R}}$, $\frac{m}{R}$, conclusisse quod sit $\frac{m}{R}$ quantitas data. Idem feci in sequentibus. " Pone " $\frac{m\underset{''}{n}}{n\underset{,}{R}} = \frac{\dot{m}}{R}$, hoc est $\frac{m\underset{''}{n}\underset{,}{n}}{\underset{\cdot}{R}} = \frac{\dot{m}\underset{\cdot}{n}\underset{,}{n}}{R}$, &c ". ubi ut uniformitas appareret inter formulas $\frac{m\underset{''}{n}\underset{,}{n}}{\underset{\cdot}{R}}$, $\frac{\dot{m}\underset{,}{n}n}{R}$, æquationem transformavi. Videtis, credo, quàm feliciter penetraverim ad *profundiora* Bernoullii. An hæc obscura dicet?

Ad primam jam partem promissi pervenio, ut ostendam pristinam illam Analysin Bernoullii esse omnino corruptissimam. Primo per substitutionem satis ridiculam, ex *profundioribus suis* nescio quibus petitam, æquationem $FO \times \Delta RO = \varphi\omega \times \Delta\rho\omega$ transformat in hanc $FO \times \Delta PF = \varphi\omega \times \Delta\pi\varphi$; quod in casu particulari (nempe quando functiones sunt ut quadrata ordinatarum) huc redit, ut sint simul $FO \times RO = \varphi\omega \times \rho\omega$ & $FO \times PF = \varphi\omega \times \pi\varphi$; unde consit $PF : RO :: \pi\varphi : \rho\omega$ Sed hoc impossibile est, quoniam est vel $PF \sqsupset RO \sqsupset \rho\omega \sqsupset \pi\varphi$, vel $PF \sqsubset RO \sqsubset \rho\omega \sqsubset \pi\varphi$; quorum neutrum cum analogia exposita conciliari potest. Nam si $PF \sqsupset RO \sqsupset \rho\omega \sqsupset \pi\varphi$, per Analogiam etiam erit $\pi\varphi \sqsupset \rho\omega$ (propter $PF \sqsupset RO$) contra hypothesin; vel si $PF \sqsubset RO \sqsubset \rho\omega \sqsubset \pi\varphi$, per Analogiam etiam erit $\pi\varphi \sqsubset \rho\omega$, contra hypothesin Secundo parum scienter fingit curvaturam in F esse ad curvaturam in φ sicut est φO ad FO; cum nihil in hac tota Analysi sit quod privilegium illud vindicet puncto O potius, quam alio cuilibet puncto ω in arculo minimo $FO\omega\varphi$ ubivis sumpto. Nec sanè Curvedo tam ridicule

dicule vult æstimari. Tertio nimis imperitè facit $mn = ddx$, $nl = ddy$, & $ml = \frac{dt\,ddy}{dx}$, cum sint $mn = \frac{1}{2} ddx$, $nl = \frac{1}{2} ddy$, & $ml = \frac{dt\,ddy}{2dx}$. Denique quod omnium pessimum est, vitiosissimis hisce principiis perfectissimam alligavit conclusionem. In problemate primo dico; nam in secundo est talium parentum magis digna proles. Errores Bernoullii veteres & exoletos me exposuisse putatis. Non ita est; ipse enim hæc habet: "*Omnia dudum inspecta accuratè rursus excutiendo ad severi examinis trutinam revocavi*[11] Notandum autem Solutionem *primi* problematis in schediasmate meo Commentariis Acad. *p.* 235 inserto. *rectissime se habere*"[12]. Errores ergo suos jam denuo adoptavit. Unde fortasse nunc quæret aliquis, Quo jure hic primas sibi in sublimiori illa Analysi tam obstinata ambitione arroget? Ut nemo sit qui in illa aliquid profecerit, quin continuo accusetur *ad profundiora Bernoullii penetrasse*[13]: Unde constet verum esse, quod quidam nuper affirmavit, regulas extantes in libro de Analysi infinitè parvarum à Bernoullio emanasse[14]? Quod laudes Excellentissimi Marchionis Hospitalii sint suo Præceptori tribuendæ? An hic sit idoneus qui alios docuerit *regulas differentiandi differentias*[15]? Cum multis aliis, quæ sigillatim enumerare non est opus. Sed istis respondeat qui volet: nos in hisce diutius non moramur.

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11 Pag 16. 12 Pag 17. 13 Pag. 18. *vide etiam Ep. pro Em. Math. & scripta ipsius Bernoullii passim.*

14 Concedit Dn. Marchionem de l'Hopital calculum istum intellexisse, nec ignorat, illustrissimum hunc virum eundem à Cel. Bernoullio didicisse: atque minime ipsum fugit, regulas in dicto libro [*de Analysi infinitè parvarum*] extantes à Cl. Bernoullio promanasse. *Act. Lips. An* 1718. *p.* 464.

15 Dum interea conjici potest, illum cum Dn. Newtono ab initio in isto errore hæsisse, donec tandem liberati fuissent usu *calculi differentialis, & regulas differentiandi differentias à Cel. Bernoullio edocti essent.* ib. p. 465.

Res

Res ipsas exposui. peroratione non utor; harum enim tædet. Nec si quidquam regesserit Bernoullius, ulterius respondere necesse habebo. A contumeliis nos semel vindicare & jus & ratio postulat; ulterius non expedit.

III. *An Account of the Impression of the almost Entire Sceleton of a large Animal in a very hard Stone, lately presented the* Royal Society, *from* Nottinghamshire. *By Dr.* William Stukely, *M. D. and* R. S. Soc.

HAVING an Account from my Friend, *Robert Darwin*, Esq; of *Lincoln's-Inn*, a Person of Curiosity, of an human *Sceleton* (as it was then thought) impress'd in Stone found lately at the Rev. Mr. *John South's*, Rector of *Elston* near *Newark*, *Nottinghamshire*, I was desirous of a Description of it, for the Entertainment of the ROYAL SOCIETY, and have at length procured the Stone it self for their Repository, where such remarkable Appearances are best preserv'd, and deservedly valued. It cannot but be matter of Regret, that so considerable a Rarity, the like whereof has not been observ'd before in this Island (to my knowledge) should be maim'd and imperfect, yet we may content our selves if enough be still visible to favour a Conjecture of what it has been. The Stone it self is a blue Clay Stone, the same as (and undoubtedly came from) the neighbouring Quarries of *Fulbeck*, or thereabouts, upon the Western Cliff of the long Tract of Hills extending quite through the adjacent County of *Lincoln*. It lay, time out of mind,

 at

at the side of a Well near the aforesaid Mr. *South's* Parsonage-House, where it had serv'd for a Landing-place to those that drew Water; but upon removal, the Under side exhibited this unusual Form, and was accordingly taken notice of by that worthy Gentleman, and laid up in his Garden for Curiosity-sake. Where the remaining part of the Stone, which contain'd the Upper-part and Continuation of the *Sceleton*, or that which was the other side, and tally'd with it, may be, is now utterly unknown: but upon view, I am perswaded, it cannot be reckon'd Human, but seems to be a *Crocodile* or *Porpoise*. There are Sixteen *Vertebræ* of the Back and Loyns very plain and distinct, with their Processes and intermediate Cartilages, Nine whole or partial Ribs of the Left-side, the *Os Sacrum, Ilium in situ*, and two Thigh-Bones displac'd a little, the Beginnings of the *Tibia* and *Fibula* of the Right-Leg; on one Corner there seem to be the *Vestigia* of a Foot with four of the five Toes, and a little way off an entire Toe, now left perfect in the Stone: there are no less than Eleven Joints of the Tail, and the Cartilages between them of a White Colour distinguishable from the rest. We should impose upon our Senses, to question, whether these be the real Reliques of an Animal; for the very Bones themselves are now to be seen as plainly, as if preserv'd in an *Egyptian* Mummy; a very little while ago, the Society had a Draught of a Crocodile, tho' a small one, found after the like manner inclos'd in Stone, from a Quarry in the Mountains of upper *Germany*. I suppose the same Reason accounts for both and all the rest of these kind of Fossils; and I please my self in an ocular Evidence, and so great a Confirmation of what I had the Honour to present to the *Royal Society*, in a late Discourse, where I hinted at a Solution of some obvious

rious and remarkable Phænomena, in the external Face of the Globe, consequent to its Formation, as set forth in the *Mosaic* Account; and of some Changes it suffer'd at the universal *Cataclysm*, and Proofs of that great *Catastrophe* of the animal and vegetable World in Plants, Shells, and Parts of living Creatures found in Rocks and Quarries.

Its remarkable, that all the Stone Pits about the Country whence this came, abound with prodigious Quantities of Shells, and the like, and the greatest part of the Substance of the Stone is a Composition of them. There are many Accounts of them in the *Transactions*, and this Stone has many Shells of different kinds in it. Sir *Hans Sloan* has a Fish-Sceleton, amongst his immense Treasure of Curiosities, found near this Place, given by the Duke of *Rutland*. If we look upon a Map of the Country, and observe the *Lincolnshire Alpes* which I spoke of before, how they run 50 Miles North and South, and on the West side are steep and rocky, we may see the Reason why these Quarries should be so stuft with them; for it is just to conceive, that upon retiring of the Waters of the Deluge from the Superficies of this Country, into the Eastern Seas, these heavy Bodies met a full stop, and were intercepted by this Cliff, which has retain'd such vast Quantities of them ever since: whilst those which fell upon common Mold are mostly rotten, and now lost.

Sir *Isaac Newton*'s Doctrine of the Attraction of the Particles of Matter, according to the Quantity of its Solidity, Proximity, and Surface, especially that it is infinitely greater in the point of Contact, upon which depends its Cohesion and all the Varieties of Physical Action, will easily direct us to a Notion of Petrifaction. We learn how a proper Degree of Heat or Cold,

Moisture, Motion, Rest and Time, promote this Principle, from the common Experiments of Chrystallization and Freezing even before the Fire, and in many Chymical Mixtures. Whence we cannot be ignorant of Stone growing in the Quarries gradually, not by any fancied Vegetation, tho' there is something like it in Corals, but generally by Apposition of Parts to Parts, as is notorious in the *Fluors* of subterraneous Grotts and Caverns. So that we have no reason to doubt but what was Clay, Sand, or Earth 3000 Years ago, may now be Stone or Marble, according to the Proportion of Concurrence of such mentioned Causes. This will persuade us that the now barren and rocky Plains of the Countries of *Syria*, *India*, and *Arabia*, are owing to Natural Causes, as well as an immediate Curse of God for the Disobedience of its ancient Possessors his peculiar People, because the same is observable of the famous Countries of *Greece* and *Africa*, warm Regions so renowned for Fertility in antient Authors. Wherefore there may be some likelyhood in the Opinion of those who think that in many Ages the whole Face of the Globe may become one great Rock. Dr. *Plott*, in his Natural History of *Oxfordshire*, gives an Account of a *Tumulus*, now a perfect Mount of Stone: and upon St. *Vincent*'s Rock near *Bristol* are Fortifications now become solid Cliff. I remember, about six Years ago, Mr. *Ralph Widdrington*, Brother to the Earl of that Name, shew'd me many human Bones taken from whole Sceletons, with Brittish Beads, Chains, Iron Rings, Brass Bitts of Bridles, and the like, which were dug up in a Quarry, near the Seat of the Family, at *Blankney*, *Lincolnshire*; which very probably was plain Mold when these old Corpses of the *Britons* were interr'd; and since then I saw many human Bones and Armour, with *Roman* Coins, *Fibulæ*, &c. found in a Stone-

a Stone-pit in the Park at *Hunstanton, Norfolk*; belonging to Sir *Nicolas L'Estrange*, in whose Custody they now are, which were conjectured to have been buried in Earth after a Battle. From whence we may judge it a vulgar Mistake, when in the Ruins of old Castles and Walls, we admire the Tenacity of the Mortar, and are apt to praise our Ancestors, for an Art which we suppose now lost; when doubtless the Strength of the Cement is owing to the Length of Time: and in future Ages our Modern Buildings may obtain the same Judgment.

From all which Instances, I only desire to infer the antient state of these Cliffs, where this Sceleton was, and Shells are daily found, intimately mixt in the Substance of the Stone, to have been formerly of a softer Consistence, capable of admitting them into its Bowels, and to have immur'd them as part of it self; and that Earth which is now manageable by the Plough, may possibly in time assume the same Density, at least very little below the Surface; for in this very Cliff the upper *Strata* are yet Clay, growing harder as deeper. What Creature this has been, for want of a Natural History of Sceletons, well worthy the Endeavours of this Society, we cannot positively determine; but generally find the like to be amphibious or marine Animals. Why such rather than many others, should chance to be thus entomb'd, may be thought, because they were able much longer than Terrestrial Animals to live in that World of Waters, even till they began to abate and fall away into their destin'd Receptacles; so that while the Bodies of the rest soon perishing, were corrupted, and their Bones separated and dispers'd much earlier; this Sceleton, with others of its like, fell entire into the Fissures of this Bed of Clay, which has since turn'd into Stone, and made this noble Monument and

pregnant

pregnant Token of that general Inundation, durable as the vain-glorious *Egyptian* Monarchs Pyramids at *Memphis*; to be perpetuated in the lasting Records of this Society. *See the Figure of this Impression, in Tab.I.*

IV. *A curious Description of the* Strata *observ'd in the* Coal-Mines *of* Mendip *in* Somersetshire; *being a Letter of* John Strachey *Esq; to Dr.* Robert Welsted, *M. D. and* R. S. Soc. *and by him communicated to the Society.*

I Now send you the Observations which I sometime since promised you, relating to the different *Strata* of *Earths* and *Minerals* found principally in the Coal-Mines in my Neighbourhood. For the better Illustration whereof, I have inclosed a Draught, which you must suppose the Section of a Coal-Country, and to take in about Four Mile from the North-West to South-East, and may be applied to the Veins of Coal as they lye at *Faringdon-Gourny*, and likewise at *Bishop-Sutton*, which last Place is near *Stowy*, but in the Parish of *Chew Magna* in this County of *Somerset*. For Discovery of Coal, they first search for the *Crop*, which is really Coal, tho' very friable and weak, and sometimes appears to the Day, as they term it; or else for the *Cliff*, which is dark or blackish Rock, and always keeps its regular Course as the Coal does, lying obliquely over it. For all Coal lies shelving like the Tyle of a House, not perpendicular nor horizontal, unless it be broken by a *Ridge*; which is a parting of Clay, Stone, or Rubble; as if the Veins by some violent Shock were disjointed and broken, so as to let in

in Rubble, *&c.* between them. The Obliquity or *Pitch*, as they term it, in all the Works hereabout, is about 22 Inches in a Fathom; and when it riseth to the Land is called the *Crop*, but in the North *Basseting*. In the Works near *Stowy*, and likewise at *Faringdon* it riseth to the North West, and pitcheth to the South East; but the farther they work to the South West, the *Pitch* enclines to the South; and *è contra*, when they work towards the North East. So likewise they observe as they work to the South West, when they meet with a *Ridg* it causeth the Coal to *trap up*, that is, being cut off by the *Ridg*, they find it over their heads, when they are thro' the *Ridg*: but on the contrary, when they work thro' a ridg to the *North East*, they say it *traps down*, that is, they find it under their feet.

Coal is generally dug in Valleys or low Grounds. The Surface in these parts is mostly a red Soyl, which under the first or second Spitt degenerates into *Malm* or *Loom*, and often yields a Rock of Reddish *Firestone*, till you come to four, five, and many times to twelve or fourteen Fathom depth, when by degrees it changeth to a Gray, then to a Dark or Blackish Rock, which they call the *Coal Clives*. These always lye shelving and regular as the Coal doth. But in these parts they never meet with *Firestone* over the Coal, as at *Newcastle* and in *Staffordshire*. These *Clives* vary much in Hardness, in some places being little harder than *Malm* or *Loom*, in others so hard as that they are forced to split them with Gunpowder: So likewise in Colour, the top inclining to red or grey, but the nearer to Coal the blacker they grow; and wheresoever they meet with them they are sure to find Coal under them. But to their disappointment 'tis not always worth the the digging. The first or uppermost Vein at *Sutton*

is

is called the *Stinking Vein*. It is hard Coal fit for Mechanick uses, but of a sulphurous Smell. About five Fathom and half, seldom more than seven Fathom under this, lyes another Vein, which from certain Lumps of Stone mixt with it like a *Caput mortuum* not Inflamable, called *Cats-head*, they ca l the *Cathead Vein*. About the same Depth under this again lyes the *Three Coal Vein*, so called because it's divided into three different Coals; Between the first and second Coal is a Stone of a foot, in some places two feet thick; but the middle and third Coal seem placed loose on each other, without any separation of a different Matter. These three Veins before-mentioned are sometimes work'd in the same Pit: But the next Vein which I am going to mention is generally wrought in a separate Pit; for tho' it lyes the like depth under the other, the *Cliff* between them is hard and subject to Water; wherefore I have represented a Pit sunk thro' the three Upper Veins at A. and another sunk upon the *three Coal Veins* only at B. and so if they sink on any of the lower Veins they go more to the North West. *See Fig. Tab.* II.

Next under the *three Coal Veins* is the *Peaw Vein*, so denominated because the Coal is figured with Eyes resembling a Peacock's Tayl, gilt with Gold, which Bird in this Country Dialect is called a *Peaw*. The *Cliff* also over this Vein is variegated with *Cockle-shells* and *Fern Branches*, and this is always an Indication of this Vein, which, as I before hinted, is always searched for about 15 Fathom to the North West of the former.

Under this again between five and six Fathom lies the *Smith's Coal Vein*, about a yard thick; And near the same depth under that again the *Shelly Vein:* And under that a Vein of 10 Inches thick, which being little valued, has not been wrought to any purpose.

Some say there is also another under the last, but that

that has not been proved within Man's Memory. At *Faringdon* they have the same Veins, which, as I am informed, agree in all Parts with those of *Bishop-Sutton* before-mention'd. But as *Faringdon* lies four Miles South-East from *Bishop-Sutton*, so, in the regular Course, they would lye a Mile and ½ deeper than those at *Sutton*. But as in fact they are dug near the same Depth, it follows there must be a *Trap*, or several *Traps down*, which in all must amount to that Depth between the said Works.

Between *Faringdon* and *High-Littleton* the same Veins seem to retain their regular Course; but at *Littleton* their undermost and deepest Vein is the best Coal, which at *Faringdon* proves small.

On the other hand, in the Parish of *Stanton-Drew*, to the North-East of the Coal-Works at *Sutton* aforesaid, about a Mile distant, and in the true Course with those at *Sutton*, the same Veins are found again. But here they wind a little, and their Course or Drift runs almost North, and they dip to the East; which Winding is attributed to *Ridges*, which the Workmen have met with on both Sides, and have occasion'd them to discontinue the Work that way. At *Stanton* they have little of the Red Earth or *Malm* on the Surface, but come immediately to an *Iron-Gritt* or *grey Tile-Stone*, which is a Fore-runner of the *Coal-Clives*; in all other Matters they agree with the Works near *Stowy*.

In the same Parish of *Stanton-Drew*, a little to the Eastward, they have another Coal-work, but the Veins are in all respects different from the former. Their Drift or Course is to the Eleven a-Clock Sun, as they term it, they *Pitch* to the Five a Clock Morning, and rise to land; consequently to the Five a-Clock Evening-Sun. They have several Veins, but as yet only three are thought worth working. The uppermost about three

L111111 Feet

Feet thick ſmall *Lime-Coal*. The next is about three Fathom under it, about two Feet and an half thick, fit for culinary Uſes: the undermoſt is about the like Depth under the former, only 10 Inches thick, but good hard Coal.

At *Clutton*, about two Mile from theſe latter, in the ſame Drift, *viz.* almoſt to the South Eaſt and by South, theſe laſt Veins appear again. The Surface here is red, and ſo continues to ten, and ſometimes to fourteen Fathom, and in other reſpects agree with the laſt-mention'd Works at *Stanton-Drew*.

At *Burnet*, *Queen-Charlton*, and *Briſleton*, they have Four Veins which Pitch to the North nearly, and conſequently the Drift lies almoſt Eaſt and Weſt. The Surface is red land generally to the Depth of four or five Fathom. The uppermoſt is from three to ſix Feet thick at *Briſleton*, but leſs at *Charlton* and *Burnet*. The next, call'd *Pot-Vein*, is ſix Fathom under the former, eighteen Inches thick, all hard Coal. *Thirdly*, The *Trench-Vein*, 7 Fathom under the other, which is from two Feet and half to three Feet thick, all ſolid Coal. *Fourthly*, *Rock-Vein*, always diſtinguiſh'd by a Rock of Paving-Stone, call'd *Penant*, lying over it, which Rock is ſometime twenty Feet thick, or more, and therefore this Vein is never wrought in the ſame Pit with the former Vein, but about 200 Yards more to the South, or to Land, as they term it. It's computed ſeven Fathom under the former.

This is all I can ſay in relation to the different Veins of Coal and Earth in the Coal-works in theſe Parts; wherein all agree in the Oblique Situation of the Veins; and every Vein hath its *Cliff* or *Clives* lying over it, in the ſame oblique manner. All of them Pitch or Riſe about Twenty two Inches in a Fathom, and almoſt all have the ſame *Strata* of Earth, Malm, and

and Rock over them, but differ in respect to their Course or Drift, as also in Thickness, Goodness, and Use.

Now as Coal is here generally dug in Valleys, so the Hills, which interfere between the several Works before mentioned, seem also to observe a regular Course in the *Strata* of Stone and Earth found in their Bowels: For in these Hills (I mean those only that are dispers'd between the Coal-Works above mention'd) we find on the Summits a stony Arable mixt with a spungy yellowish Earth and Clay; under which are Quarries of *Lyas*, in several Beds, to about eight or ten Feet deep, and six Feet under that thro' yellowish *Loom*, you have a blue Clay enclinable to *Marle*, which is about a Yard thick: Under this is another Yard of whitish *Loom*, and then a deep blue Marle soft, fat, and soapy, six Feet thick; only at about two Feet thick, it is parted by a *Marchasite* about six Inches thick. But as this swells beyond the Bounds of a Letter, I must defer the farther Description of these and some *Lead-Mines* to another Opportunity; only 'tis to be noted, that these Beds of *Stone* and *Marle*, different from *Coal*, lie all Horizontal.

Your humble Servant,

John Strachey.

V. *Some Instances of the very great and speedy Vegetation of* TURNIPS. *Communicated by the Rev. Dr.* J. Theoph. Desaguliers, *R. S. S.*

AT *Sutton Coldfield* in *Warwickshire*, a peaty Ground near a Pool (of which it was formerly a part) was sown with *Turnip-Seed* on the 2*d* Day of *July* 1702. In less than Three Days Time the Turnips were seen above Ground. At Three Weeks end the Roots were in Bigness equal to Walnuts. Within less than Five Weeks after the Sowing, the Gardener drew great Quantities of Turnips to sell, they then being as big as large Apples. At the end of Six Weeks, *viz.* on the 12th Day of *August*, a large Turnip was plucked up (though probably not so big as several others then growing upon the same Ground) which, together with its Top and long descending part of the Root, weighed above Two Pounds and Fourteen Ounces. At the same time also was weighed an Ounce of the same sort of Turnip-Seed, that the Gardener had sown his Ground with; and afterwards a Thousand of the Grains were counted singly out of the Ounce so weighed; and the rest of the Ounce was divided into Heaps, as near as could be guessed, equal to the 1000 Seeds first severed and laid together: And it was found that the whole Ounce contain'd above 14600 single Grains; which Number multiplied by 46 (*viz.* the Number of Ounces that the Turnip weighed) produceth 671,600, *viz.* the Number of single Grains of Seed required to equal the Weight of the Turnip. From whence may be gathered, that (upon supposition, that the Increase of the Turnip was all along uniform and equal, from the Time

Time it was ſown till it was pluck'd up) the Grain of Seed which it ſprung from, weighing when it was ſown but $\frac{1}{14600}$ of an Ounce, was increaſed in Weight according to the following Proportions, *viz.*

In Six Weeks time	—	671,600	times its own Weight.
Every	Week —	111,933 $\frac{1}{3}$	
	Day —	15,990 $\frac{1}{2}$	
	Hour —	666 $\frac{1}{4}$	
	Minute of an Hour —	11	

Some time after, another Ounce of the ſame ſort of Seed was exactly weighed, and the Grains were found to be in Number 14673.

Another Turnip of the ſame Crop was plucked up on the 21*ſt* Day of *October*; and being put into a Scale, was found to weigh above Ten Pounds and an half; which unuſual and truly wonderful Bulk it acquired (it being ſuppoſed, as above, that the Growth was all along alike) by increaſing the Weight of the Seed it was raiſed from, 15 times in every Minute of an Hour from the Sowing to the Drawing of it.

The Gardener neglected to thin his Turnips in due Time, elſe probably their Growth had been more conſiderable.

At another Time, in two other ſorts of Turnip-Seed, it was found by counting, that an Ounce of one ſort contained 14702 Grains; and an Ounce of the other ſort no fewer than 14905 Grains.

It's credibly reported, that of late Years, Turnips have been pretty frequently found growing in ſeveral Counties of this Kingdom, that have weighed above twice as much; one of which was ſeen at *Birmingham* about the Year 1710.

VI. *An*

VI. *An Account of some Experiments tried with Monf.* Villette's *Burning Concave, in* June 1718. *By the Rev. Dr.* J. Harris *and Dr.* J. T. Desaguliers, *Reg. Soc. SS.*

THIS Miroir is a Concave 47 Inches wide, and ground to a Sphere of 76 Inches *Radius*; so that its *Focus* is about 38 Inches distant from the *Vertex* of the Glass. The Metal of which it is made is a Mixture of Copper, Tin and Tin-Glass, whose Reflexion has something of a yellow Cast. The Concave-Surface has scarce any Flaws, and those very small; but the Convex side, which is also polish'd, has some Holes in it.

Having held several Bodies in the *Focus* of this Miroir, we observ'd what happen'd to them whilst expos'd to this great Heat; and with an half second Pendulum took notice of the Time in which any material Change happen'd to them.

The Experiments were as follow, and made from Nine till Twelve in the Morning.

N° 1. A red piece of a *Roman Patera*, which began to melt in 3 Seconds, was ready to drop in 100.

2. Another black Piece melted at 4, and was ready to drop at 64 Seconds.

3. Chalk taken out of an *Echinus Spatagus* fill'd with Chalk only, fled away in 23 Seconds.

4. A Fossile-Shell calcin'd in 7 Seconds, and did no more in 64.

5. A piece of *Pompey*'s Pillar at *Alexandria*, was vitrified in the Black Part in 50 Seconds, and in the White part in 54.

6. Cop-

6. Copper-Oar, that had no Metal in it visible, vitrified in 8 Seconds.

7. Slag, or Cinder of the ancient Iron-work said to have been wrought by the *Saxons*, ready to run in 29 Seconds and an half.

Here the Glass growing hot, burn'd with much less Force.

8. Iron-Oar fled at first, but melted in 24 Seconds.

9. Talk began to calcine at 40 Seconds, and held in the *Focus* 64.

10. *Calculus humanus* in 2 Seconds was calcin'd, and only dropp'd off in 60.

11. An anonymous Fish's Tooth melted in 32 Seconds and an half.

12. The *Asbestos* seem'd condens'd a little in 28 Seconds; but it was now something cloudy: Monf. *Villette* says that the Glass usually calcines it.

13. A Golden Marchasite broke to pieces, and began to melt in about 30 Seconds.

14. A Silver Sixpence melted in 7 Seconds and an half.

15. A King *William*'s Copper Halfpeny melted in 20 Seconds, and ran with an Hole in it in 31.

16. A King *George*'s Halfpenny melted in 16 Seconds and ran in 34.

17. Tin melted in 3 Seconds.

18. Cast Iron in 16 Seconds.

19. Slate melted in 3 Seconds, had an Hole in 6.

20. Thin Tile melted in 4 Seconds, had a Hole and was vitrifi'd thro' in 80.

21. Bone calcin'd in 4 Seconds, and vitrifi'd in 33.

An Emerald was melted into a Substance like a Turquois Stone.

A Diamond weighing 4 Grains lost $\frac{7}{8}$ of its Weight.

VII. *An*

VII. *An Account of the Extraordinary* METEOR *seen all over* England, *on the 19th of* March $171\frac{8}{9}$. *With a Demonstration of the uncommon Height thereof. By* Edm. Halley, *LL. D. and Secretary to the* Royal Society.

THIS wonderful luminous *Meteor* which was seen in the Heavens on the 19th of *March* last, as it was matter of Surprize and Astonishment to the Vulgar Spectator, so it afforded no less Subject of Enquiry and Entertainment to the speculative and curious in Physical things: Some of its *Phænomena* being exceeding hard to account for, according to the Notions hitherto received by our Naturalists; such are the very great Height thereof above the Earth; the vast Quantity of the Matter thereof; the extravagant Velocity wherewith it moved; and the prodigious Explosions thereof heard at so great a Distance, whose Sound, attended with a very sensible Tremour of the subject Air, was certainly propagated through a *Medium* incredibly rare and next to a *Vacuum*.

In *Num.* 341. of these *Transactions*, I have collected what I could find of such-like *Meteors*, and since, turning over the *Ephemerides* of *Kepler*, I accidentally hit upon another, prior to all those there described, and which was seen all over *Germany*. Of this the Words of *Kepler* are: *Die* $\frac{1}{11}$ *Nov.* 1623. *Meteorum ignitum, Globus ardens ab occasu in ortum volans totâ passim* Germaniâ *fuit conspectus. In* Austria *etiam fragorem exauditum affirmarunt quasi à fulmine; quod vanum tamen puto: nihil enim tale confirmant descriptiones quæ extant.* Yet

* neither

neither this, nor any of the other hitherto described, seem to come up in any Circumstance to this late Appearance; of which I am in hopes to give a satisfactory Account, being enabled by the very many Relations thereof communicated to the *Royal Society*, from most parts of the Kingdom; tho' it was not my good Fortune to see it my self; and tho' very few of our Countrymen who best know the *Stars*, had better luck. Some of the most perfect Descriptions we have receiv'd are the following:

First, Our very worthy Vice-President Sir *Hans Sloan*, Baronet, being abroad at that time, happen'd to have his Eyes turned towards it, in its very first Eruption; and the next Day he was pleased to give me in Writing what he had with great Exactness noted about it, in the Terms following: " On *Thursday*, *March* 19. 17$\frac{18}{19}$. " passing along Eastward by the N. E. Corner of *Sou-* " *thampton-street* in *Bloomsbury-Square*, *London*, at about " a Quarter after Eight at Night, I was surpriz'd to " see a sudden great Light, much beyond that of the " Moon, which shone then very bright. I turn'd to " the Westward where the Light was; which I appre- " hended at first to be artificial Fire-works or Rockets. " The first place I observ'd it in, was about the *Plei-* " *ades* Northerly, whence it moved after the manner, " but more slowly than a falling Star, in a seeming " direct Line, descending a little beyond, and withal " below, the Stars in *Orion*'s *Belt* then in the S. W. " The long Stream appear'd to me to be branched about " the middle, and the *Meteor* in its way turn'd Pear- " fashion'd or tapering upwards. At the lower end it " came at last to be bigger and Spherical, tho' it was " not so big as the Full Moon. The Colour of it was " whitish, with an eye of Blue, of a most vivid daz- " ling Lustre, which seem'd in Brightness very nearly

"to resemble, if not surpass that of the Body of the "Sun in a clear Day, beheld by the naked Eye. This "Brightness obliged me to turn my Eyes (which had "their Pupils adapted to the Light of the Moon) from "it several times, as well when it was a Stream, as "when it was Pear-fashion'd and a Globe; tho' I had "a great Curiosity to observe it with Attention. It "seem'd to move in about half a Minute or less, about "the Length of 20°, and to go out, as I guess'd, about "as much above the Horizon. There was left behind "it, where it had pass'd, a Track of a cloudy or faint "reddish Yellow Colour, such as red-hot Iron or glow-"ing Coals have, which remained more than a Minute, "seem'd to sparkle, and kept its Place without falling. "This Track was interrupted, or had a Chasm towards "its upper end, at about two Thirds of its Length. I "did not hear any Noise it made, but the place where "the Globe of Light had been, remain'd after it was "extinct, of the same reddish Yellow Colour with the "Stream for some time, and at first some Sparks seem'd "to issue from it, such as come from red-hot Iron bea-"ten on an Anvil. The Surprize, Brightness of the "Light, and Noise of the People upon the Variations "of the Appearance, calling to one another to observe "what they never had observ'd in their Days, and "thought to be prodigious, hinder'd me from taking "notice or remembring any thing farther about it.

It were to be wisht that Sir *Hans* had more especially regarded the Situation of the Track of this Meteor among the fixt Stars, and let us know how much it past above the *Pleiades*, and how much under the *Belt* of *Orion*, that so we might with more Certainty have determin'd its Position in respect of the Horizon of *London*; for which purpose the whole Number of Spectators there has not furnished us with one sufficient

Observa-

Obſervation. But all the Relations, however otherwiſe differing, agree in this, that the Splendour was little inferior to that of the Sun; that within doors the *Candles* gave no manner of Light, and in the Streets not only all the *Stars* diſappear'd, but the *Moon* then Nine Days old, and high near the Meridian, the Sky being very clear, was ſo far effaced as to be ſcarce ſeen, at leaſt not to caſt a Shade, even where the Beams of the Meteor were intercepted by the Houſes: ſo that for ſome few Seconds of Time, in all reſpects it reſembled perfect Day.

The Time when this happen'd was generally reckoned at a quarter paſt Eight; but by the more accurate Account of the Rev. Mr. *Pound* (who only ſaw the Light) agreeing with what has been ſent us from the *Pariſian* Obſervatory, it appears to have been at $8^h\ 8'$ apparent Time at *London*. And the Sun being then in $9\frac{1}{2}$ *gr.* of *Aries*, the Right Aſcenſion of the Mid-Heaven was 130 *gr.* 45′, whereby the Poſition of the Sphere of fixt Stars is given. Hence the *Lucida Pleiadum* will be found at that time to have been $25\frac{1}{4}$ *gr.* high, in an *Azimuth* 6 *gr.* to the Northward of the Weſt, and conſequently the Arch the Meteor moved in, was inclined to the Horizon with an Angle of about 27 *gr.* having its Node or Interſection therewith, nearly *South South Weſt*; as will be more evident by what follows.

At *Oxford* five Minutes earlier, Mr. *John Whiteſide*, *R. S. Soc.* Keeper of the *Aſhmole Muſeum*, and very skilful in both Mathematical and Phyſical Matters, immediately after the Extinction of the Meteor, made haſte out to ſee what it might be, and well conſider'd the Situation of the Track it had left in the Sky: He found it to have paſt about $1\frac{1}{2}$ Degree above the preceding Shoulder of *Orion*, and about $3\frac{1}{2}$ *gr.* above

the middle of his *Belt*, where there appear'd a luminous *Nubecula* of a reddish Light, being a Dilatation of the Track, seeming to have been occasion'd by some Explosion there; and by what he could learn from those that saw it, it was thereabout that it broke out, and first began to efface the Stars. Hence it proceeded as to sense in an Arch of a great Circle, and passing in the middle between the Tail of *Lepus* (θ *Bayero*) and β in the Fore-Foot of *Canis major*, it terminated about ξ in the Breast of the same, nearly in 95 *gr.* of Right-Ascension, with 23 *gr.* South Declination: and at the place of its Extinction there remained a large whitish *Nebula*, much broader and of a stronger Light than the rest of the Track, which he took for a certain Indication of a very great Explosion made there. By Computation it will be found that the Angle this Track made with the Horizon of *Oxford* was nearest 40 *gr.* and its Intersection due *S S W*; and that the place of its Extinction was about 9 *gr.* above the Horizon, in the Azimuth of 32 *gr.* to the *West*.

At *Worcester* Mr. *Nicolas Fatio*, a Person greatly skill'd in Astronomical Affairs, saw this Meteor descend obliquely towards the *South*, making an Angle with the Horizon of about 65°, and intersecting it about *S S W* $\frac{1}{2}$ *S*, as may be collected from a Scheme thereof sent up by him, and communicated to the *Royal Society*, seeming to be design'd with sufficient Exactness. By this the Track left all *Orion* and *Canis major* to the Westward, and divided the Distance between *Sirius* and *Procyon*, so as to be almost twice as far from *Procyon* as *Sirius*. The Time here was one Minute before Eight, this City being about 9′ of Time to the West of *London*, and consequently the Right-Ascension of the Mid-Heaven 128 $\frac{1}{2}$ *gr.*

Now

Now the Situation of the three Cities *London*, *Oxford*, and *Worcester* being nearly on the same *W.N.W.* Point, whereon the Track of the Meteor had its greatest Altitude above the Horizon, equal to the Angle of its visible Way; if we suppose it at *London* to have been 27 *gr.* high, and at the same time at *Worcester* to be 65 *gr.* high, in the Plane of the Vertical Circle passing through *London* and *Worcester*; supposing likewise the Distance between them to be 90 Geographical Miles, or one Degree and half of an Arch of a great Circle of the *Earth*, we shall by a Trigonometrical *Calculus*, too obvious to be here inserted, find the perpendicular Height to have been 64 such Miles; and the Point over which it was then perpendicular to have been 30 such Miles *W.N.W.* from *Worcester*. And the Geographical Mile to the *English* Statute Mile being as 23 to 20, this Height will be no less than 73 $\frac{1}{2}$ *English* Miles. The place also directly under it, will be found to be about *Prestain* on the Confines of *Hereford* and *Radnor*-Shires. Nor can we be much out in this Determination, the *Oxford* Observation concurring nearly in the same Conclusion.

This Altitude being added to the Semidiameter of the Earth as *Radius*, becomes the *Secant* of Eleven Degrees, so that the Meteor might be seen above the Horizon in all Places not more than 220 Leagues distant from it. Whence it will not be strange that it should be seen over all Parts of the Islands of *Great Britain* and *Ireland*, over all *Holland* and the hither Parts of *Germany*, *France* and *Spain*, at one and the same instant of Time.

This suggests a very great use that might be made of these momentaneous *Phænomena*, to determine the *Geographical Longitudes* of Places. For if in any two Places two Observers, by help of *Pendulum Clocks* duly corrected

by

by Cœleſtial Obſervation, do exactly note at what Hour, Minute and Second ſuch a Meteor as this blows up and is extinguiſht, the Difference of thoſe Times will be the Difference of Longitude of the two Places, as is well known. Nor does it require ſo much as the Uſe of a *Teleſcope*, as in the Methods hitherto put in practice for that purpoſe; ſo that if theſe Appearances could be predicted, and Notice given of their coming, that we might know when to expect them, I ſhould make no Difficulty to prefer this way of ſettling the *Geography* of a Country before all others.

Having thus fixt one Point in the Line of its Motion, let us now conſider what courſe the Meteor took from thence. And firſt at the Town of *Kirkby-Stephens*, on the Borders of *Yorkſhire* and *Weſtmoreland*, in a Meridian very little to the Weſtward of *Worceſter*, but about $2\frac{1}{2}$ *gr.* more to the North, it was obſerved to break out as from a dusky Cloud, directly under the Moon, and from thence to deſcend, nearly in a Perpendicular, almoſt to the Horizon. Now the Moon, being at that time in the third Degree of *Leo*, was about half an hour paſt the Meridian, and conſequently much about a point to the Weſt, or *S b W*: and the Situation of *Preſtain* from *Kirby-Stevens* being ſufficiently near upon the ſame Point, it follows that the Direction of the Track of the Meteor was according to the Great Circle paſſing over thoſe two Places.

And this is further confirm'd by the Obſervation of *Sam. Cruuys*, Eſq; *Reg. Soc. Soc.* who at *Tiverton*, about twelve Geographical Miles nearly due North from *Exeter*, obſerved the firſt Exploſion of this Meteor exactly in his *Zenith*, as he was aſſured by applying his Eye to the ſide of his Door, which he took to be perpendicular, and looking upwards: And from thence he ſaw it deſcend to the Southwards directly in the ſame

* Azi-

Azimuth, without declining either to the Right or Left: Hence it is plain, that the Track likewise pass'd over this place, which by our best Maps is found to lie in a Line with *Prestain* and *Kirby-Stevens* with sufficient Exactness; so that we shall take it for granted that this was the very Course it held.

On this Supposition, that the first Explosion attended with the reddish *Nubecula*, was directly over *Tiverton*, we have the *Oxford* Observation to compare with it, in order to determine more nicely the perpendicular Altitude there. At *Oxford* this *Nubecula* was found to be $3\frac{1}{2}$ *gr.* above the middle Star of *Orion's Girdle*, at $8^h\ 3'$, and was therefore $26\frac{1}{2}$ *gr.* above the Horizon; and the Distance between *Oxford* and *Tiverton*, being $1^\circ.\ 55'$ or 115 Geographical Miles, it will be as the Sine of $61^\circ.\ 35'$ to the Sine of $63^\circ.\ 30'$ So the Semidiameter of the Earth being $3437\frac{1}{2}$ such Miles, to 3498 Miles the Distance of the Meteor from the Center of the Earth; from which deducting the Semidiameter, there remains $60\frac{1}{2}$ Geographical Miles for the Height of the Meteor above *Tiverton*: And that this was so is confirmed by the Observation of the Rev. Mr. *Will. Derham*, who at *Windsor* saw the aforesaid *Nubecula* about two Degrees above the most Southerly of the Seven Stars in the Shield of *Orion*; that is (the Time being $8^h\ 6'$) in the Altitude of $23\frac{1}{4}$ *gr.* whence, the Distance between *Tiverton* and *Windsor* being 150 measured Miles, or 130 Geographical, by a like Proportion we shall find the same Height of the Meteor 60 such Miles wanting only one Quarter. So that in a round Number we may conclude it to have been just 60 Geographic or 69 Statute Miles above the Earth's Surface. Nor is it possible to come at a precise Determination of this matter, by reason of the Coarseness and Inaccuracy of our *Data*, which were only

only the Notes of Persons under the Surprize of the suddenness of the Light, and no ways pretending to Exactness; however, such as they are, they abundantly evince the Height thereof to have exceeded 60 *English* Miles, not to say 38 or 40, as some would fain have it.

I was unwilling to leave off, till I had pitcht upon some Hypothesis that might subject the Motion of this Meteor to a *Calculus*, that the Curious might be able to compute the visible way thereof, either in respect of the Horizon, or among the fixt Stars: This I found might be done with tolerable Exactness, supposing that it mov'd in the Arch of a Circle concentrick with the Earth, but 60 Geogr. Miles without it; and that the Point of the first Explosion was over the Lat. of 50° 40′ and 3° 40′ to the West of *London*; and that of the last Extinction over Lat. 47°. 40′ with 4°. 50′ *West* Longitude: The Time being fixt to 8 Minutes past Eight at *London*. Hence it will be easy, by a Trigonometrical Process, to obtain the visible Altitude and Azimuth of the Meteor at either of its Explosions, as seen from any Place whose Longitude and Latitude is known; and from the Time given, the Points in the Sphere of Stars answering to those Azimuths and Altitudes are readily deduced. Let those that contend for a much less Height of this Meteor try if they can on such their Supposition reconcile the several *Phænomena* before recited with one another, and with the Observation of the Rev. Mr. *William Ella*, Rector of *Rampton* in *Nottinghamshire*, between *Gainsborough* and *Redford*, which for its Exactness I must not omit. Here at 8^h 5′ the Meteor was seen to pass precisely in the middle between *Sirius* and the Fore-Foot of *Canis major*, moving obliquely to the Southward, in a Line whose Direction seem'd to be from the middle between

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the two Shoulders of *Orion*. The Latitude of the place being nearly 53°. 20′, and Longitude *West* from *London* 0°. 45′. Let them try how they can account for its being seen five Degrees high at *Aberdeen* in *Scotland*, and near as much at *Peterhead* half a Degree more Northerly: and then they will be better able to judge whether it did not exceed the reputed Limits of our Atmosphere. Lastly, if the apparent Altitude of the Meteor at *Paris* was not $5\frac{1}{2}$ but 11 *gr.* on the *W b N* Point, when it must have been in its greatest Lustre, there will be no pretence to bring it lower than I have made it, especially if it be allowed to have follow'd the Track I have assign'd it, over *Prestain*, *Cardiff*, *Minhead*, *Tiverton*, and *Brest* in *Bretany*.

Allowing this to have been the Path it mov'd in, it would be easy to assign the real Magnitude and Velocity of this Meteor, if the several Accounts of its apparent Diameter, and of the Time of its Passage from one of its Explosions to the other, were consistent with themselves. But some of them making its visible Appearance nearly equal to the Sun's, which in the Opinion of many it far exceeded, we may suppose with the least that, at the time when it first broke out over *Tiverton*, its Diameter was half a Degree. And its Horizontal Distance being 150 Geogr. Miles from *London*, and its Altitude 60, the Hypothenusal or real Distance from the Eye will be more than 160 such Miles; to which Radius the Subtense of half a Degree will be above an English Mile and half, being about 2800 Yards *quamproximè*. After the same manner it is difficult to assign its due Velocity, whilst some make it half, others less than a quarter, of a Minute, in passing from its first Explosion to its last Extinction: But the Distance it moved in that time being about 3 *gr.* or 180 Geogr. Miles, we may modestly compute

it to have run above 300 ſuch Miles in a Minute; which is a Swiftneſs wholly incredible, and ſuch, that if a heavy Body were projected horizontally with the ſame, it would not deſcend by its Gravity to the Earth, but would rather fly off, and move round its Center in a perpetual Orb, reſembling that of the *Moon*.

Of ſeveral Accidents that were reported to have attended its Paſſage, many were the effect of pure Fancy; ſuch as the hearing it hiſs as it went along, as if it had been very near at hand: others imagined they felt the Warmth of its Beams; and ſome there were that thought, at leaſt wrote, that they were ſcalded by it. But what is certain, and no way to be diſputed, is the wonderful Noiſe that follow'd its Exploſion. All Accounts from *Devon* and *Cornwal* and the neighbouring Counties are unanimous, that there was heard there, as it were the Report of a very great Cannon, or rather of a Broad ſide, at ſome diſtance, which was ſoon follow'd by a rattling Noiſe, as if many ſmall-Arms had been promiſcuouſly diſcharged. What was peculiar to this Sound was, that it was attended with an uncommon Tremour of the Air, and every where in thoſe Counties, very ſenſibly ſhook the Glaſs-Windows and Doors in the Houſes, and according to ſome, even the Houſes themſelves, beyond the uſual Effect of Cannon, though near; and Mr. *Cruwys* at *Tiverton*, on this occaſion, loſt a Looking-Glaſs, that being looſe in its Frame, fell out on the ſhock, and was broken. Nor do we yet know the Extent of this prodigious Sound, which was heard, againſt the then Eaſterly Wind, in the Neighbourhood of *London*, as I am inform'd; and by the Learned Dr. *Tabor*, who diſtinctly heard it beyond *Lewis* in *Suſſex*: So that I cannot help thinking, that ſuch a Meteor as this might have occaſion'd that famous Ode of *Horace*: *Parcus Deorum cultor*, &c.

* ——— *Namque*

———*Namque Diespiter*
Igne corusco nubila dividens
Plerumque, per purum tonantes
Egit equos volucremque currum,
Quo bruta tellus, &c. Concutitur.———

But whether the Report heard near *Lewis* were of that Explosion right over *Devonshire*, or rather of that latter and much greater at the Extinction over *Britany*, I shall not undertake to determine, till we have some further Accounts from *France*, whence hitherto we have only had, that at *Paris* the Time of the Appearance was at 17 Minutes past Eight.

It remains to attempt something towards a Solution of the uncommon *Phænomena* of this Meteor; and by comparing them with things more familiar to us, to shew at least how they might possibly be effected. And first the unusual and continu'd Heats of the last Summer in these Parts of the World, may well be suppos'd to have excited an extraordinary Quantity of Vapour of all sorts; of which the aqueous and most others, soon condens'd by Cold, and wanting a certain Degree of Specifick Gravity in the Air to buoy them up, ascend but to a small Height, and are quickly returned in Rain, Dews, *&c.* whereas the inflammable sulphureous Vapours, by an innate Levity, have a sort of *Vis centrifuga*, and not only have no need of the Air to support them, but being agitated by Heat, will ascend in *Vacuo Boileano*, and sublime to the top of the Receiver, when most other Fumes fall instantly down, and lie like Water at the bottom; the Experiment whereof was first shewn me by the Reverend Mr. *Whiteside* at *Oxford*, and was very lately made before the *Royal Society*. By this we may comprehend how the matter of the Meteor might have been raised from a large Tract of the Earth's Surface, and ascend far above the reputed Limits of the *Atmosphere*; where, being disengaged from all other Particles, by that principle of Nature that congregates *Homogenia* visible in so many Instances, its Atoms

Atoms might in length of time coalesce and run * *fortuitously* together, as we see Salts shoot in Water; and gradually contracting themselves into a narrower compass, might lie like a Train of *Gunpowder* in the *Ether*, till catching fire by some internal Ferment, as we find the Damps in Mines frequently do, the Flame would be communicated to its continued parts, and so run on like a Train fir'd.

This may explain how it came to move with so unconceivable a Velocity; for if a continu'd Train of Powder were no bigger than a Barrel, it is not easy to say how very fast the Fire would fly alongst it; much less can we imagin the Rapidity of the Accension of these more inflammable Vapours, lying in a Train of so vast a Thickness. If this were the Case, as it is highly probable, it was not a Globe of Fire that ran along, but a successive kindling of new Matter: and as some parts of the Earth might emit these Vapours more copiously than others, this Train might in some parts thereof, be much denser and bigger than in others, which might occasion several smaller Explosions, as the Fire ran along it, besides the great ones which were like the blowing up of Magazines. Thus we may account for the rattling Noise like small-Arms, heard after the great Bounce on the Explosion over *Tiverton*; the Continuance of which for some time, argues that the Sound thereof came from Distances that encreased.

What may be said to the Propagation of the Sound thro' a *Medium*, according to the receiv'd Theory of the Air above 300000 times rarer than what we breath, and as I said before, next to a *Vacuum*, I must confess I know not. Hitherto we have concluded the Air to be the Vehicle of Sound; and in our artificial *Vacuum* we find it greatly diminish'd: but we have this only Instance of the effect of an Explosion of a Mile or two diameter, the immensity of which may perhaps compensate the extream Fineness of the *Medium*.

FINIS.

* Dele *fortuitously*.

... are *Tables for the ready Computing of the*
pses of the first Satellite *of* Jupiter *by Addition only.*
he Reverend Mr. James Pound, R. S. S.

Philosoph. Transact. N°. 361.

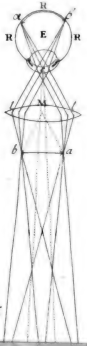


Fig: I.

Fig: II.

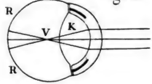
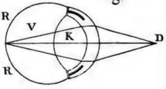


Fig: III.



of an Explosion of a Mile or two diameter, the imm[...]ty of which may perhaps compensate the extream[...]ness of the *Medium*.

FINIS.

* Dele *fortuitously*.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *June*, *July* and *August*, 1719.

The CONTENTS

I. *An Obſervation of the end of the Total Lunar Eclipſe on the 5th of* March 1718. *obſerved near the Cape of* Good Hope, *ſerving to determine the Longitude thereof. With Remarks thereon. By* E. Halley, *R. S. Secr.*

'TIS now better than thirty Years ſince I had a Diſpute with ſome of the *French Geographers* about the Longitude of the Cape of *Good Hope*, ſaid to have been obſerv'd by the Religious Miſſionaries ſent to *China* in the Year 1685. By an Emerſion of the firſt Satellite of *Jupiter*, they determined that Cape to be $1^h. 11'$ or $17\frac{3}{4}$ *grad.* more Eaſterly than *Paris*, that is 20 *grad.* from *London*; which for the reaſons I then gave, I concluded could not be more than 17 *grad.* See *Phil. Tranſact.* N° 185. Very lately I have fallen upon an Obſervation which I believe will determine the Controverſy in my favour: for I had accidentally a Journal of an Officer of the Ship *Emperor* put into my Hands, who in his return from *India*, on the fifth of *March* 1718. obſerv'd the End of a Lunar Eclipſe, when the viſible altitude of the Moons Centre was $13^\circ. 25'$. he being then in the Latitude of $34^\circ. 23'$ *South* and as they found afterwards, juſt 180 Leagues to the Eaſtwards of Cape *Bonne Eſperance*. By Calculation I find that in that Latitude the Moon had that height at $7^h. 17'\frac{1}{2}$ *P. M.* and by comparing this Eclipſe with that we obſerv'd with great exactneſs on *Febr.* 11°. 1682. (which agrees perfectly well with our Numbers) I conclude the middle of this to have been at *London* at $3^h. 48'$ *P. M.* To which adding $1^h. 46'$ for the

the Semiduration (this being very certain from the observ'd Continuance of the Eclipse of 1682.) the End will be found to have been at *London* at $5^h. 34'$. The Ship was therefore in a Meridian 26° to the Eastwards of *London*: But she was at that time 180 Leagues to the Eastwards of the *Cape*, which distance in that Latitude gives eleven Degrees of Longitude; this therefore being deducted from the Longitude of the Ship, leaves just 15 *grad.* or one Hour, for the difference of Meridians between *London* and the *Cape*. So that by this account the *Cape* is yet nearer our Meridian than I had formerly made it, and near six Degrees nearer than M. *De la-Hire* places it in his *Tables*.

This Eclipse was attended with all the Circumstances requisite to make the Conclusion as certain as the nature of the thing will admit of: For the Moon was nearly *in Perigao*, and the Eclipse almost central; for which reasons she emerged out of the Shadow as swiftly as possible: The Sea was very smooth, there having been little Wind for above 30 Hours before; and the Moon was not too high to be well observed with a Forestaff. Nor were they long at Sea before they made the Land, for in less than five Days, on the tenth of *March* at Noon, they had past Cape *d'Agulhas* the most Southerly Promontory of *Africa*, which then bore from them *North East*, about seven Leagues distant. The End of this Eclipse, though not visible here might have been seen in *Germany*, both at *Nurenburg*, *Leipsick* and *Berlin*, but we cannot learn that it was any where observed there; however our Numbers in this Case may be securely relied on.

On this occasion it may not be amiss to insert an Observation or two I procured to be made at the *Cape*, by Mr. *Alexander Brown* a *Scotch* Gentleman, who went to reside in *India* on our Companies account. He carried with

with him a very good *Brass Quadrant* of above two Foot Radius, and at the *Dutch* Settlement at *Table Bay*, having rectify'd his *Pendulum-Clock* by correspondent Altitudes, on the 4th of *August* 1694, at 5^{h}. 59′ *Manè*, the distance of the bright Limb of the Moon from the right Shoulder of *Orion* was observ'd to be 25° 3′. And the next Morning *Aug.* 5. at 5^{h}. 21′. 12″, the same Limb was distant from *Procyon* 25°. 57′, and at 5^{h}. 36′. 48″ from the *Lucida Arietis* 58°. 29′.

It were much to be wisht that the Moon had, either of these Mornings, been accurately observ'd at *Greenwich* or *Paris*, or at some Place in *Europe*, whose Longitude from them is well known. But that failing us, I had recourse to the Period of the Lunar Motions, which is perform'd in 18 Years and ten or eleven Days, after which the Errors of our Lunar Computations return very nearly the same; and I found among my own old Observations, one that tallyed well with that of the 4th of *August*. *Viz.* Anno 1676. *July* 23°. 13^{h}. 11′. 35″. at *Oxford*, I observ'd the Moon to apply to the Star *in medio Collo Tauri*, by *Bayer* markt *A*. The Star at that time was distant from the Southern and nearest Cusp of the Moon by the Micrometer 20′. 32″. and at 13^{h}. 17′. 15″. when it seem'd to immerge upon the bright Limb of the Moon, it was distant from the Northern Cusp 23′. 20″; but this less certain by reason of the hazey Air. The Star at that time was in ♉ 28°. 56′. with 1°. 13′. 20″. *North* Lat. whereby I found that our Lunar Tables, founded on Sir *Isaac Newton*'s correct Theory of her Motion, gave her place at that time only two Minutes too slow; which Error being allowed on the 4th of *August* 1694. the result was, that 5^{h}. 59′. at Cape *Bonne Esperance* was at *London* 4^{h}. 53′. whence the difference of Longitude $16\frac{1}{2}$ degrees, sufficiently near what we had before determin'd.

II. JACOBI KEILL, *M. D.* *De Viribus Cordis Epistola.*

Viro Celeberrimo

RICHARDO MEAD, *M. D.*

S. P. D.

Jacobus Keill.

EPistolam D. *Jurin*, Tibi, Vir Clarissime, inscriptam, in Actis Philosophicis nuper publicatam legi; in quâ, ea quæ à me traduntur de viribus Cordis, infirmare conatur vir Doctissimus. Cum æstimatio virium, quibus Cor Sanguinem expellit, à *Borellio* facta, fere omnibus valde incredibilis videbatur, non me temerarium, non *Borelli* nomini injuriosum, non orbi literato ingratum facturum existimavi, si ad verum propius accedere tentarem. In quo Tentamine, non accuratam virium Cordis definitionem mihi propositum erat dare, sed potius methodum, quâ hæ vires forte inveniri possent, indicare; & Geometriæ peritiores ad Problematis valde desiderati investigationem incitare. Quo animo tentamen illud primùm suscepi, eodem ea, quæ in eo reprehendit vir Doctissimus, nunc defendam. Nequaquam

quaquam enim mihi honorem quæro (utcunque parva mea existimatio sit, est certe debito major) sed Genti Medicæ lucem undecunque illatam gaudeo. Idcirco non ut Decreta mea sustineam, sed ut vir huic negotio plusquam par, suas demonstrationes secum reputare, & Reipublicæ Literariæ correctiores denuo reddere dignetur, hanc Tibi scripsi literam. Quem enim Adversarius Patronum sibi ambivit, ego Te judicem Controversiæ intelligentem & æquissimum mehercule exopto.

Præcipuum quod *Borellio*, D. *Moreland*, mihi objicit vitium est, quod in Potentiâ Cordis æstimandâ, quam rationem ad pondus iners, vel corporis gravitatem obtineat, determinare suscepimus. " Sed Cor, inquit, " cum & ipsum inter contrahendum movetur, & corpora " opposita, Sanguinem nempe & arteriarum tunicas " in motum impellit, patet ejus *Potentiam* non aliâ " ratione sciri posse quanta sit, quam ut motus hujus " quantitatem cognitam teneamus. *Motus* autem qui- " libet cum pondere quiescente comparari non magis " potest, quàm linea cum rectangulo." At à nemine certè nostrûm, quod scio, est *Motus* Cordis cum pondere quiescente comparatus. Potentiam autem Cordis, seu vim Cordis motricem & Sanguinem impellentem cum pondere conferre, quid prohibet non video. Quamquam enim inter *Pondus* & *Motum* corporis solidi nulla sit relatio, vis tamen motrix, si in fluidum agit, ad vim gravitatis quandam certe rationem habet. Et revera vis corporis motrix, certam in fluido motûs quantitatem in dato tempore efficiens, æqualis est ponderi, quod vi gravitatis cadendo, in eodem tempore, eandem motûs quantitatem sibi acquirit. Hinc vis, quâ ex orificio aliquo aqua exprimitur, certo ponderi æqualis esse dicitur: quia pondus datum, & vis aquam exprimens æquales motus in temporibus æqualibus generant,

nerant. Hic genuinus Corollarii *Newtoniani* ſenſus mihi videtur eſſe, nec ab hoc ſenſu diſcrepant, quæ de Cordis viribus explicui. Verba *Newtoni* ſunt, *Vis, qua totus aquæ exilientis motus generari poteſt, æqualis eſt ponderi &c.* quæ non ſatis attendiſſe videtur *Jurinius*, cum dicit *Pondus autem illud quo motus aquæ ex vaſe effluentis generari poteſt*, &c.

Sed ſi hâc re à nobis peccatum eſt, cum ſummis certe hujus ſæculi Geometris *Hugenio* & *Newtono* peccavimus, quorum uterque vim fluidorum per vim gravitatis exponit. Nec in *Corol.* prædicto id ſolummodo facit *Newtonus*, ſed in aliis etiam locis oſtendit Methodum, quâ ratio reſiſtentiæ Medii, id eſt, actionis fluidi in corpus ſolidum, ad vim Gravitatis vel centripetam inveniri poteſt, ut videri licet in *Prop. 4ta* & *5ta* Libri ſecundi, eorumque Corollariis. Alia profecto eſt actio fluidorum in ſolidum, & alia ſolidorum in ſe invicem. Fluidum datâ velocitate motum, datum pondus ſuſtinere poteſt, cum fluidi partes ſibi mutuo continuò ſuccedentes in pondus impingunt, adeoque vis fluidi eſt revera ponderi æqualis; ſed cum Solidorum non par eſt ratio, eorum Vis cum Gravitate comparari nequit.

Me inſuper reprehendit Vir ingenioſiſſimus, quòd velocitatem Sanginis è corde detruſi, per totam ſyſtolem æqualem poſui, quam ille valde inæqualem eſſe demonſtravit. Verum à me nuſquam Sanguini æqualis data eſt velocitas, ſed pro ſummâ omnium velocitatum mediam poſui. Sed utrum æqualis vel inæqualis eſt Sanguinis è Corde ejecti celeritas, nondum ſatis mihi conſtat; certe quæ pro æquali velocitate ſtat ratio, ea mihi in præſens firmior videtur.

Sic emaculatis vitiis quæ in primâ noſtrâ methodo reprehendit vir Cl: quid in alterâ quæ ſubjuncta eſt, illi diſplicet, videamus. Et eſt certe aſſumptio illa quæ à *Borellio*, aliisque viris doctis ſæpius uſurpata eſt,

 nempe,

nempe, quod similium musculorum vires sunt in ratione ponderum. Aliam virium rationem in Theoremate *5to* stabilire conatur *Jurinius*: sed cum ex communi omnium suorum Theorematum Principio oritur demonstratio, communi etiam eorum fato involvetur: Si enim principium illud fallax est, (ut mihi videtur) nec ad casus ad quos adhibetur, congruit; corruunt certe omnia, quæ hâc basi innituntur. Supponit Vir Clar: Vasorum tunicas in Sanguinem intùs contentum impetu irruere, & motûs sui partem Sanguini ictu communicare: & hîc in motu Cordis, vult Ventriculum tanquam solidum Corpus, datâ velocitate motum, in Sanguinem impingere, & ictu motûs sui partem illi impertire: quæ suppositio nec Sanguinis nec Cordis, nec Aeris è Pulmone expressi motui competit, nec ullâ minimorum ictuum reiteratione, horum motibus ita accommodari potest, quin quæ inde deducentur conclusiones pro incertis & omnino falsis haberi debeant.

Cum inter Sanguinem & Cordis intimum nullum intercedit spatium, sed est alter alteri contiguus, non ictu hoc in illum, sed pressu agit: nec ullam in initio suæ contractionis celeritatem ventriculi habent, sed se contrahendo velocitatem tempore acquirunt, tanquam gravia cadendo, vel ut fluida rarescendo, ex quo forte omnis vis Cordis oritur. Adeoque non æquabilis est motus contractionis, ut vult Vir Doctissimus, sed est motus instar cadentis acceleratus. Idem igitur est discrimen inter ictum quo Cor Sanguinem ferire vult *Jurinius*, & pressuram quâ Cor revera in Sanguinem agit, quod est inter actionem corporis solidi moti & vim gravitatis: sed ipso fatente, hæc comparari nequeunt, adeoque pressura seu actio cordis in Sanguinem per ictum nec à Viro laudato exposita est, nec unquam exponi potest. Hanc sententiam confirmat ipsa Cordis potentia à Viro Cl. inventa. Si enim pondus datâ velocitate motum, cordis

cordis potentiæ æquale esset, tunc Sanguis omni vi Cordis in pondus illud directe impulsus motum ponderis temporis momento destrueret: sed quocunque magno impetu ponderi occurrat Sanguis, nunquam illi omnem motum in instanti eripiet, adeoque est hoc pondere potentia Cordis minor, nec recte per motum ponderis vires Cordis exponuntur.

Fluidorum vires in corpora solida, ubique eodem prorsus modo quo solidorum vires in se invicem, *Jurinius* æstimat & perpendit, cum tamen maxima intersit differentia; & ab hoc capite fluit quicquid est in illius Propositionibus erroris. Ubi enim corpus solidum, cujus partes firmiter inter se cohærent, in aliud impingit, unaquæque corporis particula simul & semel suam alteri vim impertit: at res aliter se habet in fluidis, in queis nulla est partium cohærentia, nulla fluidi pars, nisi in ipso tactu, in corpus sibi oppositum agit: idcirco cum columna aquæ adversus corpus solidum sursum vertitur, partes columnæ à corpore remotiores nullam illi vim imprimunt. Corpus etiam solidum unicum solummodo ictum alteri communicat; at columna fluidi in corpus sibi oppositum continue agit, & minima columnæ pars minimo temporis momento, ictum infinite parvum illi imprimit, eodem prorsus modo quo gravia cadendo agunt, quibus igitur fluidorum motus recte comparatur. Porro omnis motus corporis solidi in alterum directe impingentis in temporis momento destrui potest: sed motus solidi vim fluido imprimentis, non nisi gradatim imminuitur, & in dato tempore evanescit, pari ratione, quâ Gravitas in corpus sursum missum vim suam exerit. Ex quibus satis abunde constat, inter vim fluidi in motum acti, & vim gravitatis magnam esse affinitatem, & unam per alteram recte exponi posse; vim autem corporis solidi ad vim gravitatis referri non posse. Cumque hanc differentiam

rentiam non satis attendisse videtur Doctissimus *Jurinius*, à vero multum aberrasse mihi videtur. Si igitur sepositâ suâ, de Vasorum ictu, hypothesi, & vi pressuræ, quâ Natura utitur, pro Principio adhibitâ, alia Theoremata de Cordis & Sanguinis motu & viribus, eleganter suâ demonstrationis methodo, construere dignabitur, sese dignum, mihi certe gratum, nec eruditis inutile præstiterit. Tu, qui Rei Medicæ principatum tenes, Vir Amplissime, dissentientium disputationes tuâ prudentiâ ita moderari digneris, ne Indoctis ludibrio, sed ut Doctis emolumento esse possint. Dabam *Northamptoniæ* 23. die *Junii* 1719.

III. *An account of some Experiments relating to the Specifick Gravity of Human Blood. By* James Jurin, M. D. *and* F. R. S.

IT is well known from the Observations of Mr. *Leeuwenhoek* and others, that Human Blood consists of red globular Particles, swimming in a pellucid *Lympha*, or *Serum*. Which two different Substances, tho' of unequal Specifick Gravities, yet so long as they continue to circulate in the Veins and Arteries, are prevented from separating by their Motion and Warmth. But when the Blood comes to stagnate and cool in a Porringer, the globular Particles uniting together by their attractive Power, and sinking by their Weight, which is greater than that of the *Serum*, form the *Coagulum*, or *Crassamentum*, at the bottom of the Porringer, the *Serum* swimming above it.

Things always happen in this manner, when the *Crassamentum* is at liberty to subside: but it often falls out, that, either by its adhesion to the sides of the Vessel, or by the bubbles of Air, which the Blood

gathers upon falling into the Porringer, and which ſtick to it's Surface, the *Craſſamentum* is kept from ſinking, and ſeems to float upon the top of the *Serum.*

Theſe Accidents ſeem to have given the firſt occaſion to that Opinion, which, I think, has been generally entertain'd by thoſe who have writ upon this Subject, namely, that the globular part of the Blood is ſpecifically lighter than the *Serum*, in which it ſwims.

But that which has ſo fully eſtabliſht this perſuaſion, is the Authority of the late excellent Mr. *Boyle*, who, among the many valuable and curious Experiments he has given us in his Natural Hiſtory of Human Blood, has left the following ones upon this Subject.

The ſpecifick Gravity of *Serum* of Human Blood was found by weighing a piece of Sealing Wax firſt in *Serum*, and afterwards in Water, to be to the ſpecifick Gravity of Water, as 1024 to 1000.

In a ſecond Experiment, which for greater accuracy was made with an Inſtrument contriv'd on purpoſe, the ſpecifick Gravity of *Serum* was found to be to that of Water, as 1194, to 1000.

In a third Experiment made by the ſame Inſtrument, and with *Serum* from the Blood of another Perſon, it's ſpecifick Gravity appear'd to be 1186.

The Medium between theſe two laſt Experiments is 1190, which has ſince been univerſally receiv'd for the ſpecifick Gravity of *Serum* of Human Blood, the firſt Experiment being declar'd by Mr. *Boyle* himſelf to be leſs exactly made than the other.

The ſpecifick Gravity of Human Blood was found by Mr. *Boyle*, to be to that of Water, as 1040 to 1000; though on account of difficulties by him mention'd, he was far from being ſatisfy'd with this Experiment, and recommended the thing to farther tryals.

Theſe

These Experiments however having hitherto past uncontroverted, and it appearing from them, that the specifick Gravity of *Serum* was greater than that of Blood in the proportion of 1190 to 1040, or of 8 to 7 nearly; it was a necessary consequence of this, that the Blood Globules were specifically lighter than the *Serum*, and that in a very great degree, considering the small proportion that the bulk of the *Crassamentum* was found to bear to that of the *Serum*, from other Experiments.

From this it was not improbably conjectured, that these Globules were thin Vesicles fill'd with an Aereal substance: and this Opinion seem'd to receive a great confirmation, upon it's being observ'd, in viewing the Circulation by a Microscope, that a Blood Globule, in passing through a very narrow Vessel, would change its shape from a Globular to an Oval Form, and would recover it's former Figure, as soon as it was got thro' the narrow Passage; which appearance seem'd to be naturally accounted for from the Elasticity of the included *Aura*.

Upon this conjecture have been built a great many Solutions of the Phænomena observable in the Animal Oeconomy, and the disorders of it; particularly a late ingenious account of Muscular Motion. It it not my business at present to examine any of these, nor is it my design to cast any reflection upon their Authors, who were led into this mistake by the natural consequence of a matter of Fact, for the truth of which they had so great an Authority, as that of the excellent Person above-mentioned. But I hope, I shall easily be pardon'd for enquiring into the soundness of the Foundation, when the Superstructure erected thereupon is so considerable; and the following Experiments, however trivial in themselves, will not appear unworthy the

the consideration of the *Royal Society*, if it be found, that they may prevent us from running into Errors of the greatest consequence.

Exp. I. I have several times cut off a small part of the *Crassamentum*, when by its adhesion to the sides of the Porringer it has seem'd to swim upon the Surface of the *Serum*, and have put it into another Vessel fill'd with *Serum*: upon which it has immediately sunk to the Bottom.

Exp. II. When the *Coagulum* has been buoyd up in the *Serum* by the bubbles of Air adhering to its Surface, I have separated a small part of it, where those Bubbles have been thickest, and put it into a Glass of *Serum*, in which it has swom, as before. Then setting the Glass upon the Air-Pump, those Bubbles burst after one another, as the Receiver was exhausting, and the Air being again let into the Receiver, the lump of *Crassamentum* sunk to the bottom of the Glass.

Exp. III I have often placed a drop of *Serum* upon a clean Glass before a Microscope, in which I had dissolv'd a very small quantity of Blood, and observ'd, that when the Glass was held in a perpendicular Posture, the Blood-Globules subsided to the bottom of the Drop; and inverting the Glass, the Globules again descended thro' the *Serum* to the Bottom. I had the same success with a small quantity of *Serum* and Blood in a Capillary Tube. And the same thing has been long since observ'd by the famous Mr. *Leeuwenhoek*.

These Experiments undeniably demonstrate, that the *Crassamentum*, or globular part of the Blood, is specifically heavier than the *Serum*; and consequently it is by no means probable, that the Blood Globules are Vesicles fill'd with Air, or any other Fluid lighter than *Serum*. And that they are not fill'd with any sort of

 Elastick

Elaſtick Fluid, will appear from the following Experiment.

Exp. IV. In a ſmall quantity of *Serum* of Human Blood, I diſſolv'd ſo much Blood, as that the Globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall drop of this Liquor on the inſide of a thin Glaſs Tube, I fitted the Tube on to the Air-Pump, and placed a Microſcope by it, ſo that I could ſee the Blood-Globules through the Tube. This being done, I caus'd the Tube to be exhauſted, keeping my Eye upon the Globules all the time, in order to obſerve whether they dilated themſelves, as the Air was withdrawn; but could not perceive the leaſt alteration, they appearing exactly of the ſame bigneſs in the *Vacuum*, as they had done before. Whereas if they had been fill'd with an Elaſtick Fluid, they would either have burſt, or have been dilated to at leaſt 70 or 80 times their former Magnitude. The Stop-cock being afterwards turn'd, and the Air ſuffer'd to re-enter the Tube, the Blood-Globules ſtill retain'd the ſame bigneſs, as in *Vacuo*.

To this it will perhaps be objected, that a learned Member of this Society, in a Book lately publiſh'd, has aſſerted the direct contrary to what I here affirm, and has aſſur'd us, that the Blood-Globules in an exhauſted Receiver, inſtantly ſwell, and dilate themſelves ſo, as to become incredibly large. But as that Gentleman does not tell us, upon what Experiment this aſſertion is grounded, it may not be unreaſonable to ſuppoſe, that he was miſled by the common Hypotheſis, which he there maintains, of the Blood-Globules being fill'd with Air, and by what he has heard or ſeen of the bubbles of Air, which ariſe from Blood in the Air Pump in the ſame manner as from other Liquors, and which not eaſily breaking out from ſo viſcid a Fluid, occaſion

the

the appearance he mentions. However this may be, to prevent any dispute, and avoid the coming to *Utri creditis, Quirites*? I shall offer a Testimony, that every body will be satisfy'd with, namely that of the learned and ingenious Mr. *Machin*, Professor of *Astronomy* in *Gresham Colledge*, and one of our Secretaries, who having honour'd me with his Company at a repetition of this Experiment, in order to be witness to the Event, was fully satisfied upon repeated tryals, that there was no perceivable difference between the Magnitude of the Blood Globules in the Air, and in *Vacuo*. Upon this occasion the two first Experiments were likewise repeated in his presence, with the same Success, as above related.

Though what has been already said is a sufficient proof of the Opinion above-mention'd, yet however to prevent the Objections, which may arise for want of Experiments made in the same manner with Mr. *Boyle*'s, as well as for the satisfaction of the Curious, who may be desirous to know the true Specifick Gravities of *Serum* and Blood, I shall proceed to demonstrate the same thing by Hydrostatical Experiments.

Exp. V. *Novemb*. 13. 1713. Having suffer'd a quantity of my own Blood to stand about 24 Hours in the Porringer, and then drawing off the *Serum* carefully with a small *Siphon* into a convenient Glass, I found by the Hydrostatical Balance it's Specifick Gravity to be to that of Water, as 1029,8 to 1000.

Exp. VI. *Feb*. 21. 1716-7. I examin'd the *Serum* from the Blood of another Person in the same manner and found it's Specifick Gravity to be 1028, 6.

Exp VII. VIII and IX. *April* 8th, 1717. I obtain'd three several quantities of *Serum* from the Blood of different Persons. The first of these was of a deep

Colour, inclining ſomething to red, and a little Turbid. It's Specifick Gravity was 1029, 7.

The ſecond was likewiſe a little Turbid, and of a pale whitiſh Colour. The Specifick Gravity of this was 1030, 2.

The third quantity of *Serum* was perfectly clear, and of the colour of Canary. It's Specifick Gravity was found to be 1030.

Though theſe five ſeveral Experiments were all carefully made, and with a Balance whoſe accuracy I was well aſſur'd of, yet for farther Evidence, I thought it proper to make that which follows, after another manner.

Exp. X. *Jan.* 15th. 1718-9. I drew off all the *Serum* from five or ſix ſeveral Porringers, containing the Blood of different Perſons. This I found to be a little tinged with Blood, which was occaſion'd by my being oblig'd to draw it off pretty near to the bottom of the Porringers, in order to obtain a quantity ſufficient for my purpoſe. For this reaſon I ſuffer'd it to ſtand about two Days, in which time the Globular part of the Blood was entirely precipitated to the Bottom, and the *Serum* was become perfectly fine and tranſparent. I then drew it off with a *Siphon* into a Glaſs Vial with a narrow Neck, which I fill'd to a certain mark made in the Neck for that purpoſe. This done, I plac'd my Vial in a nice pair of Scales, in which I had a counterpoiſe for the weight of the Vial, and found that quantity of *Serum* to weigh $2284\frac{1}{4}$ Grains.

Then pouring out the *Serum*, I fill'd the Vial with common Water to the ſame mark, and found the weight of the Water to be 2219 Grains.

From which it follows, that the Specifick Gravity of this *Serum* was 1029, 4.

Exp.

Exp. XI. *July* 14. 1719. I procur'd a quantity of Blood taken from the temporal Artery, from which I drew off the *Serum* the next Day, and weighing it in the same manner found it's Specifick Gravity to be 1028, 8.

These Experiments agree so nearly together, that the little difference between them may very well be attributed to that which is between the *Serum* of different Persons; or to the variations occasion'd by heat and cold in the several Seasons of the Year, in which they were made. So that from them we may safely determine the Specifick Gravity of *Serum* of Human Blood at a Medium to be 1029,5, or in a round number 1030. From which the greatest Variation in any of these Experiments is little more than one in 1000; whereas the difference between Mr. *Boyle*'s Experiments and mine amounts to 160 in 1000.

Exp. XII. *April* 6. 1717. In order to find the Specifick Gravity of Human Blood, which, by reason of it's tenacity, and sudden alterations upon standing, cannot be determin'd by the Hydrostatical Balance; I took a narrow-neck'd Vial, and fill'd it to a Mark, with Blood pour'd immediately out of the Porringer, as soon as the Person was blooded. This I weigh'd, as I had done the *Serum* before, and found it's Specifick Gravity to be 1051.

Exp. XIII. *Aug*. 5th. 1717. Having fill'd the same Vial with the Blood of another Person, running immediately out of the Vein through a Funnel, it's Specifick Gravity was determin'd at 1053.

Suffering this to stand till it was cold, I found the Blood was sunk a small matter below the Mark in the neck of the Vial. This being fill'd up with the Water, which in so small a quantity could make no sensible

ble difference from Blood, I found the Specifick Gravity of cold Blood to be 1055.

Exp. XIV. *Aug.* 6*th.* 1718. The last Experiment being repeated in the same manner as the Year before, the Specifick Gravity of cold Blood was again found to 1055.

Exp. XV. *July* 14th. 1719. The Arterial Blood, from which the *Serum* was afterwards drawn off for the 11th Experiment, being weigh'd in the same manner, it's Specifick Gravity was 1052, 5.

As this Arterial Blood and it's *Serum*, differ no more in Specifick Gravity from Venal Blood and *Serum*, than the several Portions of these do from one another, it's plain, that the difference in this respect between Arterial and Venal Blood is wholly inconsiderable. The Animal Oeconomy indeed teaches us, that the *Serous* Liquor is perpetually drawing off from the Arterial Blood by the several Secretions, but as the quantity separated in one Circulation is very small, the Blood must arrive in the Veins nearly of the same density, as when it runs through the Arteries.

In the 13th Experiment we observ'd, that the Blood alter'd it's Specifick Gravity upon cooling from 1053 to 1055; from which we may infer, that if the Blood made use of in the 12th Experiment had been suffered to stand till it was cold, it's Specifick Gravity would have been 1053; wherefore, taking a Medium between the four last Experiments, we may allow the Specifick Gravity of cold Human Blood to be 1054.

The difference of 14 Parts in a 1000, between this and the Specifick Gravity determined by Mr. *Boyle*, is easily accounted for, if we consider, that that Gentleman did not make use of a Vessel with a narrow Neck, as plainly appears from the circumstances mentioned

tioned in his Experiment; and consequently a small error in the height of the Liquor would make a considerable alteration in the Specifick Gravity.

Since therefore the Specifick Gravity of Human Blood is 1054, and that of its *Serum* 1030, it is plain, that Blood is heavier than *Serum* by about one part in 43. From which it manifestly follows, that the Globular part of the Blood is specifically heavier than the *Serum*, since the Globular part being separated from the Blood leaves the remainder, or the *Serum*, specifically lighter than the intire Mass.

But in order to determine the exact Specifick Gravity of the Blood Globules, it is first necessary to know the Proportion, which the whole quantity of the *Crassamentum* contained in Blood bears to the *Serum*. To this end Mr. *Boyle* has given us two several Observations of the weights of the *Crassamentum* and *Serum*, after they have separated one from another in the Porringer. But besides the difficulty of making this Experiment with any tolerable exactness, it is to be consider'd, that there is a great deal of *Serum* contain'd in the interstices of the Globules, that compose the *Crassamentum*.

This difficulty however is in some measure answer'd by two other Experiments, which Mr. *Boyle* made for this purpose, after the following manner. He put a quantity of the *Crassamentum*, already separated from the *Serum*, into an Alembick, and distill'd off the remaining *Serum* to dryness, but without drawing off the Oil, or Volatile Salt; after which he weigh'd the distill'd Liquor, and the dry Mass left behind.

By comparing these Experiments with the two former, it will be found that the entire weight of *Serum* contain'd in Blood is nearly [illegible] of the whole, and consequently

consequently the weight of the dry'd *Crassamentum* is only two fifteenths of the Blood.

But for farther satisfaction, an Analysis was made at my desire with a large quantity of Blood, amounting to four Pounds fourteen Ounces, by that ingenious and skilfull Chymist, Mr. *John Brown*.

From this was obtain'd, with a very gentle heat, two Pounds, fourteen Ounces, and six Drachms of a Phlegmatick Liquor, that had scarce any thing of the fœtid Scent, which is usual in the distillation of Animal Substances; and its Specifick Gravity was nearly the same with that of common Water, being but 1000,8. This being mixt with a strong solution of Alum, scarce afforded any *Coagulum*; but exhibited a considerable one upon mixture with a solution of *Roman* Vitriol.

The distillation being continued with the same Heat, we had seven Ounces more of Phlegm considerably impregnated with Volatile Salt, as was manifest from the Smell. The Specifick Gravity of this was 1007, and having mix'd it with *Tinctura Martis optima*, Solution of Alum, and of *Roman* Vitriol, a large *Coagulum* was precipitated. In distilling these there was lost by Evaporation, two Ounces and two Drachms.

The third portion of Liquor, being rais'd with a stronger Fire, amounted to seven Ounces six Drachms. This was reddish, and turbid, and so strongly charg'd with Volatile Salts, that it might very well deserve the name of Spirit. Its Specifick Gravity was 1080, 1.

Besides these we had seven Drachms of Volatile Salt, an ounce of Oil, and eight Ounces four Drachms of *Caput Mortuum*, which still retain'd some small remainder of the Oil, as was manifest from its taking Fire at the flame of a Candle. In this latter part of the Operation was lost three Ounces, seven Drachms.

Upon

Upon making due allowance for the difference between the Specifick Gravities of the three first Portions of Liquor and that of *Serum*, as likewise for what was lost in the two several parts of the Operation, which we may reasonably conclude to have been of a Specifick Gravity nearly the same with that of the Liquor drawn off, it will be found, that the quantity of *Serum* contain'd in this Mass of Blood was about $\frac{15}{17}$ of the whole Weight, and consequently that the quantity of *Crassamentum* was $\frac{2}{17}$ of the same Weight.

If we calculate therefore upon this Supposition, that the weight of the Globular part of the Blood is $\frac{2}{17}$ of the whole, we shall find the Specifick Gravity of a Blood Globule to be to that of Water as 1277 to 1000.

If we follow the proportion of $\frac{2}{15}$, which results from Mr. *Boyle*'s Experiments, the Specifick Gravity of a Blood Globule will be 1242.

But this computation is in all appearance a great deal too large; for we cannot be assur'd, that our whole quantity of aqueous Liquor was rais'd from the *Serum* of the Blood. On the contrary it is more than probable, that a considerable part of it was afforded by the Blood Globules themselves, especially in the latter part of the Operation, when their texture must of necessity have been broken and dissolv'd by the strong Fire that was made use of. To prove this, we need only consider the condition of the dry'd *Crassamentum*, after the Phlegm is drawn off, that being now a hard and brittle Substance: whereas the Globules in their natural State are soft and yielding. For which reasons it may perhaps be more satisfactory, if we attempt to find the quantity of the Globular part of the Blood after another manner.

It appears therefore from Mr. *Boyle*'s Observations, that the quantity of *Serum*, which may be pour'd off

from the *Crassamentum*, is about one half of the whole Mass. The remaining *Crassamentum* consists of the Blood Globules, and a quantity of *Serum* filling up the Interstices between them; which, if the Globules keep their Spherical Form, may easily be found by the principles of common Geometry, to be nearly one half of the bulk of the *Crassamentum*: but if the Globules by their pressure against one another change their Figure, the quantity of *Serum* will be something less.

If this quantity of *Serum* lying between the Blood Globules be added to that pour'd off, it appears, that the *Serum* contain'd in Blood is about $\frac{3}{4}$ of the whole bulk, and consequently that the Blood Globules make about $\frac{1}{4}$ of the whole. From which we shall find the Specifick Gravity of the Blood Globules to be to that of Water as 1126 to 1000.

If we suppose the Blood Globules to make $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{3}$, or $\frac{1}{2}$ of the whole bulk, their Specifick Gravity will be respectively 1174, 1150, 1102, or 1078. So that upon any of these Suppositions, the Specifick Gravity of the Blood Globules will be considerably greater than that of the *Serum*, and consequently they cannot be suppos'd to be Vesicles fill'd with an Aereal Substance.

It will therefore perhaps be askt, What do they really consist of?

In order to come to a Solution of this Question, it may be proper to take notice,

That Blood is compos'd of Phlegm, Oil, Volatile and fixt Salts, and Earth. For as to the Spirit, we look upon it with Mr *Boyle*, to consist of the Phlegm and Volatile Salt united together.

That the *Serum*, upon a Chymical Analysis, exhibits a great deal of the first of these, and the others in a very small quantity.

That

That on the contrary the *Crassamentum* yields much less Phlegm, but the other Principles much more copiously than the *Serum*.

From which *Data*, I think, we may safely conclude, that the *Crassamentum*, or Globular part of the Blood, consists of some Phlegm united with the Oil and Salts, and a small quantity of Earth.

But what is the exact proportion of these several Principles to one another; what alterations are produced in the Body by a change of this proportion; how, and in what part these Globules are form'd; by what means they preserve their Figure, without dissolving in the *Serum*, or uniting with one another; what variations are made in their Specifick Gravities by Heat and Cold; and what are the effects of those Variations, are Questions not very easy to be solv'd, and yet of so much importance to the Animal Oeconomy, that it were greatly to be wisht, we had a number of *Data* sufficient to determine them.

P. S. Since this Paper was sent to the Press, I made the following Experiments, which serving to confirm the Method last made use of, for finding the Specifick Gravity of the Blood Globules, it may not be improper to relate them.

August 6 1719. I took a lump of the *Crassamentum* and wash'd it gently in fair Water, to free it from the loose Globules, which precipitating out of the *Serum*, after the *Coagulum* is form'd, do not unite into one Body with it. This done, I laid it on a spungy brown Paper, in order to drain off the superfluous Moisture. After which, weighing it first in Air, and then in Water, I found its Specifick Gravity to be 1083.

Another lump of the same *Crassamentum* being weigh'd in the same manner, its Specifick Gravity was 1082.9.

Sept. 18. 1719. I found the Specifick Gravity of another piece of *Crassamentum* to be 1082.1.

A second piece from the Blood of a different Person gave me 1086,1.

A third from the same Person gave 1086,6.

From this it follows that the Specifick Gravity of the Blood Globules is at least 1084, which is the *Medium* between these five Experiments.

But if we allow one half of the bulk of the *Crassamentum* to consist of *Serum*, filling up the Spaces between the Blood Globules, we shall find their Specifick Gravity to be 1138.

From this we must make a small abatement, because some part of the *Serum* must have been squees'd out from between the Globules, by their yielding to one anothers Pressure, when the lump of *Crassamentum* lay upon the Paper: and this will reduce their Specifick Gravity sufficiently near to 1126, as we had before determin'd it.

IV. *An Acconnt of the* Sunk *Island in* Humber, *some Years since recover'd from the Sea. Being an Extract of a Letter Communicated to the* Royal Society *by* John Chamberlayne, *Esq*; R. S. S.

THis Island goes by the name of the *Sunk Island*, so called I suppose from the sinking Marsh Ground about it. As for its Original, one may make pretty sure Conjectures of that I believe, because 'tis yet within

in

in the memory of Man, ſince it began to raiſe its Head above the Ocean, there being ſeveral old People here alive, who can remember when there appeared nothing of it but a waſt and barren Sand; and that only at Low-Water too, when for the ſpace of a few Hours it ſhewed its Head, and then was buried again till the next Tides Retreat: thus ſucceſſively it liv'd and died until the Year 1666, when it began to maintain its ground againſt the inſult of the Waves; about which time it began to be reſcued wholly from future danger, by the Care and Induſtry of Colonel *Gilby*, who having, as I am imform'd, a Leaſe or Gift of it from the Crown, did raiſe Banks about the riſing Grounds of it, and ſo defending it from the Encroachments of the Water, it became Firm and Solid; and in a ſhort time afforded good Paſturage for Sheep and other Cattel. The Expences at firſt to improve it to what it is, muſt needs have been very conſiderable; it being encompaſs'd with high Banks, and deep Canals for receiving and diſcharging the Liquid Element, which every now and then notwithſtanding threatens to re-poſſeſs it ſelf of its ancient Hereditament, but hitherto in vain; for I now acquaint you of its preſent Safety.

This Iſland is now about 9 Miles in Circumference, within the Banks, which ſeem to render it impregnable againſt all future attacks of the Sea, and is of a very fat and fertile Soil, affords good Graſs, Corn and Hay, and is repleniſhed with numerous flocks of Sheep, which are of a larger Size and finer Wool than thoſe in *Holderneſs*, from which it is divided by about two Miles in Water; and from *Lincolnſhire* by about four. It is ſtor'd with vaſt numbers of Rabits, that ſeem innumerable, they appearing through all Parts in prodigious Swarms; their Skins are counted the fineſt in *England*, of a dark Mouſe Colour, Shagg'd, and ſoft as Silk.

There

There are also Cows and Horses feeding constantly in the Place, with great plenty of Wild Foul.

The Inhabitants are not so numerous, there being only three Families that live constantly upon the Place; however they are never too solitary, there being abundance of Workmen and Labourers that continually resort thither, sometimes I am told to the number of a Hundred and upwards, for the repairing of the Banks, *&c.*

The Yearly Income of the Proprietor Mr *Gilby*, amounts to about 800*l.* and pays the King's Taxes to those who Collect for the *East-Riding*, and is usually uplifted by those of the Liberty and Township of *Ottringham*, from the Marshes of which there is a Passage over the Sands to the *Sunk* at Low-water. But this Custom of paying the King's Cess to them, proceeds only from the conveniency, not Necessity; for it never belong'd to that or any other Parish, so that I cannot resolve you in what Diocese this Island lyes, unless it had been united to some neighbouring Parish, or converted to one of it self; which if effected, the Tyth of Lambs, Wool and Rabits, *&c.* would make up a handsome Benefice. It lyes nearer indeed to the Diocese of *York*, by at least two Miles, than to that of *Lincoln*, being two Miles South of *Holderness*, in the River *Humber*, and four Miles North of *Lincolnshire*, *&c.*

Welwick, *April* 14.
1717.

V. *A*

V. *A Way for* Myopes *to use* Telescopes *without Eye-Glasses, an Object-Glass alone becoming as useful to them, and sometimes more than a Combination of Glasses. Communicated to the* Royal-Society, *by the Reverend* J. T. Desaguliers, LL. D. *and* F. R. S.

Lemma 1:

WHat is requir'd of a Telescope is to give large, and distinct Vision; that is, to make the Object (as in *Galilæo's* Telescope) or its Image (as in the Telescopes made up of Convex *Lentes*) appear under a great Angle, and to have all the Rays of those Pencils that enter the Eye, meet in a point upon the *Retina* of the Eye, on their respective Axes.

The first Figure represents the Combination of two Convex *Lentes* for the Astronomical or inverting Telescope; where the above-mentioned Requisites are obtain'd. A B is the Object suppos'd at a vast distance from the Ojective *Lens* L L, so that Rays coming from the extremity A of the Object. will fall upon the *Lens* L L, in the same manner as if they were parallel to their Axis A X; and after passing the Glass unite at *a*, where they project the Image of the Point A; from whence diverging, they fall on the Eye-Glass *l l*, and having pass'd through it, go on parallel to each other, and enter the *Cornea* of a common Eye E, which unites those parallel Rays upon its *Retina* R R R at *α*, where the Image of *a* is projected: The same may be said of the Rays that come from B, and after their

their ſeveral refractions through the two Glaſſes and the Coats and Humours of the Eye, meet upon the *Retina* at β, where they project the diſtinct Image of the Point *b*. The Rays that come from all the Points of the Object A B being affected after the ſame manner, give a diſtinct Image of thoſe Points upon the *Retina*, and therefore the Object does appear diſtinct.

The Object will alſo appear magnified in the ſame proportion as the Angle $l C l =$ to $b M a$ (under which its Image is ſeen,) is greater than the Angle A C B under which the Object A B would be ſeen by the naked Eye; as is more at large demonſtrated by Dioptrical Writers.

Lemma 2.

If parallel Rays fall upon the *Cornea* of a *Myops*, or ſhort-ſighted Perſon, they will unite in the Eye, before they come to the *Retina*, the farther from it the more Convex the Eye is; but if the Rays which fall upon the *Cornea* diverge in proportion to the too great Convexity of the Eye, as from D, ſuch Rays will be ſo refracted by the Coats and Humours of the Eye as to meet in one point upon the *Retina* R R, ſee *Fig.* 2 and 3. Where I have in the Scheme neglected the Refraction of the Rays paſſing out of the *Cryſtalline* K into the *Vitreous* Humour V, as I do in the other Caſes.

This *Lemma* is alſo demonſtrated by Dioptrical Writers.

Lemma 3.

If two Pencils of Rays (in each whereof all the Rays are parallel to the Axis, as *a* C) fall upon different Parts of the *Cornea*, at the greateſt diſtance from one another that can be allow'd for thoſe Rays to enter the Pupil P P, their *Axes* will, after entring the *Aqueous*

Aqueous Humour, converge, and meet either in the Vitreous, or Cryſtalline Humour, according to the Convexity of the *Cornea* thro' which they paſs'd, and diverge again before they come to the *Retina*; the Rays of each Pencil converging upon their reſpective Axes, to the place where the ſaid Axes croſs one another, *Fig.* 4.

Demonſtration.

The Axes $a\,C\,a$, $\alpha\,C\,\alpha$, falling obliquely upon the *Cornea* at C, C, and entring from *Air* into the *Aqueous* Humour, will be refracted towards the Perpendicular to K: where ſtriking more directly upon the *Cryſtalline*, they will go on to α, a, upon the *Retina* R R R R, decuſſating at V within the *Vitreous* Humour. The other Rays r, r; ρ, ρ, after their Refraction in the *Aqueous* Humour, fall more obliquely on the *Cryſtalline*, and therefore are refracted again ſo as to meet at V, where the Axes alſo meet, and thence go on to the *Retina* R R R R, *Fig.* 4.

Lemma 4.

But if the Axes of the above-mention'd Pencils are Parallel, the Rays that accompany them diverging from a Point ſo near the Eye, that the divergence may be proportionable to the too great Convexity of the Eye; then only the Axes will meet in the Eye before they come to the *Retina* (by *Lemma* 3.) but the other Rays will not unite upon their reſpective Axes, till they come to the *Retina*, (by *Lemma* 2.)

Propoſition.

I ſuppoſe the Eye of the *Myops* ſo Convex that he can ſee no farther than a common Eye, with the Eye-Glaſs of a Teleſcope before it: then the Eye of the

9 S *Myops*

lished. for computing the Eclipses of the *first Satellite* of *Jupiter*, without the help of any other Numbers. The ease of this *Calculus* gave great satisfaction to those that delight in Telescope observations; and has been of good use to encourage Astronomers to ascertain the Geographical Longitudes of many places, by help of these Eclipses; whose frequency seems to afford us the properest means for that purpose.

But it being now 26 Years since those Tables were published, length of Time has discovered that this *Satellites* motion is a small matter swifter than M. *Cassini* had supposed it; and the Reverend Mr. *Pound* being provided with all the Qualifications requisite for such a Work, has of late apply'd himself to rectify by frequent Observation what he found amiss in the aforesaid *Calculus*; and withal has put it into another Form yet much more easy and compendious, by bringing what M. *Cassini* had given us in odd Numbers, to the Millesimals of a Circle, both as to *Numb.* I. which he calls *Numb.* A. being the mean Anomalie of *Jupiter* in such parts; as also to *Numb.* II. or our *Numb.* B. which is the distance of the mean place of *Jupiter*, from the true place of the *Sun*, and which with the addition of the Equation of *Numb.* B. gives the true angle of *Commutation* in the same Millesimals of a Circle. And having deducted from the *Epoches* the greatest Equations both of *Numb.* A and B. he restores them by adding as much to the Equations themselves, by which means they all become Affirmative, so that the whole computation is performed by Addition only.

The Reader is supposed to be acquainted with the Method of M. *Cassini's* *Calculus*, which is at large explain'd in the aforesaid *Transaction, Num.* 214. For which reason this shorter Description may suffice at present.

EPOCHÆ

Epochæ Conjunctionum Primi Satellitis Cum Jove.

An. Jul. Curr.	Conjunct. D.	H.	'	"	Num. A.	Num. B.
1719	1	6	11	13	872	396
1720	0	20	22	40	956	310
21	1	5	2	44	40	229
22	0	19	14	11	125	143
23	0	9	25	38	209	57
1724	1	18	5	42	293	971
1725	0	8	17	10	377	889
26	1	16	57	13	462	808
27	1	7	8	41	546	722
28	0	21	20	8	630	636
1729	1	6	0	12	715	554
1730	0	20	11	39	799	468
31	0	10	23	7	883	382
32	0	0	34	34	967	296
33	0	9	14	38	52	215
1734	1	17	54	41	136	133
1735	1	8	6	9	220	47
36	0	22	17	36	305	961
37	1	6	57	40	389	880
38	0	21	9	7	473	794
1739	0	11	20	35	557	708
1740	0	1	32	2	642	622
41	0	10	12	6	726	540
42	0	0	23	33	810	454
43	1	9	3	37	895	373
1744	0	23	15	4	979	287
1745	1	7	55	8	63	205
46	0	22	6	35	148	119
47	0	12	18	3	232	33
48	0	2	29	30	316	947
1749	0	11	9	34	400	866
1749	0	11	9	34	400	866
1750	0	1	21	1	485	780
51	1	10	1	5	569	698
52	1	0	12	33	653	612
53	1	8	52	37	738	531
1754	0	23	4	4	822	445
1755	0	13	15	32	906	359
56	0	3	27	0	990	273
57	0	12	7	3	75	191
58	0	2	18	30	159	110
1759	1	10	58	34	243	24
1760	1	1	10	1	328	938
61	1	9	50	35	412	856
62	1	0	1	2	496	770
63	0	14	13	0	580	684
1764	0	4	24	27	665	598
1765	0	13	4	31	749	517
66	0	3	15	58	833	431
67	1	11	56	2	918	349
68	1	2	7	29	2	263
1769	1	10	47	33	86	182
1770	1	0	59	0	171	96
71	0	15	10	28	255	10
72	0	5	21	56	339	924
73	0	14	2	0	423	842
1774	0	4	13	27	508	761
1775	1	12	53	31	592	675
76		3	4	58	676	589
77	1	11	45	1	761	507
78	1	1	56	28	845	421
1779	0	16	7	56	929	335

Revolu-

Revolutiones Primi Satellitis Jovis in mensibus.

D.	h.	′	″	N. A.	Nu. B.
Januarii.					
1	18	28	36	0	5
3	12	57	12	1	9
5	7	25	48	1	14
7	1	54	24	2	18
8	20	23	0	2	23
10	14	51	36	2	27
12	9	20	12	3	32
14	3	48	48	3	37
15	22	17	24	4	41
17	16	46	0	4	46
19	11	14	36	4	51
21	5	43	12	5	55
23	0	11	47	5	60
24	18	40	23	6	64
26	13	8	59	6	69
28	7	37	35	7	73
30	2	6	11	7	78
31	20	34	47	7	82
Februarii					
0	20	34	47	7	82
2	15	3	23	8	87
4	9	31	59	8	92
6	4	0	35	9	96
7	22	29	11	9	101
9	16	57	47	9	105
11	11	26	23	10	110
13	5	54	59	10	114
15	0	23	35	11	118
16	18	52	11	11	123
18	13	20	47	11	128
20	7	49	23	12	132
22	2	17	59	12	137
23	20	46	35	13	141
25	15	15	11	13	146
27	9	43	47	13	150
Martii.					
1	4	12	23	14	155
2	22	40	59	14	159
4	17	9	35	15	164
6	11	38	10	15	168
8	6	6	46	16	173
10	0	35	22	16	177
11	19	3	58	16	182
13	13	32	34	17	186
15	8	1	10	17	190
17	2	29	46	18	195
18	20	58	22	18	199
20	15	26	58	18	204
22	9	55	34	19	208
24	4	24	10	19	213
25	22	52	46	20	217
27	17	21	22	20	221
29	11	49	58	20	225
31	6	18	34	21	230

Aprilis.

Revolutiones Primi Satellitis Jovis in mensibus.

D.	h.	′	″	N. A.	Nu. B.
Aprilis.					
0	6	8	34	21	230
2	0	47	10	21	235
3	19	15	46	22	239
5	13	44	22	22	244
7	8	12	58	22	248
9	2	41	34	23	252
10	21	10	10	23	257
12	15	38	46	24	261
14	10	7	22	24	265
16	4	35	58	25	270
17	23	4	33	25	274
19	17	33		25	279
21	12	1	45	26	283
23	6	30	21	26	287
25	0	58	57	27	292
26	19	27	33	27	296
28	13	56	9	27	300
30	8	24	45	28	304
Maii.					
0	8	24	45	28	304
2	2	53	21	28	309
3	21	21	57	29	313
5	15	50	33	29	317
7	10	19	9	29	322
9	4	47	45	30	326
10	23	16	21	30	330
12	17	44	57	31	335
14	12	13	33	31	339
16	6	42	9	31	343
18	1	10	45	32	348
19	19	39	21	32	352
21	14	7	57	33	356
23	8	36	33	33	361
25	3	5	9	33	365
26	21	33	45	34	369
28	16	2	21	34	373
30	10	30	57	35	378
Junii					
0	10	30	57	35	378
1	4	59	32	35	382
2	23	28	8	36	386
4	17	56	44	36	391
6	12	25	20	36	395
8	6	53	56	37	399
10	1	22	32	37	403
11	19	51	8	38	408
13	14	19	44	38	412
15	8	48	20	38	416
17	3	16	56	39	420
18	21	45	32	39	425
20	16	14	8	40	429
22	10	42	44	40	433
24	5	11	20	40	438
25	23	39	56	41	442
27	18	8	32	41	446
29	12	37	8	42	450

Revolutiones Primi Satellitis Jovis in mensibus.

D.	h.	′	″	N. A.	Nu. B.
Julii					
1	7	5	44	42	455
3	1	34	20	42	459
4	20	2	56	43	463
6	14	31	32	43	468
8	9	0	8	44	472
10	3	28	44	44	476
11	21	57	20	45	480
13	16	25	55	45	485
15	10	54	31	45	489
17	5	23	7	46	493
18	23	51	43	46	498
20	18	20	19	47	502
22	12	48	55	47	506
24	7	17	31	47	510
26	1	46	7	48	515
27	20	14	43	48	519
29	14	43	19	49	523
31	9	11	55	49	528
Augusti.					
0	9	11	55	49	528
2	3	40	31	49	532
3	22	9	7	50	536
5	16	37	43	50	541
7	11	6	19	51	545
9	5	34	55	51	549
11	0	3	31	51	554
12	18	32	7	52	558
14	13	0	43	52	562
16	7	29	19	53	567
18	1	57	55	53	571
19	20	26	31	54	575
21	14	55	7	54	580
23	9	23	43	54	584
25	3	52	18	55	588
26	22	20	54	55	593
28	16	49	30	56	597
30	11	18	6	56	602
Septembris.					
1	5	46	42	56	606
3	0	15	18	57	610
4	18	43	54	57	615
6	13	12	30	58	619
8	7	41	6	58	624
10	2	9	42	58	628
11	20	38	18	59	632
13	15	6	54	59	637
15	9	35	30	60	641
17	4	4	6	60	646
18	22	32	42	60	650
20	17	1	18	61	655
22	11	29	54	61	659
24	5	58	30	62	663
26	0	27	6	62	668
27	18	55	42	62	672
29	13	24	18	63	677

Revolutiones Primi Satellitis Jovis in mensibus.

Octobris. D.	h.	′	″	N. A.	Nu. B.
1	7	52	54	63	681
3	2	21	30	64	686
4	20	50	6	64	690
6	15	18	41	65	695
8	9	47	17	65	699
10	4	15	53	65	704
11	22	44	29	66	708
13	17	13	5	66	713
15	11	41	41	67	717
17	6	10	17	67	721
19	0	38	53	67	726
20	19	7	29	68	730
22	13	36	5	68	735
24	8	4	41	69	739
26	2	33	17	69	744
27	21	1	53	69	749
29	15	30	29	70	753
31	9	59	5	70	758
Novembris.					
0	9	59	5	70	758
2	4	27	41	71	762
3	22	56	17	71	767
5	17	24	53	71	772
7	11	53	29	72	776
9	6	22	5	72	781
11	0	50	41	73	785
12	19	19	17	73	790
14	13	47	53	74	794

Novembris. D.	h.	′	″	N. A.	Nu. B.
16	8	16	29	74	799
18	2	45	5	74	804
19	21	13	40	75	808
21	15	42	16	75	813
23	10	10	52	76	817
25	4	39	28	76	822
26	23	8	4	76	827
28	17	36	40	77	831
30	12	5	16	77	836
Decembris.					
0	12	5	16	77	836
2	6	33	52	78	840
4	1	2	28	78	845
5	19	31	4	78	849
7	13	59	40	79	854
9	8	28	16	79	859
11	2	56	52	80	863
13	21	25	28	80	868
14	15	54	4	80	873
16	10	22	40	81	877
18	4	51	16	81	882
19	23	19	52	82	886
21	17	48	28	82	891
23	12	17	4	82	897
25	6	45	40	83	900
27	1	14	16	83	905
28	19	42	52	84	909
30	14	11	28	84	914

Prima Æquationes Conjunctionum Primi Satellitis cum Joue.

Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B.
0	39	8	15	128	12	7	26	256	0	1	31	384	11	52	26
4	38	12	16	132	11	27	26	260	0	0	31	388	12	37	26
8	37	16	16	136	10	47	26	264	0	1	31	392	13	23	25
12	36	21	16	140	10	9	27	268	0	3	31	396	14	11	25
16	35	26	17	144	9	31	27	272	0	7	31	400	14	59	25
20	34	30	17	148	8	45	27	276	0	12	31	404	15	48	24
24	33	35	17	152	8	19	27	280	0	19	31	408	16	38	24
28	32	40	18	156	7	44	28	284	0	28	30	412	17	30	24
32	31	45	18	160	7	10	28	288	0	38	30	416	18	22	23
36	30	50	19	164	6	38	28	292	0	50	30	420	19	15	23
40	29	56	19	168	6	7	28	296	1	3	30	424	20	9	23
44	29	3	19	172	5	37	28	300	1	17	30	428	21	4	22
48	28	10	20	176	5	8	29	304	1	33	30	432	22	59	22
52	27	16	20	180	4	41	29	308	1	50	30	436	22	55	22
56	26	23	20	184	4	15	29	312	2	8	30	440	23	53	21
60	25	30	21	188	3	49	29	316	2	28	30	444	24	51	21
64	24	38	21	192	3	24	29	320	2	51	30	448	25	49	21
68	23	47	21	196	3	1	29	324	3	15	29	452	26	48	20
72	22	56	22	200	2	40	30	328	3	40	29	456	27	48	20
76	22	5	22	204	2	20	30	332	4	6	29	460	28	48	19
80	21	15	22	208	2	1	30	336	4	34	29	464	29	49	19
84	20	26	23	212	1	42	30	340	5	3	29	468	30	50	19
88	19	37	23	216	1	25	30	344	5	34	29	472	31	51	18
92	18	48	23	220	1	10	30	348	6	5	28	476	32	53	18
96	18	0	24	224	0	58	30	352	6	38	28	480	33	55	17
100	17	14	24	228	0	47	30	356	7	13	28	484	34	57	17
104	16	28	24	232	0	36	30	360	7	50	28	488	35	59	17
108	15	42	24	236	0	26	30	364	8	27	27	492	37	1	16
112	14	57	25	240	0	18	30	368	9	6	27	496	38	5	16
116	14	13	25	244	0	12	31	372	9	46	27	500	39	8	15
120	13	30	25	248	0	7	31	376	10	27	27	504	40	11	15
124	12	48	26	252	0	4	31	380	11	9	26	508	41	15	14
128	12	7	26	256	0	1	31	384	11	52	26	512	42	17	14

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num. A.	Æquat. Conjunct. Adde. ′ ″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′ ″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′ ″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′ ″	Æq Nu. B.
512	42 17	14	640	70 26	3	768	77 40	0	896	61 48	6
516	43 19	14	644	71 3	3	772	77 29	0	900	61 2	7
520	44 21	13	648	71 38	3	776	77 18	0	904	60 15	7
524	45 23	13	652	72 11	2	780	77 6	0	908	59 28	7
528	46 25	13	656	72 42	2	784	76 51	1	912	58 39	8
532	47 26	12	660	73 13	2	788	76 34	1	916	57 50	8
536	48 27	12	664	73 42	2	792	76 15	1	920	57 1	8
540	49 28	11	668	74 10	2	796	75 56	1	924	56 11	9
544	50 28	11	672	74 36	1	800	75 36	1	928	55 20	9
548	51 28	11	676	75 1	1	804	75 15	1	932	54 29	9
552	52 27	10	680	75 25	1	808	74 52	1	936	53 38	10
556	53 25	10	684	75 48	1	812	74 27	1	940	52 46	10
560	54 23	9	688	76 8	1	816	74 1	2	944	51 53	10
564	55 21	9	692	76 26	1	820	73 35	2	948	51 0	11
568	56 17	9	696	76 43	0	824	73 8	2	952	50 6	11
572	57 12	8	700	76 59	0	828	72 39	2	956	49 13	11
576	58 7	8	704	77 13	0	832	72 9	2	960	48 20	12
580	59 1	8	708	77 26	0	836	71 38	3	964	47 26	12
584	59 54	7	712	77 38	0	840	71 6	3	968	46 31	12
588	60 46	7	716	77 48	0	844	70 32	3	972	45 36	13
592	61 38	6	720	77 57	0	848	69 57	3	976	44 41	13
596	62 28	6	724	78 4	0	852	69 21	3	980	43 46	13
600	63 17	6	728	78 9	0	856	58 45	4	984	42 50	14
604	64 5	5	732	78 13	0	860	68 7	4	988	41 55	14
608	64 53	5	736	78 15	0	864	67 29	4	992	41 0	14
612	65 39	5	740	78 16	0	868	66 49	4	996	40 4	15
616	66 24	5	744	78 15	0	872	66 9	5	1000	39 8	15
620	67 7	4	748	78 12	0	876	65 28	5	1004	38 12	16
624	67 49	4	752	78 9	0	880	64 46	5	1008	37 16	16
628	68 30	4	756	78 4	0	884	64 3	5	1012	36 21	16
632	69 10	4	760	77 58	0	888	63 19	6	1016	35 26	17
636	69 49	3	764	77 50	0	892	62 34	6	1020	34 30	17
640	70 26	3	768	77 40	0	896	61 48	6	1024	33 35	17

Secundæ

Secundæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Addendæ.

Num. B. Æqu.	0	100	200	300	400	500	600	700	800	900
	Æqu.	Æqu.	Æq	Æq.	Æq.	Æq.	Æq	Æq.	Æqu.	Æqu.
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	14 0	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52
4	14 0	12 46	9 36	5 20	1 30	0 0	1 44	5 40	9 54	12 58
8	13 59	12 40	9 26	5 9	1 23	0 1	1 52	5 51	10 3	13 2
12	13 59	12 35	9 17	4 59	1 16	0 2	1 59	6 1	10 12	13 7
16	13 58	12 29	9 7	4 48	1 9	0 3	2 7	6 11	10 21	13 11
20	13 57	12 23	8 58	4 38	1 3	0 4	2 15	6 22	10 31	13 16
24	13 56	12 17	8 48	4 28	0 57	0 5	2 24	6 33	10 40	13 20
28	13 54	12 11	8 38	4 18	0 52	0 7	2 32	6 44	10 49	13 25
32	13 53	12 4	8 28	4 8	0 46	0 10	2 41	6 55	10 57	13 29
36	13 51	11 56	8 17	3 58	0 40	0 13	2 50	7 5	11 5	13 33
40	13 49	11 49	8 7	3 58	0 35	0 16	2 59	7 16	11 13	13 36
44	13 47	11 42	7 57	3 38	0 31	0 19	3 9	7 26	11 20	13 38
48	13 44	11 34	7 47	3 29	0 27	0 23	3 19	7 36	11 27	13 41
52	13 41	11 27	7 36	3 19	0 23	0 27	3 29	7 47	11 34	13 44
56	13 38	11 20	7 26	3 9	0 19	0 31	3 38	7 57	11 42	13 47
60	13 36	11 13	7 16	2 59	0 16	0 35	3 48	8 7	11 49	13 49
64	13 33	11 5	7 5	2 50	0 13	0 40	3 58	8 17	11 56	13 51
68	13 29	10 57	6 55	2 41	0 10	0 46	4 8	8 28	12 4	13 53
72	13 25	10 49	6 44	2 32	0 7	0 52	4 18	8 38	12 11	13 54
76	13 20	10 40	6 33	2 24	0 5	0 57	4 28	8 48	12 17	13 56
80	13 16	10 31	6 22	2 15	0 4	1 3	4 38	8 58	12 23	13 57
84	13 11	10 21	6 11	2 7	0 3	1 9	4 48	9 7	12 29	13 58
88	13 7	10 12	6 1	1 59	0 2	1 16	4 59	9 17	12 35	13 59
92	13 2	10 3	5 51	1 52	0 1	1 23	5 9	9 26	12 40	13 59
96	12 58	9 54	5 40	1 44	0 0	1 30	5 20	9 36	12 46	14 0
100	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52	14 0

Tenu

Tertiæ Æquationes Addendæ.

Nu. A.	Æquationes ' "	Num. A.
0	3 30	1000
20	3 29	980
40	3 28	960
60	3 25	940
80	3 19	920
100	3 12	900
120	3 4	880
140	2 56	860
160	2 46	840
180	2 34	820
200	2 22	800
220	2 10	780
240	1 57	760
260	1 44	740
280	1 30	720
300	1 17	700
320	1 5	680
340	0 53	660
360	0 41	640
380	0 31	620
400	0 22	600
420	0 14	580
440	0 8	560
460	0 4	540
480	0 2	520
500	0 0	500

Semidurationes Eclipsium Primi Satellitis Jovis.

Nu. A.	Semidurationes H ' "	Nu. A	Semidurationes H ' "	Nu. A	Semidurationes H ' "	Nu. A	Semidurationes H ' "
0	1 5 9	250	1 7 0	500	1 5 9	750	1 7 46
10	1 4 56	260	1 7 15	510	1 4 53	760	1 7 57
20	1 4 44	270	1 7 31	520	1 4 39	770	1 8 7
30	1 4 33	280	1 7 45	530	1 4 26	780	1 8 15
40	1 4 23	290	1 7 57	540	1 4 15	790	1 8 22
50	1 4 13	300	1 8 7	550	1 4 7	800	1 8 26
60	1 4 7	310	1 8 15	560	1 4 3	810	1 8 28
70	1 4 4	320	1 8 22	570	1 4 1	820	1 8 30
80	1 4 2	330	1 8 27	580	1 4 0	830	1 8 28
90	1 4 0	340	1 8 28	590	1 4 3	840	1 8 26
100	1 4 2	350	1 8 29	600	1 4 7	850	1 8 22
110	1 4 3	360	1 8 27	610	1 4 13	860	1 8 16
120	1 4 6	370	1 8 24	620	1 4 23	870	1 8 8
130	1 4 12	380	1 8 17	630	1 4 35	880	1 8 0
140	1 4 21	390	1 8 9	640	1 4 49	890	1 7 50
150	1 4 31	400	1 7 58	650	1 5 4	900	1 7 37
160	1 4 42	410	1 7 46	660	1 5 19	910	1 7 22
170	1 4 55	420	1 7 31	670	1 5 36	920	1 7 8
180	1 5 9	430	1 7 14	680	1 5 54	930	1 6 55
190	1 5 23	440	1 6 58	690	1 6 10	940	1 6 40
200	1 5 39	450	1 6 40	700	1 6 28	950	1 6 23
210	1 5 55	460	1 6 20	710	1 6 46	960	1 6 8
220	1 6 11	470	1 6 2	720	1 7 2	970	1 5 54
230	1 6 26	480	1 5 45	730	1 7 17	980	1 5 37
240	1 6 43	490	1 5 26	740	1 7 33	990	1 5 22
250	1 7 0	500	1 5 9	750	1 7 46	1000	1 5 9

The Use of the foregoing TABLES.

THE Eclipses of the first Satellite of *Jupiter*, as has been already said, afford the best means of determining the Longitude of places on the Land, where Telescopes of a convenient length may be used; thirteen of these Eclipses happening every 23 Days; but it is requisite that the Observer know near the matter when these opportunities offer themselves, least on the one hand he let them slip, or else grow weary by a too long attendance on them.

Those therefore who are curious to observe them, may readily compute the times of the Immersions or Emersions of this Satellite, and that with great exactness, by the following very short Precepts, which admit of no Exception or Caution, *viz.*

Out of the first Table take the *Epoche* for the Year, with its corresponding *Numb.* A and *Numb.* B; and to them add, out of the Tables of Months, the Day, Hour, Minute and Second, nearest less than the time of the Eclipse you seek for, together with its *Num.* A and B: the Sum of the times is the mean time of the middle of the Eclipse. 2. With *Num.* A thus collected take out the first Æquation of the Conjunctions; as also the Æquation of *Num.* B. always to be added to *Num* B. before found. 3. With *Num* B so equated, take out the second Æquation of the Conjunctions; and in the last Table, the third Æquation, as also the Semi-duration of the Eclipse answering to *Num.* A. 4. To the mean time of the middle of the Eclipse, add all those three Æquations; the Sum shall be the true equated time of the middle of the Eclipse sought. 5. If *Num.* B. equated be less than 500, subtract the

the Semiduration, and you will have the time of the Immersion, or if it be more than 500, adding the same, it will give the time of the Emersion.

But Note, the times thus found are equal time, still to be reduced to the Apparent: and that in the *Bissextile* Year, after *February*, one Day is to be deducted from the Day of the Month.

The less skilful may perhaps be pleas'd with an Example or two, which may serve them to imitate. Let it be required to find the time of the Immersion of this Satellite into *Jupiter*'s shadow, *November* the 9*th* 1719. in the Morning. The Work stands thus,

	D. h. ′ ″	Nu. A.	Nu. B.	
1719.	1. 6.11.13	872	396	
Novemb.	7.11.53.29	72	776	
Conj. Med.	8.18. 4.42	944	172	
Æquat. I.	51.53		10	Æq. B.
Æquat. II.	10.26		182	B. Æquat.
Æquat. III.	3.26			
	8.19.10.27			
	1. 6.33	*Semidur. Subst.*		
Novemb.	8.18. 3.54			

So that by this *Calculus*, on the ninth of *Novemb.* at 4 Minutes after 6 in the Morning, equal Time, may be seen the *Immersion* of this Satellite into *Jupiter*'s shadow.

Another Example shall be of the *Emersion* on the fifth of *April* 1720. *viz.*

1720.

	D. h. ′ ″		Nu. A.	Nu. B.	
1720.	0.20.22.40		956	310	
April	4.13.44.22	Biss.	22	244	
Conj. Med.	5.10.07.02		978	554	
Æquat. I.	44.13			13	Æq. B.
Æquat. II.	0.45			567	B. Æquat.
Æquat. III.	3.29				
	1. 5.40	*Semidur. Add.*			
April	5.12.01.09				

Hence it appears that at one Minute after Midnight following the fifth of *April*, equal Time, will happen the Emersion required. Nor do we doubt but that the Event will very nearly answer.

Lastly, it may not be amiss here to inform the Reader, that we have learnt, by the experience of many Years Observation, that the second inequality of this Satellite proceeds from the progressive Propagation of Light, and is common to all the rest of the Satellites: Light, being found to proceed in about seven Minutes of time as far as from the Sun to the Earth, whether with an equable motion or otherwise is still a question. For this reason we have added a *Third Æquation*, whereby to account for the greater distance of *Jupiter* from the *Earth* in *Aphelio* than in *Perihelio*, as the *Second Æquation* answers to the greater distance of the Planet when near the Conjunction of the Sun, than when near his Opposition.

F I N I S.

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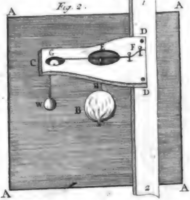
Fig. 2.



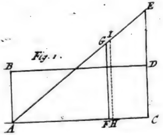


Fig. 1.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *September* and *October* 1719.

The CONTENTS

I. *A Letter of the curious Mr.* Henry Barham, R. S. Soc. *to Sir* Hans Sloan, *Bart.* Vice-President *of the* Royal Society; *giving several Experiments and Observations on the production of* Silk-Worms, *and of their* Silk *in* England, *as made by him last Summer.*

Worthy Sir,

AS you are the Patron of Industry and encourager of Natural Experiments, I think you justly claim the first View of these small ones, I made upon *Silk Worms*, the last Summer.

And altho' they may have been done before by others in some other parts of the World; yet in all the Authors I have Read I do not find they make use of the same Method; and I dare be bold to say, that these following Observations and Experiments were never made in *England* with that Nicety, as I have done, and shall do if I live. It being the first Attempt of this kind, it may come short of that compleat Methodical Manner it may be brought to hereafter; the which I hope you'll excuse.

After you have perused it your self, if you see any thing in it worthy the Communicating to the *Royal Society* (it being design'd for the Publick) you may do me the Honour. But I wholly submit to your Judgment and Opinion in this, as I do in all other things whatsoever. *I am,*

Your humble Servant.

Henry Barham.

EXperiments made in *Chelsea Park*, in the Months of *May June* and *July* 1719.

April 27. I receiv'd a small parcel of *Silk-Worms* Eggs from *Languedoc.*

May 6. Early in the Morning I found them Hatcht of themselves, the Wind shifting in the Night from *East Northerly* to the *West Southerly*, changing the Air of a sudden to Warm, two Days before the change of the Moon.

After Feeding and Managing them according to Art, through the whole Course of their four Sicknesses, they were come to their State of Perfection, being then as thick as a Man's little Finger, and from 4 to 5 Inches long, of a yellowish Colour, and when held against the Light, they

might

might be seen through as you may an Egg, being of the same Colour and Consistence (fill'd with the matter that makes the Silk) This is a certain Sign that they will begin to Spin in 24 Hours or less. They then forsake their Food (being very Voracious before) and hunt about for a convenient Place to fix their first Hold-fasts, for supporting the Balls or Cones that they are to make, which they do in a most wonderful Mathematical Manner, with a Mixture of a Gummy Substance that tyes all together; and when the loose furzy Substance is taken off, and some of the Silk is wound off, the remainder is so Smooth and Compact, shining like *Sattin*, that they are made use of for Artificial Flowers, and esteemed the best of any thing yet known for that purpose, for which (only) they are generally kept in *Boarding Schools*. I weighed many hundreds of these Silk-Balls or Cones, which I found to weigh from 35 to 40 Grains, with their *Aurelia's* or *Chrysalis* within them.

June 27. They begun to Spin, having been Hatcht 7 Weeks and 3 Days; and in 4 or 5 Days finished their laborious and curious Work: but their Balls were not fit to be removed until 8 or 10 Days.

July 7. *Monf. Lachiure* began to wind off their Silk-Balls with a Machine that made great dispatch, winding much fine Silk in a Day: I found that an Ounce of Silk-Balls would make about a Dram of fine Silk; but to be more certain, I weighed out to the Winder 12 Pounds of Silk-Balls at 4 times, and told the Balls in every 3 Pound as followeth, *viz.*

The first 3 Pound contained	812 Balls
The second 3 Pound contained	842
The third 3 Pound contained	797
The fourth 3 Pound contained	868
So that the whole 12 *lib.* Weight contained	3319 Balls.

Which when wound off, was found to yield and make one Pound and one Ounce, or 17 Ounces of fine Silk, and about 7 Ounces of coarse Refuse unwound, in all a Pound and half of *Averdupois* Weight, or 2 Pounds *Troy*; which is as great or greater making or yielding as in any part of the World, and the Silk as fine. I shewed it to a noted *Silk Broker*, who said it was *Italian* Silk, (not knowing it was made in *England*) and worth about 20 *Shillings per* Pound, if I had never so many Bales of it, *&c.*

Now

• Now upon this Experiment finding that 3319 Silk-Balls would make one Pound and one Ounce of fine Silk. I was desirous to know what quantity of Silk might be expected from the Worms Hatched from one Ounce of Eggs.

Of which to obtain the Knowledge, I made use of the following Method: by often weighing and telling I found that one hundred Eggs weighed but one Grain, so that if one Grain contains 100, a Scruple must contain 2000, and a Dram 6000, and an Ounce at 8 Drams to the Ounce, must contain 48000 Eggs. Now if every Egg hatch a Worm, and every Worm makes a Silk-Ball, there must be from one Ounce 48000 Silk-Balls; and if 3319 Balls will make one Pound and one Ounce of fine Silk, (which by Experience I found they did) then 48000 Silk-Balls will make 15 Pounds and 6 Ounces of *Averdupois* Weight in fine Silk, or 18 Pounds and eight Ounces of *Troy* Weight, which is very considerable. And in the same Proportion one Pound of *Silk Worms* Eggs, will produce Worms sufficient to make above 180 Pounds of Silk. But allowing for Casualties, and supposing but 12 Pound of fineSilk made from the Worms and their Silk-Balls produced from an Ounce of *Silk Worms* Eggs; it will be found much to exceed most Countries, according to *Augustino Gallo*'s Computation: For he sayeth, that in the *Southern* parts of *France, viz. Languedoc* and *Provence*, they make but 7 or 8 Pound of Silk from *Silk Worms* hatched from an Ounce of Eggs; and in *Brescia* in *Italy*, but 8, 9, or 10 Pound of Silk from an Ounce; only in *Calabria*, where the *Silk Worms* and their Eggs are larger, they make 11 or 12 Pounds of Silk from an Ounce of Eggs; which still doth not exceed, nay hardly comes up to, what we make in *England*.

As to the Charge and Expences of making the aforesaid quantity of Silk in *England*, different from that of other Places, I shall be able to give you a more particular Account in my next Experimental Observations.

I have only this to add, that Experience hath taught me how to hatch *Silk Worms* twice in a Year, so as to have two good Crops of Silk in one Year. And that the *Mulberry* Trees will have Leaves in *England* twice in a Year, without prejudice to either Tree or Fruit, is most certainly true. But more in my next.

II. VIRO

Viro Celeberrimo,

Richardo Mead, *M.D.*

Collegii Medicorum *Londinensium* & Societatis Regiæ Socio. *S. P. D.*

Jacobus Jurin, M.D. *&* Reg. Soc. S.

Apologiam Præstantissimi Viri, *Jacobi Keilii*, qui acerbâ nuper & immaturâ morte præreptus magnum sui desiderium Eruditis reliquit, studiose pervolvimus. Quam inter legendum singularem simul candidissimi Ingenii humanitatem, quâ nosmet, utut à placitis suis dissentientes, excipere dignatus est, simul Animi magnitudinem, & studium in Rempublicam Literariam tanto Viro dignum, magnâ admiratione prosequebamur. Huic enim ille non solum per omnem vitæ cursum diligentem & strenuam operam navavit, sed etiam pulcherrimo Exemplo, confectus jam atroci morbo & se perire sentiens, eruditam illam Epistolam tanquam supremi amoris pignus, eidem legavit. Cui tamen necessario nobis respondendum est, non sane quod acerrimo tuo Judicio diffidamus, sed ne aliis Lectoribus minus idoneis impedimento esse possit, ad rectam sententiam ferendam, Viri illius Doctissimi Auctoritas. Accipe igitur, Vir Clarissime, quæ in ejus defensione minus recte tradita censemus, & tuum simul Arbitrium esto, utrum contentionis abrepti studio iniquiores

ſimus ipſius Manibus, an ita diſputemus, ut qui de Veritate potius quam de Victoriâ ſimus ſolliciti.

Queritur primo Vir Clariſſimus, quod ſeſe una cum Doctiſſimis Viris *Borello*, & *Morlando*, tanquam Cordis Motum cum pondere inerti conferentem, injuſte perſtrinxerim. Ego certe, cum prius notaſſem Motum quendam Sanguinis & Arteriarum ex Cordis Vi oriri, dixi tandem ſciri non poſſe Cordis Potentiam quanta ſit, niſi Motus hujuſce quantitatem cognitam teneamus: Motum vero quemlibet cum pondere quieſcente comparari non magis poſſe, quam Lineam cum Rectangulo. Quibus verbis id ſignificare volui, Doctiſſimos Viros non quidem diſerte Motum Cordis cum pondere quieſcente comparare, ſed ipſos, cum Cordis Potentiam per pondus exponerent, nullam oſtendiſſe rationem, quâ Motûs quantitas ex Cordis Potentiâ oriundi poſſet æſtimari. Ex hâc Objectione, ſi recte aſſequor mentem Viri Clariſſimi, ita ſeſe expedire conatur. Cordis Potentia in preſſione conſiſtit, eamque æquabiliter in Sanguinem impendit, eodem prorſus modo, quo Gravitatis vis deorſum pondus impellit, & actione perpetuâ in motum accelerat. Proinde, cum Cordis Potentia ponderi per Corollarium *Newtonianum* definito æqualis eſt, ea Motum eundem durante Syſtole in Sanguinem imprimet, quem pondus iſtud eodem tempore cadendo per Gravitatis Vim comparabit. Ita vero cum mentem ſuam exponit Vir Cl. ſublatum iri penitus Objectionem iſtam noſtram confitemur; ſi nimirum Cordis Potentia prædicto ponderi æqualis ſit, eademque conſiſtat in æquabili preſſione per totam Syſtolen continuatâ. Atqui ex duabus iſtis Propoſitionibus poſteriorem neutiquam probare conatur Vir Doctiſſimus, ſed Hypotheſεωs loco ponit; quamvis nos rationibus quibuſdam adductis contrariam Sententiam conati ſumus veriſimiliorem reddere; nempe, quod Cordis Potentia nequaquam æquabiliter agat

in

in Sanguinem per totam Syſtolen, ſed cum totas vires exiguâ temporis particulâ collegerit, inde uno impetu in Sanguinem irruat, eumque ex Ventriculis expellat, eo modo quem in Diſſertatione noſtrâ Epiſtolari fuſius expoſuimus. Priorem vero Propoſitionem, etiam conceſſâ Viro Cl. iſtâ Hypotheſi, falſam eſſe mox demonſtrabimus.

Corollarii *Newtoniani* ſenſum quod attinet, nolumus Lectori moleſtiam nimiam faceſſere, cum neque putemus ejus intereſſe uter *Newtoni* mentem rectius acceperit; neque ita perſpicue ſententiam ſuam expoſuerit Doctiſſimus Adverſarius, quin periculum ſit, ne aliquem ei ſenſum affingamus, quem ipſe forſitan, ſi poſſet adhuc ſe defendere, foret repudiaturus. Id vero adnotaſſe operæ pretium erit, quod cum loquatur *Keillius* de Vi quâ ex Orificio aliquo aqua exprimitur, *Newtonus* nullum omninò verbum in iſto Corollario poſuerit, quo Aqua per Vim aliquam exprimi ſignificetur; ſed pondus ſolum determinaverit æquale iſti Vi, quâ totus Aquæ effluentis Motus generari poteſt, ſive quod Gravitatis Vi cadendo Motum comparare poteſt Motui aquæ eodem tempore effluentis æqualem.

Quod autem Corollarium illud, ſi non male intellexerit Vir Cl. certe non ſatis apte uſurpârit, facile perſpiciet Lector Eruditus, qui animum adverterit, quid interſit diſcriminis inter effluxum aquæ ex foramine in fundo vaſis ſemper pleni, quomodo à *Newtono* conſideratur in eo Corollario, & effluxum Sanguinis ex Corde in Aortam. In caſu enim priori aqua jam totam velocitatem comparavit, & per datum temporis ſpatium æquabiliter effluit ex foramine. At Cordis Vis per Hypotheſin *Keillianam*, applicatur Sanguini in Ventriculo quieſcenti, & cum primo temporis momento velocitate infinite parvâ verſus Aortam propellit; continuatâ vero æquabili preſſione tandem ei finitam ve-

 locitatem

locitatem imprimit, eamque perpetim auget, donec omnem Sanguinem ex Ventriculo expulerit.

Rursum in casu *Newtoniano* consideratur Motus non quidem totius aquæ Cataractâ contentæ, quæ omnis in motu constituta est, & diversa velocitate versus exitum tendit, sed aquæ solum in ipso foramine positæ & jam exilientis. Vis autem Cordis toti Sanguinis moli Ventriculo contentæ Motum imprimit, totamque Aortam versus propellit.

Denique negamus pondus quinque unciarum, à Viro Cl. determinatum, posse eam Motus quantitatem durante Cordis Systole per Gravitatis Vim comparare, quam Cordis Potentia producit, concessâ etiam ei Hypothesi istâ, quod Cordis Potentia in æquabili pressione consistat. Per hanc enim Hypothesin erit Motus à Ventriculi sinistri Potentiâ productus, ex Calculo nostro [*Transf.* Numb. 359. p. 932, 934.] æqualis Motui Ponderis Octodecim librarum circiter, quod singulis minutis secundis longitudinem uncialem percurrat. Motus autem, quem pondus quinque unciarum durante Cordis Systole, si tollatur omnis Arteriarum & Sanguinis præcedentis resistentia, sive decimâ parte minuti secundi, per Gravitatis Vim comparabit, æquabitur fere Motui Ponderis duodecim librarum, quod suprapositâ velocitate moveatur. Quod si cui libuerit adsumptâ hâc Hypothesi verum pondus definire, quod Cordis Potentiæ æquale est, is posito Calculo eliciet pondus unciarum circiter septem cum semisse. Hoc enim durante Systole Cordis eundem fere Motum cadendo comparabit, quem producit ipsa Cordis Potentia.

Sed inquiet forsitan aliquis discrimen modo expositum inter Motum à *Keilliano* pondere acquisitum, & Motum ex Potentia Cordis oriundum inde proficisci potuisse, quod forte minus accuratæ fuerint positiones illæ, quibus Characteres Algebraicos in Calculo nostro ad numeros

revo-

revocavimus. Cui dubio ut occurramus, & ostendamus simul nos longe majus discrimen inventuros fuisse, nisi contigisset ut positiones istæ *Keillio* faverent; operæ pretium erit casum aliquem simpliciorem adsumere, quo data moles aquæ, per datum orificium, dato tempore, per vim aliquam sive pressionem æquabilem exprimatur, quæ sunt conditiones ab Adversario positæ ad Potentiam Cordis definiendam.

In eo autem casu demonstrabimus neque Motum aquæ effluentis, neque Motum toti tandem moli aquæ per Vim illam impressum, Motui aquæ in Corollario *Newtoniano*; neque Vim eam sive pressionem, ponderi per istud Corollarium definito, æquari. Quod si præstare licuerit, corruat funditus necesse est tota demonstratio *Keilliana*.

Adsumemus igitur Cylindrum aquæ datum, tubo Cylindrico infinitæ longitudinis contentum; eritque pro orificio ista sectio tubi ad quam pertingit utralibet aquæ superficies, alteri autem superficiei Vis applicabitur ope Emboli eâdem Diametro cum ipso tubo. Perfluat jam dato tempore data quævis aquæ quantitas per dictam sectionem tubi; tum alia quantitas æqualis per foramen pari Diametro factum in fundo vasis, quod more *Newtoniano* usque plenum conservatur: & primo loco dispiciamus, utrum pares futuri sint in utroque casu Motus aquæ effluentis.

* Exponatur tempus effluxus aquæ per rectam A C, velocitas autem æquabilis, qua aqua effluit ex foramine in fundo vasis per rectam A B. Unde moles aquæ effluentis ex foramine, cum sit in ratione temporis & velocitatis conjunctim, exponetur per Rectangulum A B C D; & Motus ejusdem exponetur per solidum Parallelepipedon, ex eodem Rectangulo ducto in altitudinem A B, quippe qui sit in ratione composita ex rationibus molis & velocitatis.

* *Vide Fig.* II.

In

In casu altero, ubi aqua per tubum Cylindricum fluit, tempus, ut prius, exponetur per eandem rectam A C; velocitas autem aquæ erit in ratione temporis, quippe cum vis adhibita, ex Hypothesi, in datam aquæ molem æquabiliter agat, & proinde repræsentabitur per rectam mutabilem F G, rectæ A F, sive tempori ab initio effluxus, proportionalem. Molecula autem aquæ, particulâ temporis F H prædictam Sectionem præterfluens, exponetur per Rectangulum ex ipsâ F H ductâ in exponentem velocitatis F G; vel si evanescere intelligatur recula F H, per Trapezium F G I H, & moles aquæ toto tempore A C præterfluens significabitur per Triangulum rectangulum A C E. Et quoniam ex Hypothesi moles ista moli aquæ in casu priore effluenti æqualis est, erit Triangulum A C E æquale Rectangulo A B D C; unde C E, sive velocitas acquisita in fine temporis A C, dupla erit velocitatis C D sive A B, quâ aqua ex foramine in fundo vasis effluebat. Motus autem aquæ particulâ temporis F H præterlabentis, cum sit in ratione molis & velocitatis conjunctim, exponetur per Prisma evanescens, quod fit ex Trapezio F G I H ducto in velocitatem F G: Unde totus Motus aquæ toto tempore A C præterfluentis exponetur per Pyramidem, cujus basis est Quadratum rectæ C E, cujusque altitudo perpendicularis est ipsa A C. Quæ Pyramis cum sit ad Parallelepipedon casu priore definitum, ut 4 ad 3, erunt quoque Motus aquæ effluentis in utroque casu in eâdem ratione, & proinde inæquales, quod primo loco demonstrandum susceperamus.

Proximum est, ut ostendamus Motum tandem impressum toti aquæ tubo contentæ non esse æqualem Motui in exemplo primo determinato. Hic autem, cum tota ista moles aquæ per positiones supra scriptas neutiquam definita sit, adsumemus eam æqualem moli expositæ per Rectangulum A B C D, quæ in casu

Vide Fig. II.

primo

primo effluit ex foramine, quæque in secundo sectionem dictam præterfluit. Unde cum totus Motus ei tandem impressus sit in ratione molis & velocitatis in fine acquisitæ, idem exponetur per Parallelepipedon ex Rectangulo A B D C ducto in rectam C E. Hoc autem est ad Parallelepipedon, primo casu definitum, ex eodem Rectangulo & rectâ C D, ut altitudo C E ad altitudinem C D, sive in ratione duplâ. Porro, cum molem aquæ tubo contentæ per quodvis aliud Rectangulum, loco Rectanguli A B G D, exponere licuisset, patet inde Motum hunc posse quamlibet rationem ad Motum primo casu definitum obtinere, & idcirco nequaquam eidem esse æqualem. Quod erat secundo loco demonstrandum.

Superest, ut ostendamus Vim in hoc casu adhibitam ponderi per Corollarium *Newtonianum* definito non esse æqualem. Hæc autem Vis & vis Gravitatis agens in istud pondus, cum ambæ sint æquabiles, erunt in ratione Motuum ex iisdem dato tempore productorum. Quos cum inæquales esse modò demonstratum sit, erunt illæ Vires itidem inæquales. Quod erat demonstrandum postremo.

Pergit Vir Cl. ad alterum illud vitium, quod ego in ejus solutione reprehenderam, nempe quod velocitatem Sanguinis ex Corde effluentis æquabilem posuerit, quam insigniter inæqualem fieri à me demonstratum est. Negat autem se æquabilem velocitatem Sanguini tribuisse, sed pro summâ diversarum omnium velocitatum velocitatem mediam usurpasse. Præterea nondum satis sibi constare dicit, utrum æqualis vel inæqualis sit Sanguinis ejecti velocitas, sed quæ pro æquali velocitate stat ratio, eam sibi firmiorem videri. Utrum vero, qui velocitatem Sanguinis inventurus molem Sanguinis expulsi ad orificium Aortæ applicat, nullâ factâ mentione neque diversarum velocitatum, neque velocitatis

tatis mediæ, velocitatem Sanguinis æquabilem ponat, penes æquum Lectorem sit Judicium. Idem quoque facile æstimabit, utrum Vis aliqua sive pressio fluido in vase quiescenti applicata, quæ est Hypothesis Viri Doctissimi, id fluidum primo temporis momento, eâdem velocitate quâ in fine, propulsura sit.

Postquam ita satisfactum putat Vir Cl. iis Objectionibus, quas contra priorem suam Methodum attuleram, jam ad alteram illam faciliorem vindicandam accedit. In hac Ego animadverteram Virum Cl. adsumere istam Propositionem, quod Vires Cordis in diversis Animalibus sint in ratione ponderum, item ponere velocitatem Sanguinis ex sectâ Iliacâ Arteriâ profluentis æqualem ei, quâ Sanguis ex Corde in Aortam emittitur; quas ambas positiones falsas esse nobis demonstratum est. Vitium posterius non defendit Vir Cl. prius vero tuetur *Borelli* & aliorum Doctorum Virorum auctoritate, qui assumptionem istam sæpius usurparunt. Ita quidem, & nos ejusmodi assumptionem in *Borello* reprehendimus, neque valet cujusquam auctoritas contra legitimam demonstrationem. Superest ergo Viro Cl. ad examen revocanda nostra demonstratio. Hanc autem fallaci quodam Principio inniti putat, quo cum omnia Theoremata nostra superstructa sint, communi ruinâ omnia involvit. Ait enim me ponere, quod Ventriculi Cordis, tanquam solidum corpus datâ velocitate motum, in Sanguinem impingunt, eoque ictu Motus sui partem eidem communicant. Quam Hypothesin Motui neque Sanguinis, neque Cordis, neque Aeris ex Pulmone expressi, competere censet Vir Clarissimus.

Quod Pulmonem attinet, quoniam hoc obiter attingere voluit Vir D. agnosco me considerasse Pulmonem inter contrahendum tanquam datâ velocitate impingentem in Aerem contentum, idque consulto fecisse profiteor.

Quum

Quum enim tum *Bellinus*, tum alii multi Viri Doctissimi, quos inter eminet Cl. Adversarius, multa protulerint de Vi illâ, quâ Aer inter exspirandum in Sanguinem Pulmones præterfluentem agit, ejusque moleculas dissolvit; quam solutionem ipso exspirationis initio censent accidere; mihi propositum erat hanc ipsorum sententiam ad trutinam revocare. Videbam autem, quod, si aerem per Vim æquabilem sive pressionem expelli statuerem, Motus aeri à Pulmone impressus initio exspirandi, sive reactio aeris in Pulmonem, adeoque in Sanguinem præterfluentem, pro quantitate infinite parvâ habenda erat, adeoque nihil omnino eorum effectuum, quæ ipsi adscribebantur, præstare poterat. Ita vero si fecissem, jure questuros putabam *Bellini* sequaces, quod inique secum ageretur; quippe cum rejiceretur ipsorum sententia propter demonstrationem ex Hypothesi arbitrariâ & eâdem omnium adversissimâ deductam. Malui igitur ex illâ Hypothesi demonstrationem deducere, quæ omnium maxime ipsis faveret, maximamque Motus quantitatem exspirandi initio aeri tribueret. Hæc autem erat, quâ ponebatur Pulmo initio exspirationis datâ velocitate in Aerem impingere.

Cæterum in Potentiâ Cordis definiendâ istam quidem Hypothesin, quâ ipsius Ventriculi, omni impetu momento temporis concepto, tanquam solidum corpus datâ velocitate præditum, in Sanguinem irruunt, primo loco propono, tanquam omnium simplicissimam, ex eâque solutionem deduco. Atqui deinde considero tum eam Hypothesin, quâ Ventriculi Cordis Motum omnem suum particulâ temporis admodum parvâ concipiunt, quæque mihi veri simillima videtur, tum ipsam Hypothesin *Keillianam*, atque alias infinitas, iisque omnibus solutionem meam accommodo. Adeo ut, sive istud Principium incertum & fallax, sive verum & stabile

reperiatur, nihil exinde ſolutionis noſtræ certitudini detrahatur.

Non tamen videmus aliquid argumenti allatum, quo minus iſtam poſitionem nobis adhibere, pari jure atque Viro Cl. contrariam illam de Vi ſive preſſione uſurpare licuerit. Nihil ſane ſpatii inter parietes Ventriculorum & Sanguinem intercedere non diffitemur, & tamen quare res ictu peragi nequeat nondum liquet. Certe, ſi Cubo Globum contingenti ictus imprimatur, Cubus partem Motus ſibi impreſſi Globo communicabit pari facilitate, ac ſi ſpatium inter eos interceſſerit.

At hæc ſunt corpora ſolida, & ubi de fluidorum Motu agitur, longe alia res eſt. Diſcrimen ſane inter ictus corporum ſolidorum, & actionem ſive ſolidi in fluidum, ſive fluidi in ſolidum, fuſius exponit Vir Cl. quod diſcrimen cum me minus advertiſſe cenſeat, ex eo fonte fluere pronunciat quicquid Erroris in meis Propoſitionibus continetur. Ego vero differentiam iſtam ut recte traditam a Viro Cl. lubens admitto, & aio me communem illam doctrinam neutiquam ignoraſſe, cum nihil frequentius in Mechanicis ſcriptoribus occurrat, ſed caſus quoſdam novos expoſuiſſe, quibus ea doctrina cum adhiberi nequiret, alia erat ineunda ratio atque hactenus fuerat uſurpata. Ea tribus verbis abſolvi poteſt. Nam, ut exemplo facillimo utamur, quieſcere ponatur Cylindrus aquæ datæ longitudinis in dato tubo, & moveatur per iſtum tubum Cylindrus alius ſolidus pari diametro, ac datâ velocitate in Cylindrum aqueum impingat. Quid inde futurum eſt? Nempe totus Cylindrus aquæ eo ictu in motum ciebitur, pari ratione, ac ſi fuiſſet & ipſe ſolidus Cylindrus: alter vero Cylindrus Motus ſui partem momento temporis deperdet, & ambo Cylindri communi velocitate per tubum deferentur. Simili modo res eveniet, ſi Cylindrus aqueus per tubum fluens Cylindro ſolido quieſcenti

centi impegerit. Quod si Cylindrus aqueus datâ velocitate per tubum feratur, eique occurrat Cylindrus solidus aliâ velocitate, ita ut quantitates Motuum Cylindri aquei & solidi utrinque pares sint, jam momento temporis destruetur utriusque Cylindri Motus, pariter ac si duo solida corpora æquali Motu prædita sibi mutuo occurrant. Casus magis compositos quoscunque ex dissertatione nostrâ de Motu Aquarum fluentium facile eruet Lector Eruditus, idemque simul videbit, quomodo id fieri possit, quod Adversarium Cl. præcipue torsisse videtur, nempe, quod Sanguinem toto impetu ex Ventriculo ruentem sisti posse docuerim, occurrente in contrarium corpore solido datâ Motus quantitate prædito.

Quod autem nos amicè admodum hortatur Vir Candidissimus, ut sepositâ nostrâ de Vasorum ictu Hypothesi, & Vi pressuræ, quâ Naturam uti censet, pro Principio adhibitâ, Theoremata alia construamus; id profecto, nisi gravi morbo impeditus perfunctorie prorsus evolvisset nostram Dissertationem, dudum à nobis præstitum animadvertere potuisset. Quum enim ponimus Motum Cordis in ratione temporis augeri, eâdem utique Hypothesi utimur, ac si Vim pressionis adhibeamus. Hoc autem posito, Motum ex Cordis Potentiâ oriundum determinavimus, duplo scilicet majorem quam ubi Ventriculorum ictu res peragitur. Calculum vero ipsum, ut satis facilem & priori nostro similem, Lectori reliquimus instituendum. Quæ autem sequuntur Theoremata & in iis Theorema quintum, quod rejiciendum statuit Vir Cl. tanquam ex Hypothesi de Ventriculorum ictu deductum, neutiquam pendent ex istâ Hypothesi, sed ex ipsâ Hypothesi Doctissimi Adversarii pari facilitate demonstrantur.

Nequaquam dubitamus, quin ipse Vir Cl. quid ista veri habeant, si in vivis adhuc ageret, pro suâ sagaci-

 tate

tate facile perspecturus foret; jam vero, quoniam egregium illud Rei Medicæ Lumen amisimus, eadem aliis Eruditis perpendenda simul proponimus & dijudicanda. Tibi præsertim, Vir Doctissime, cujus auctoritatem & ille plurimi fecit, & nos præcipuam habemus, Judici simul integerrimo & maxime idoneo, totam istam disputationem lubentissime subjicimus:

III. *Methodus Differentialis* Newtoniana *Illustrata. Authore* Jacobo Stirling, *è Coll. Balliol.* Oxon.

ARithmeticæ pars præcipua consistit in inveniendâ in numeris quantitate quâcunque determinatâ; cum vero quantitatum & numerorum natura non patiatur ut omnes quantitates exhibeantur in numeris accurate, necesse habemus ad Approximationes confugere. Hoc est, ubi quantitatum valores mathematice accurati neqeunt obtineri, quærendi sunt ii qui ab accuratis distant minus datâ quâvis differentiâ.

Quicquid hâc de re à Veteribus ad nos pervenit, vel est particulare, ut Methodus eorum reducendi Æquationes Quadraticas; vel saltem usibus generalibus male destinatum, ut Methodus Exhaustionum. *Vieta* quidem primus erat qui aliquid generale in hâc arte assequutus est: quippe invenit methodum reducendi Æquationes Rationales, quæ solæ tunc in usu erant. In hâc acquievêre omnes Geometræ ex ejus temporibus usque ad ea *Newtoni*. Hic ex Interpolationibus primo pervenit ad Series: quas postea ad reductionem Æquationum omnium omnino generum universaliter applicuit. Hæc autem methodus procedit per quantitatum nascentium & evanescentium rationes primas & ultimas, seu si ita loqui liceat, per quantitatum coincidentium

cidentium differentias infinite parvas. Sed & ulterius promovit *Newtonus* hanc methodum; docuitque qua ratione approximandum sit ad quantitates quæ determinantur per regularem seriem terminorum, non per Æquationem ut vulgo fit. Atque sic posuit fundamenta calculi hujus Differentialis, qui procedit per quantitatum differentias cujuscunque magnitudinis: ideoque est methodo Serierum universalior. Per hasce artes *Newtonianas* universa doctrina Approximationum reducitur ad solutionem Problematis, *Invenire Lineam Geometricam quæ per data quotcunque puncta transibit.* Ex hujus inquam solutione inveniuntur radices Æquationum quarumcunque, & etiam quantitates quarum relationes ad alias datas per nullas Æquationes hactenus notas possunt exprimi. Existimo igitur *Newtonum* perduxisse methodum Approximandi ad summum perfectionis fastigium; dum ex unico simplicissimo principio totam hanc doctrinam longe lateque patentem deducit. Quapropter credendum est animum *Newtoni* non satis perspectum fuisse iis, qui ejus methodos appellant particulares, & alias tanquam suas & solas genuinas atque generales venditant, quæ aliæ non erant quam Corollaria facillima à *Newtonianis.*

Author noster, in Epistola ad *Oldenburgum, Octob.* 24. 1676. data, mentionem fecit de methodo expeditâ ducendi Lineam Parabolicam per data quotcunque puncta; qua dixit se usum fuisse ubi Series simplices non sunt satis tractabiles. Et hanc methodum primo publicavit in Lemmate quinto Libri tertii *Principiorum.* Atque in Lectionibus publicis, circa idem tempus quo dicta Epistola scripta est, *Cantabrigiæ* habitis, exposuit modum generalem determinandi Curvas cujuscunque generis quæ transibunt per totidem data puncta quot earum natura patitur. Hæ Lectiones sub titulo *Arithmetica Universalis* anno 1707. publicatæ sunt, ubi habetur

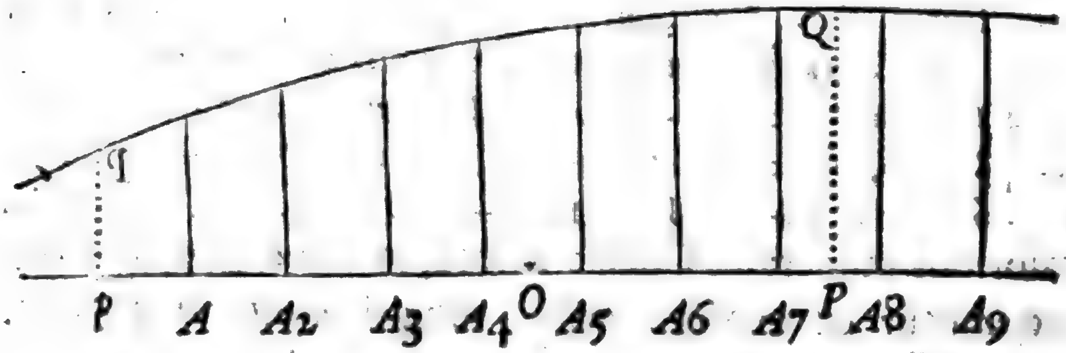
betur methodus exemplis illustrata in sectionibus Conicis. Anno vero 1711. tandem prodiit, inter alios ejusdem Authoris tractatus, ipsa Methodus Differentialis plenius quam ante exposita, cum fundamento ejus demonstrato.

Archimedes in methodo Exhaustionum, *Cavallerius* in methodo Indivisibilium, & *Wallisius* noster in Arithmetica Infinitorum, posuerunt fundamenta doctrinæ de determinanda quantitate quæsitâ per locum quem obtinet inter terminos in data Serie: at qua ratione approximandum esset ad valores quantitatum sic determinatarum horum nemo docuit; Hoc primus & solus perfecit *Newtonus:* atque exinde haud parum ampliata est universa Analysis. Nam sicut ante hoc inventum, ea Problemata Arithmetica sola pro solutis habebantur, ubi relatio quantitatis quæsitæ ad alias datas definiebatur Æquatione, jam pro solutis habenda sunt non minus ea, in quibus quantitas quæsita locum datum sortitur inter terminos datæ Seriei; siquidem numeri desiderati non minus accurate obtinentur per Methodum Differentialem, quam per extractionem Radicum: hisce vero habitis, parum interest quomodo ad eos deventum est. Et experientia multiplex docuit, quod plurima Problemata ad Æquationes ægre deducuntur, dum ad methodum Differentialem facillime. Qualis est ex multis aliis toties decantata Circuli Quadratura; quam tam perfectam, mea opinione, *Wallisius* in Arithmetica Infinitorum exhibuit quam *Archimedes* illam Parabolæ.

Propositio

Propoſitio.

Invenire Lineam Parabolicam quæ tranſibit per extrema Ordinatorum quotcunque æquidiſtantium.



B B2 B3 B4 B5 B6 B7 B8

C C2 C3 C4 C5 C6 C7

D D2 D3 D4 D5 D6

E E2 E3 E4 E5

F F2 F3 F4

G G2 G3

H H2

I

Caſus Primus.

Deſignent *A*, *A* 2, *A* 3, *A* 4, *A* 5, *A* 6, *A* 7, *A* 8, *A* 9, *&c.* Ordinatas æquidiſtantes inſiſtentes Abſciſſæ in dato angulo. Collige earum differentias *B*, *B* 2, *B* 3, *B* 4, *B* 5, *B* 6, *B* 7, *B* 8, *&c.* harumque differentias *C*, *C* 2, *C* 3, *C* 4, *C* 5, *C* 6, *C* 7, *&c.* harumque differentias *D*, *D* 2, *D* 3, *D* 4, *D* 5, *D* 6, *&c.* harumque differentias *E*, *E* 2, *E* 3, *E* 4, *E* 5, *&c.* harumque *F*, *F* 2, *F* 3, *F* 4, *&c.* Et ſic porro. Differentiæ autem colligi debent auferendo

(1054)

ferendo priores semper de posterioribus. Hoc est po- betur method... cis. A... d... nempe $B=A2-A$, $B2=A3-A2$, $B3=A4-A3$, $B4=A5-A4$, $B5=A6-A5$, &c. Tum $C=B2-B$, $C2=B3-B2$, $C3=B4-B3$, $C4=B5-B4$, &c. deinde $D=C2-C$, $D2=C3-C2$, $D3=C4-C3$, &c. Et similiter sunt omnes differentiæ sequentes colligendæ. Vel sint $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta$, &c. æquales $A, A2, A3, A4, A5, A6, A7$, &c. Eritque $A=\alpha$, $B=\beta-\alpha$, $C=\gamma-2\beta+\alpha$, $D=\delta-3\gamma+3\beta-\alpha$, $E=\varepsilon-4\delta+6\gamma-4\beta+\alpha$, $F=\zeta-5\varepsilon+10\delta-10\gamma+5\beta-\alpha$, $G=\eta-6\zeta+15\varepsilon-20\delta+15\gamma-6\beta+\alpha$, &c. In hisce valoribus numerales Coefficientes ipsorum $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. generantur ut in dignitatibus integris Binomii $\overline{1-z}|^0$, $\overline{1-z}|^1$, $\overline{1-z}|^2$, $\overline{1-z}|^3$, $\overline{1-z}|^4$, &c. Scribendo numeros 1, 2, 3, 4, 5, &c. in Serie $1\times\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}\times\frac{n-4}{5}\times$ &c. successive pro n. Sit jam PQ quælibet Ordinata reliquis intermedia, & AP ejus distantia ab Ordinata prima A appelletur z, tum erit

$$
\begin{aligned}
PQ = {} & A + \\
& B\times\frac{z}{1} + \\
& C\times\frac{z}{1}\times\frac{z-1}{2} + \\
& D\times\frac{z}{1}\times\frac{z-1}{2}\times\frac{z-2}{3} + \\
& E\times\frac{z}{1}\times\frac{z-1}{2}\times\frac{z-2}{3}\times\frac{z-3}{4} + \\
& F\times\frac{z}{1}\times\frac{z-1}{2}\times\frac{z-2}{3}\times\frac{z-3}{4}\times\frac{z-4}{5} + \\
& G\times\frac{z}{1}\times\frac{z-1}{2}\times\frac{z-2}{3}\times\frac{z-3}{4}\times\frac{z-4}{5}\times\frac{z-5}{6} + \text{\&c.}
\end{aligned}
$$

Adeoque

Adeoque ſignum ipſius z mutandum eſt, quando PQ cadit ad alteras partes Ordinatæ primæ, ut pq.

Caſus Secundus.

Sit jam $A5$ Ordinata in medio omnium; pone $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, *&c.* & $a = C4$, $b = E3$, $c = G2$, $d = I$, *&c.* id eſt, ſi ſint $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, *&c.* $A4 = \kappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, *&c.* Pone $A = \alpha - \kappa$, $B = \beta - 2\alpha + 2\kappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\kappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\kappa - 14\lambda + 6\mu - \nu$, *&c.* $a = \alpha - 2A5 + \kappa$, $b = \beta - 4\alpha + 6A5 - 4\kappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20A5 + 15\kappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70A5 - 56\kappa + 28\lambda - 8\mu + \nu$, *&c.* Et dicatur $A5P$, z, tum erit

$$PQ = A5 + \frac{Az + azz}{1.2} +$$
$$\frac{2Bz + bzz}{1.2} \times \frac{zz-1}{3.4} +$$
$$\frac{3Cz + czz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} +$$
$$\frac{4Dz + dzz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} +$$
$$\frac{5Ez + ezz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} \times \frac{zz-16}{9.10} +$$
&c.

Caſus Tertius.

Sint jam $A4$, $A5$, Ordinatæ duæ in medio omnium: Pone $A = \frac{A4 + A5}{2}$, $B = \frac{C3 + C4}{2}$, $C = \frac{E2 + E3}{2}$, $D =$

ferendo priores femper de pofterioribus. Hoc eft ponendo $B = A2 - A$, $B2 = A3 - A2$, $B3 = A4 - A3$, $B4 = A5 - A4$, $B5 = A6 - A5$, &c. Tum $C = B2 - B$, $C2 = B3 - B2$, $C3 = B4 - B3$, $C4 = B5 - B4$, &c. deinde $D = C2 - C$, $D2 = C3 - C2$, $D3 = C4 - C3$, &c. Et fimiliter funt omnes differentiæ fequentes colligendæ. Vel fint $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta$, &c. æquales A, $A2$, $A3$, $A4$, $A5$, $A6$, $A7$, &c. Eritque $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \epsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\epsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\epsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In hifce valoribus numerales Coefficientes ipforum $\alpha, \beta, \gamma, \delta, \epsilon$, &c. generantur ut in dignitatibus integris Binomii $\overline{1-z}|^0$, $\overline{1-z}|^1$, $\overline{1-z}|^2$, $\overline{1-z}|^3$, $\overline{1-z}|^4$, &c. Scribendo numeros 1, 2, 3, 4, 5, &c. in Serie $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. fucceffive pro n. Sit jam PQ quælibet Ordinata reliquis intermedia, & AP ejus diftantia ab Ordinata prima A appelletur z, tum erit

$$
\begin{aligned}
PQ = A &+ \\
& B \times \tfrac{z}{1} + \\
& C \times \tfrac{z}{1} \times \tfrac{z-1}{2} + \\
& D \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} + \\
& E \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} + \\
& F \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} \times \tfrac{z-4}{5} + \\
& G \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} \times \tfrac{z-4}{5} \times \tfrac{z-5}{6} + \text{\&c.}
\end{aligned}
$$

Adeoque

Adeoque signum ipsius z mutandum est, quando PQ cadit ad alteras partes Ordinatæ primæ, ut pq.

Casus Secundus.

Sit jam $A5$ Ordinata in medio omnium; pone $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, &c. & $a = C4$, $b = E3$, $c = G2$, $d = I$, &c. id est, si sint $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, &c. $A4 = \kappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c. Pone $A = \alpha - \kappa$, $B = \beta - 2\alpha + 2\kappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\kappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\kappa - 14\lambda + 6\mu - \nu$, &c. $a = \alpha - 2A5 + \kappa$, $b = \beta - 4\alpha + 6A5 - 4\kappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20A5 + 15\kappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70A5 - 56\kappa + 28\lambda - 8\mu + \nu$, &c. Et dicatur $A5P$, z, tum erit

$$
\begin{aligned}
PQ = A5 + \frac{Az + azz}{1.2} + {} \\
& \frac{2Bz + bzz}{1.2} \times \frac{zz-1}{3.4} + \\
& \frac{3Cz + czz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} + \\
& \frac{4Dz + dzz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} + \\
& \frac{5Ez + ezz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} \times \frac{zz-16}{9.10} + \\
& \&c.
\end{aligned}
$$

Casus Tertius.

Sint jam $A4$, $A5$, Ordinatæ duæ in medio omnium: Pone $A = \frac{A4 + A5}{2}$, $B = \frac{C3 + C4}{2}$, $C = \frac{E2 + E3}{2}$, $D =$

 $G + G2$

$\frac{G+G2}{2}$, &c. $a=B4$, $b=D3$, $c=F2$, $d=H$, &c. Vel sint $A5=\alpha$, $A6=\beta$, $A7=\gamma$, $A8=\delta$, &c. $A4=\varkappa$, $A3=\lambda$, $A2=\mu$, $A=\nu$, &c. Deinde erunt $2A=\alpha+\varkappa$, $2B=\beta-\alpha-\varkappa+\lambda$, $2C=\gamma-3\beta+2\alpha+2\varkappa-3\lambda+\mu$, $2D=\delta-5\gamma+9\beta-5\alpha-5\varkappa+9\lambda-5\mu+\nu$, &c. Et $a=\alpha-\varkappa$, $b=\beta-3\alpha+3\varkappa-\lambda$, $c=\gamma-5\beta+10\alpha-10\varkappa+5\lambda-\mu$, $d=\delta-7\gamma+21\beta-35\alpha+35\varkappa-21\lambda+7\mu-\nu$, &c. Et sit O punctum medium inter $A4$, $A5$, atque appelletur OP, z; eritque Ordinata

$$
\begin{aligned}
PQ = {} & \frac{A+az}{4^0} + \\
& \frac{3B+bz}{4^1}\times\frac{4zz-1}{2\cdot 3} + \\
& \frac{5C+cz}{4^2}\times\frac{4zz-1}{2\cdot 3}\times\frac{4zz-9}{4\cdot 5} + \\
& \frac{7D+dz}{4^3}\times\frac{4zz-1}{2\cdot 3}\times\frac{4zz-9}{4\cdot 5}\times\frac{4zz-25}{6\cdot 7} + \\
& \frac{9E+ez}{4^4}\times\frac{4zz-1}{2\cdot 3}\times\frac{4zz-9}{4\cdot 5}\times\frac{4zz-25}{6\cdot 7}\times\frac{4zz-49}{8\cdot 9} + \text{\&c.}
\end{aligned}
$$

In hisce duobus etiam casibus z est negativa, quando Ordinata PQ cadit ad alteras partes initii Abscissæ. Et in omnibus tribus casibus distantia communis Ordinatarum ponitur unitas.

Omnes tres casus demonstrantur facillime per calculum. In casu primo pro PQ scribo successive $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. & pro z interea 0, 1, 2, 3, 4, &c. quæ sunt longitudines Abscissæ ordine sequentes; & provenient æquationes

$$\alpha=A,\ \beta=A+B,\ \gamma=A+2B+C,\ \delta=A+3B+3C+D,$$
$$\varepsilon=A+4B+6C+4D+E, \text{ \&c.}$$

$\beta-\alpha$

$\beta - \alpha = B$, $\gamma - \beta = B + C$, $\delta - \gamma = B + 2C + D$, $\varepsilon - \delta = B + 3C + 3D + E$, &c.

$\gamma - 2\beta + \alpha = C$, $\delta - 2\gamma + \beta = C + D$, $\varepsilon - 2\delta + \gamma = C + 2D + E$, &c.

$\delta - 3\gamma + 3\beta - \alpha = D$, $\varepsilon - 3\delta + 3\gamma - \beta = D + E$, &c.

$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E$, &c.

Hæ Æquationes, capiendo earum differentias, nullo labore resolvuntur, uti videre est. Et dant eosdem ipsorum A, B, C, D, &c. valores, qui antea positi sunt in solutione. Et ad eundem modum demonstrantur casus duo reliqui.

Harum trium serierum unaquæque converget ad valorem Ordinatæ PQ, ubi Ordinatarum datarum differentiæ sunt justæ magnitudinis. At ubi non convergunt, aliæ artes adhibendæ sunt. Sed impræsentiarum de hujus Propositionis usu pauca adjiciamus.

Designent $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \varkappa, \lambda$, &c. terminos quoscunque æquidistantes, quorum differentiæ sunt perexiguæ; & relationes quas inter se obtinent definientur quamproxime per Æquationes sequentes, quæ oriuntur capiendo differentias & differentias differentiarum continuò, & ponendo eas æquales nihilo.

$$\alpha - \beta = 0$$
$$\alpha - 2\beta + \gamma = 0$$
$$\alpha - 3\beta + 3\gamma - \delta = 0$$
$$\alpha - 4\beta + 6\gamma - 4\delta + \varepsilon = 0$$
$$\alpha - 5\beta + 10\gamma - 10\delta + 5\varepsilon - \zeta = 0$$
$$\alpha - 6\beta + 15\gamma - 20\delta + 15\varepsilon - 6\zeta + \eta = 0$$
$$\alpha - 7\beta + 21\gamma - 35\delta + 35\varepsilon - 21\zeta + 7\eta - \theta = 0$$
$$\alpha - 8\beta + 28\gamma - 56\delta + 70\varepsilon - 56\zeta + 28\eta - 8\theta + \varkappa = 0$$
$$\alpha - 9\beta + 36\gamma - 84\delta + 126\varepsilon - 126\zeta + 84\eta - 36\theta + 9\varkappa - \lambda = 0.$$

&c.

Hæc

Hæc Tabula in uſum reſervanda eſt, ut conſulatur quoties opus ſit. Quod autem hæ Æquationes vel obtinent accurate, vel ad verum approximant, ubi differentiæ terminorum ſunt parvæ, patet ex demonſtratione caſus primi Propoſitionis.

Aſſumatur quælibet Series $\frac{1}{101}$, $\frac{1}{102}$, $\frac{1}{103}$, $\frac{1}{104}$, $\frac{1}{105}$, $\frac{1}{106}$ &c. Et quæratur terminus qui ſtat proximus ante $\frac{1}{101}$: patet quod ille eſt $\frac{1}{100}$; videamus ergo qualem hæc methodus exhibebit eundem. Repræſentet α terminum quæſitum, eritque

$$\begin{array}{l} \frac{1}{101}=\beta=0099,0099,0099,0, \\ \frac{1}{102}=\gamma=0098,0392,1568,7, \\ \frac{1}{103}=\delta=0097,0873,7864,1, \\ \frac{1}{104}=\varepsilon=0096,1538,4615,4, \\ \frac{1}{105}=\zeta=0095,2380,9523,8, \\ \frac{1}{106}=\eta=0094,3396,2264,2. \end{array} \qquad \text{Æquatio} \left\{ \begin{array}{c} \text{1ma} \\ \text{2da} \\ \text{3tia} \\ \text{4ta} \\ \text{5ta} \\ \text{6ta} \end{array} \right\} \text{dat } \alpha \left\{ \begin{array}{l} 0099,0099,0099,0, \\ 0099,9805,8629,2, \\ 0099,9994,3455,0, \\ 0099,9999,7824,8, \\ 0099,9999,9895,8, \\ 0099,9999,9993,1. \end{array} \right.$$

Patet ergo quod hæc methodus continue approximat. Si terminorum differentiæ fuiſſent minores, valores acceſſiſſent citius ad verum, & contra tardius quando differentiæ ſunt majores. Hinc ſi in Tabulis numericis deſit terminus, poteſt is per hanc methodum inſeri.

Hoc modo etiam prodeunt ipſiſſimæ Series Specioſæ, quæ per alias methodos prodire ſolent. Proponatur $\overline{1+zz}|^{-1}$ Ordinata Curvæ quadrandæ: Ea eſt prima in ſerie regulari $\overline{1+zz}|^{-1}$, $\overline{1+zz}|^{0}$, $\overline{1+zz}|^{1}$, $\overline{1+zz}|^{2}$, $\overline{1+zz}|^{3}$, &c. Ordinatarum, quæ omnes præter primam dant ſuas areas z, $z+\frac{1}{3}z^3$, $z+\frac{2}{3}z^3+\frac{1}{5}z^5$, $z+\frac{3}{3}z^3+\frac{3}{5}z^5+\frac{1}{7}z^7$, &c. conſtituentes novam ſeriem cujus primus terminus erit Area quæſita: quæ ideo invenietur ponendo pro eâ α, & pro reliquis in ſuo Ordine β, γ, δ, ε, &c. Prima Æquatio dat $\alpha=z$, ſecunda $\alpha=z-\frac{1}{3}z^3$, tertia $\alpha=z-\frac{1}{3}z^3+\frac{1}{5}z^5$, quarta $\alpha=z-\frac{1}{3}z^3+\frac{1}{5}z^5-\frac{1}{7}z^7$, &c.

&c. Est ergo universim area quæsita $z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \frac{1}{11}z^{11}$ &c. Estque hæc Series arcus ad Tangentem z, in circulo radium habente unitati æqualem. Eam invenit *Jacobus Gregorius* noster, & cum *Collinio* communicavit initio anni 1671. à quo, mediante *Oldenburgo* ad *Leibnitium* delata est.

Sit jam &c, e, d, c, b, a, P, α, β, γ, δ, ε, &c. Series utrinque excurrens in infinitum, ubi dantur omnes termini præter P in medio omnium. Sit $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. atque erit

$$P = \frac{A}{2} +$$

$$\frac{A-B}{6} +$$

$$\frac{5A-8B+3C}{60} +$$

$$\frac{7A-14B+9C-2D}{140} +$$

$$\frac{42A-96B+81C-32D+5E}{1260} +$$

$$\frac{66A-165B+165C-88D+25E-3F}{2772} +$$

$$\frac{429A-1144B+1287C-832D+325E-72F+7G}{24024} + \text{\&c.}$$

Investigatur hæc Series ex Æquationibus, excerpendo alternas in quibus numerus terminorum est impar. Nam earum differentiæ relinquent terminos in hac Serie; quæ itaque ad libitum produci potest.

Sit $\overline{1+z}|^{-1}$ Ordinata Hyperbolæ, & quæratur Area ejus quæ jacet supra Abscissam z, quando ea evadit unitas. Hæc Ordinata est media in Serie Ordinata-

rum

rum, &c. $\overline{1+z}|^{-5}$, $\overline{1+z}|^{-4}$, $\overline{1+z}|^{-3}$, $\overline{1+z}|^{-2}$, $\overline{1+z}|^{-1}$, $\overline{1+z}|^{0}$, $\overline{1+z}|^{1}$, $\overline{1+z}|^{2}$, $\overline{1+z}|^{3}$ &c. æquidiſtantium, hinc inde excurrente in infinitum. Adeoque Areæ ab hiſce Ordinatis genitæ conſtituent ſeriem conſimilem, cujus medius terminus erit Area quæſita; quæ proinde obtinebitur per Seriem modo expoſitam. Quando z eſt unitas, ut in caſu præſente, areæ curvarum evadunt &c. $\frac{15}{64}$, $\frac{7}{24}$, $\frac{3}{8}$, $\frac{1}{2}$, & 1, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{15}{4}$, &c. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}$, $C = \frac{7}{3} + \frac{7}{24} = \frac{21}{8}$, $D = \frac{15}{4} + \frac{15}{64} = \frac{255}{64}$, &c. Hiſce in Serie ſubſtitutis, prodit P, id eſt, area Hyperbolæ, $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{3}{4480} +$ &c. id eſt, $\frac{3}{4} - \frac{A}{4 \cdot 3} - \frac{2B}{4 \cdot 5} - \frac{3C}{4 \cdot 7} - \frac{4D}{4 \cdot 9} - \frac{5E}{4 \cdot 11} -$ &c. Ubi jam A, B, C D, &c. more *Newtoniano*, deſignant terminos in ſuo ordine ab initio. Calculum appono.

TERMINI.

Affirmativi	*Negativi.*
7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47,5
5,7	7
+7563,2539,3930,7494,1	−0631,7821,3370,8041,1

Summam negativam ſubducens ab affirmativâ, habeo pro Areâ, id eſt, pro Logarithmo Hyperbolico Binarii, numerum 6931,4718,0559,9453.

Pro

Pro constructione Tabularum quarumvis numericarum percommoda est Series quæ sequitur. Designent &c. e, d, c, b, a, α, β, γ, δ, ε, &c. terminos alternos in Serie utrinque serpente in infinitum; Pone $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. Et terminus inter α & a erit

$$\frac{A}{2} +$$
$$\frac{1}{1} \times \frac{A-B}{2^4} +$$
$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A-3B+C}{2^7} +$$
$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A-9B+5C-D}{2^{10}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A-28B+20C-7D+E}{2^{13}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A-90B+75C-35D+9E-F}{2^{16}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A-297B+275C-154D+54E-11F+G}{2^{19}} +$$
&c.

Hæc Series sequitur ex casu tertio Propositionis, ponendo $z = 0$. Coefficientes numerales literarum sic producuntur; exempli gratiâ, in quarto termino coefficiens literæ penultimæ C est 5; pone $5 + 1 = n$, & numeri qui proveniunt ex multiplicatione terminorum $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. erunt 1, 6, 15, 20, &c. Horum differentiæ 5, 9, 5, sunt numeri quæsiti. Atque adeo Series ad libitum produci potest.

Datis Logarithmis numerorum 46, 48, 50, 52, 54, 56, 58 & 60; invenire Logarithmum numeri 53, qui consistit in medio omnium. Pone $l,52 + l,54 = A = 3{,}4483{,}9710{,}34$, $l,50 + l,56 = B = 3{,}4471{,}5803{,}13$,

$l,48$

$l, 48 + l, 58 = C = 3,4446,6923,08$, $l, 46 + l, 60 = D = 3,4409,0908,19$. Hisce valoribus in Serie scriptis, primi quatuor termini dabunt 1,7242,2586,96 pro Logarithmo numeri 53. Et eadem ratione invenire licet quemvis alium intermedium.

In Constructione ergo Tabularum sufficit primo quærere aliquos terminos in debitis distantiis, nam reliqui possunt hoc modo interseri. Etenim continuo sunt intercalandi termini primo inventi, usque dum perventum fuerit ad ultimos qui desiderantur. Hoc modo habebitur tota Tabula ex datis paucis terminis sub initio pro fundamento operationis. Sed non convenit ut termini quos primo quærimus, sint omnes per totam Tabulam æquidistantes; nam si omittimus alternos ubi eorum differentia est maxima, possumus alibi per saltum omittere duos, tres, viginti aut forte plures terminos. Numerus autem terminorum inter duos datos consistentium, qui omittuntur, debet semper esse aliquis sequentium 1, 3, 7, 15, 31, 63, &c. dummodo volumus inserere eos per hanc Seriem; hoc vero neutiquam incommodabit opus.

Possunt autem pro Praxi termini in unam summam colligi, ut factum vides in hac Tabella. Prima expressio est primus terminus; secunda est summa primi & secundi; tertia est summa primi, secundi & tertii: & sic porro.

2	$\frac{A}{2}$
4	$\frac{9A-B}{16}$
6	$\frac{150A-25B+3C}{256}$
8	$\frac{1225A-245B+49C-5D}{2048}$
10	$\frac{39690A-8820B+2268C-305D+35E}{65536}$

Sic

Sic datis aliquibus terminis alternis, intermedii confestim dabuntur per hasce expressiones, nullâ ratione habitâ naturæ Tabulæ particularis. Nam hæ regulæ sunt eædem in omnibus. Areæ curvarum sunt proxime æquales areis Parabolicæ figuræ quæ transit per extrema Ordinatarum suarum. Sed quoniam laboriosum nimis esset semper recurrere ad Parabolam, computavi Tabulam sequentem, quâ Areæ directe exhibentur ex datis Ordinatis.

1	$\frac{A}{1}R$
3	$\frac{A+4B}{6}R$
5	$\frac{7A+32B+12C}{90}R$
7	$\frac{41A+216B+27C+272D}{840}R$
9	$\frac{989A+5888B-928C+10496D-4540E}{28350}R$
11	$\frac{16067A+106300B-48525C+272400D-260550E+427368F}{598752}R.$

Hic numerus Ordinatarum est impar, A est summa primæ & ultimæ, B secundæ & penultimæ, C tertiæ & antepenultimæ ; & sic porro, usque dum deventum sit ad eam in medio omnium, quæ per ultimam literam in quâque expressione repræsentatur. R est basis seu pars Abscissæ inter primam & ultimam Ordinatam interceptæ. Expressiones sunt Areæ contentæ inter Curvam, basin & Ordinatas hinc inde extremas. Tabulam pro pare numero Ordinatarum non apposui, quoniam Area cæteris paribus ex impare earum numero accuratius definitur.

Quæratur area quæ generatur ab Ordinatâ $\overline{1+zz}|^{-1}$ & jacet supra Abscissam z quando ea evadit unitas. In

$\overline{1+zz}|^{-1}$, pro z scribe $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; & prodibunt undecim Ordinatæ $1, \frac{100}{101}, \frac{25}{26}, \frac{100}{109}, \frac{25}{29}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{181}, \frac{1}{2}$. Hinc est $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28200}{18281}$, $C = \frac{25}{26} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Hisce valoribus substitutis in ultimâ expressione, & unitate pro R, invenies aream esse 785398187. Justus est hic numerus in septimâ figurâ, in octavâ verum superans Binario.

Si undecim Ordinatæ non dent aream satis exactam, erige plures; & concipe aream divisam esse in plures partes, quarum quamque seorsum quærens habebis pro lubitu justam.

Valor ipsius $\overline{1+Q}|^{n}$ exprimi potest per quamcunque trium serierum sequentium.

$$\overline{1+Q}|^{n} = 1 +$$

$$Q \times \frac{n}{1} +$$

$$Q^2 \times \frac{n}{1} \times \frac{n-1}{2} +$$

$$Q^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} +$$

$$Q^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} +$$

$$Q^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \text{\&c.}$$

Vel $\overline{1+Q}|^{n} = \overline{Q+1}|^{n} =$

$$R^n \times$$

$$R^n \times \frac{n}{1} +$$

$$R^n \times \frac{n}{1} \times \frac{n-1}{2} +$$

$$R^n \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} +$$

$R^4 \times$

$$R^4 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} +$$

$$R^5 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \times \frac{n+4}{5} + \&c.$$

posito scilicet $R = \frac{1+Q}{Q}$. Vel

$$\overline{1+Q}|^n = 1 +$$

$$\frac{2+\overline{n+1}\times Q}{\overline{1+Q}|^1} \times Q \times \frac{n}{1 . 2} +$$

$$\frac{4+\overline{n+2}\times Q}{\overline{1+Q}|^2} \times Q^3 \times \frac{n}{1 . 2} \times \frac{nn-1}{3 . 4} +$$

$$\frac{6+\overline{n+3}\times Q}{\overline{1+Q}|^3} \times Q^5 \times \frac{n}{1 . 2} \times \frac{nn-1}{3 . 4} \times \frac{nn-4}{5 . 6} +$$

$$\frac{8+\overline{n+4}\times Q}{\overline{1+Q}|^4} \times Q^7 \times \frac{n}{1 . 2} \times \frac{nn-1}{3 . 4} \times \frac{nn-4}{5 . 6} \times \frac{nn-9}{7 . 8} +$$

$$\frac{10+\overline{n+5}\times Q}{\overline{1+Q}|^5} \times Q^9 \times \frac{n}{1 . 2} \times \frac{nn-1}{3 . 4} \times \frac{nn-4}{5 . 6} \times \frac{nn-9}{7 . 8} \times \frac{nn-16}{9 . 10}$$

$$+ \&c.$$

Primæ duæ Series demonſtrantur per Caſum primum Propoſitionis. Nam ſi $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* deſignent Ordinatas totidem æquidiſtantes in Parabolicâ figurâ, erit $\overline{1+Q}|^n$ ejuſdem Ordinata, cujus diſtantia à $\overline{1+Q}|^0$ eſt n. Et ſic prodit Series prima. At ſi in alia Parabola $\overline{1+Q}|^0$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-4}$, *&c.* ſint æquidiſtantes Ordinatæ, erit $\overline{1+Q}|^n$ Ordinata in eadem, cujus diſtantia à $\overline{1+Q}|^0$ eſt $-n$; ſic proveniet Series ſecunda. Sit jam in tertia Parabola &c. $\overline{1+Q}|^{-4}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* Series Ordinatarum

æquidistantium hinc inde progrediens in infinitum, eritque in eadem $\overline{1+Q}|^{n}$ Ordinata, distantiâ n à termino medio $\overline{1+Q}|^{0}$ remota. Et sic provenit Series tertia per Casum Secundum Propositionis. Prima abrumpit quando est n integer & affirmativus, secunda quando est n integer & negativus, & tertia in casu utroque abrumpit. Per harum quamque radices numerales commode evolvuntur in Series. Tertia reliquis multo citius convergit: ejus terminus secundus adhiberi potest pro correctione, ubi fit extractio per repetitionem calculi.

Halleius in sua methodo construendi Logarithmos, ex prima harum serierum demonstrat Seriem *Mercatoris* pro Quadratura Hyperbolæ. Sit ejus Ordinata $\overline{1+z}|^{-1}$, vel $\overline{1+z}|^{n-1}$, existente n numero infinite parvo; unde per methodos Quadrandi, area quæ jacet supra Abscissam z, id est, Logarithmus numeri $1+z$, erit $\frac{\overline{1+z}|^{n}-1}{n}$. Est vero per primam Seriem $\overline{1+z}|^{n} = 1 + \frac{n}{1}z + \frac{n}{1}\times\frac{n-1}{2}z^2 + \frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}z^3 + \&c.$ adeoque in casu præsente, ubi est n infinite parvus, est $\overline{1+z}|^{n} = 1 + \frac{n}{1}z - \frac{n}{2}z^2 + \frac{n}{3}z^3 - \frac{n}{4}z^4$ &c. quo substituto in valore areæ, ea prodit $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \frac{1}{5}z^5 -$ &c. quæ est Series *Mercatoris*.

Similiter per Seriem secundam prodit hæc regula; Sit datus numerus $1+z$, pone $R = \frac{z}{1+z}$, eritque ejus Logarithmus $R + \frac{1}{2}R^2 + \frac{1}{3}R^3 + \frac{1}{4}R^4 + \frac{1}{5}R^5 +$ &c.

Per Seriem tertiam provenit sequens regula. Sit quilibet numerus R, pone $z = \frac{\overline{R-1}|^{2}}{2R}$, eritque ejus Logarithmus

garithmus $\frac{RR-1}{2R} - \frac{1}{3}Az - \frac{2}{5}Bz - \frac{3}{7}Cz - \frac{4}{9}Dz - \frac{5}{11}Ez$ — &c. Ubi A, B, C, D, E, &c. more *Newtoniano* designant terminos Seriei ſicut ab initio. Hæc Series, ut ea ex qua deducitur, reliquis duabus multis vicibus celerius approximat: eſtque eadem generalius expreſſa quam, ex fundamento haud abſimili, pro inventione Logarithmi Binarii prius dedimus.

Methodus inveniendi valores Serierum Arithmeticarum utcunque tarde convergentium.

In aliquibus Seriebus ſumma terminorum haberi nequit niſi ad pauciſſima figurarum loca, dummodo præter ſimplicem eorum additionem aliæ artes non adhibeantur. Proponatur jam Series quælibet cujus termini omnes iiſdem ſignis afficiuntur, & quorum proximi continue tendunt eſſe inter ſe æquales; quales ſunt ſequentes $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} +$ &c. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} +$ &c. Collige ſummam aliquot terminorum ſub initio, ii proxime addendi ſint α, β, γ, δ, ε, ζ, &c. In numeris proximis ſit $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, & quantitatum $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\alpha + \beta \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\alpha + \beta + \gamma \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\alpha + \beta + \gamma + \delta \times \frac{\delta + r\varepsilon}{\delta - \varepsilon}$, $\alpha + \beta + \gamma + \delta + \varepsilon \times \frac{\varepsilon + r\zeta}{\varepsilon - \zeta}$, &c. differentiæ ſint a, b, c, d, e, &c. Deinde in numeris proximis ſit $s = \frac{ac - bb}{ab - 2ac + bc}$, & ipſorum $a \times \frac{a + sb}{a - b}$, $a + b \times \frac{b + sc}{b - c}$, $a + b + c \times \frac{c + sd}{c - d}$, $a + b + c + d \times \frac{d + se}{d - e}$, &c. differentiæ ſint A, B, C, D, &c. & ſit $t = \frac{AC - BB}{AB - 2AC + BC}$: atque ſic procede

cede quoad libuerit. Tum erit $\alpha + \beta + \gamma + \delta + \varepsilon$ $+ \&c. = \alpha \times \frac{\alpha + r\beta}{\alpha - \beta} + a \times \frac{a + sb}{a - b} + A \times \frac{A + tB}{A - B} + \&c.$ atque ultra duos primos terminos hujus novæ Seriei raro opus erit progredi.

Ut si desideretur valor Seriei $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8}$ + &c. collige primos 21 terminos, quorum summam reperio fore 6813,8410,1885. Termini proxime addendi sunt $\alpha =$,0005,2854,1226, $\beta =$,0004,8309,1787. $\gamma =$,0004,4326,2411, $\delta =$,0004,0816,3265, &c. Hinc fit $r = 1$ proxime, & $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$ =,0117,6449,6282, $a = -$,0000,0017,5096, $b = -$,0000,0014,7410, $c = -$,0000,0012,4986, &c. Unde $s = \frac{1}{3}$ prope, & $a \times \frac{a + sb}{a - b} = -$, 0000,0141,8111, quem propter signum negativum subduco ab $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, & remanet,0117,6307, 8171: hic additus summæ primo inventæ 6813,8410, 1885, dat pro summa totius Seriei numerum 6931, 4718,0056, qui justus est in nonâ decimali; at ante duas hasce correctiones summa erat justa in primâ figura solâ. Si animus sit propius scopum attingere, pergendum erit ad approximationes sequentes. Si termini Seriei diversa habeant signa, conjungendi sunt, ut omnes eadem tandem habeant, ut in Serie $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. conjunctis terminis ea evadit $\frac{2}{1 \cdot 3} + \frac{2}{5 \cdot 7} + \frac{2}{9 \cdot 11} + \frac{2}{13 \cdot 15} +$ &c. Sed hic notandum est quod differentiæ $a, b, c, d, e,$ &c. ut & $A, B, C, D,$ &c. colligi debent subducendo quantitates antecedentes de subsequentibus. Et in omnibus hujusmodi Seriebus si $p, q, r,$ repræsentent tres terminos ordine sequentes, p primum,

mum, q ſecundum, r tertium, & rectangulum $\frac{p+r}{2} \times q$ non ſit majus pr, valor Seriei erit infinite magnus: at magnitudinis ſemper finitæ ubi accidit contrarium. Poteſt hæc regula nonunquam fallere, ubi termini p, q, r parum diſtant ab initio Seriei, at ſi conſiſtant inter eos ab initio aliquantum remotos, evadet regula certiſſima.

Ad alia Serierum genera debent aliæ regulæ adhiberi. Sit Series regularium Polygonorum Circulo Incriptorum, exiſtente Radio unitate.

$$
\begin{array}{r|l}
H = 2,0000,0000,0000,000 & 4 \\
G = 2,8284,2712,4746,190 & 8 \\
F = 3,0614,6745,8920,718 & 16 \\
E = 3,1214,4515,2258,051 & 32 \\
D = 3,1365,4849,0545,938 & 64 \\
C = 3,1403,3115,6954,752 & 128 \\
B = 3,1412,7725,0932,772 & 256 \\
A = 3,1415,1380,1144,299 & 512
\end{array}
$$

Dicatur jam ultimum Polygonum A, penultimum B, antepenultimum C, & reliqua in ſuo ordine retrorſum D, E, F, &c. atque area Circuli quæſita erit $A + \frac{A-B}{3} + \frac{4A-5B+C}{3 \cdot 15} + \frac{64A-84B+21C-D}{3 \cdot 15 \cdot 63} + \frac{4096A-5440B+1428C-85D+E}{3 \cdot 15 \cdot 63 \cdot 255}$ + &c. Ubi ſi pro A, B, C, D, E, &c. ſcribantur proprii valores, primi quatuor termini dabunt 3,1415,9265,3589,790 pro area Circuli. Hæc autem Series eſt generalis, ex natura Circuli neutiquam dependens: applicabilis eſt quotieſcunque numerorum approximantium differentiæ priores ſunt poſteriorum quaſi quadruplæ. Factores in Denominatoribus ſunt dignitates integræ numeri 4 unitatibus minutæ:

nutæ: quibus datis, coefficientes literarum in diversis terminis formantur ex multiplicatione continua numerorum 1, $\frac{n}{3}$, $\frac{n-3}{15}$, $\frac{n-15}{63}$, $\frac{n-63}{255}$, &c. Ubi pro n substituendus est ultimus Factorum in Denominatore.

Ultima quantitatum $x-1$, $2\sqrt[2]{x}-2$, $4\sqrt[4]{x}-4$, $8\sqrt[8]{x}-8$, $16\sqrt[16]{x}-16$, &c. æqualis est Logarithmo numeri x. Pro x scribe 2, & per repetitam extractionem radicis quadratæ exibunt numeri

$M=$ 1,0000,0000,0000,0000.
$L=$ 8284,2712,4746,1901.
$I=$ 7568,2864,0010,8843.
$H=$ 7240,6186,1322,0613.
$G=$ 7083,8051,8838,6214.
$F=$ 7007,0875,6931,7337.
$E=$ 6969,1430,7308,8294.
$D=$ 6950,2734,2438,7611.
$C=$ 6940,8641,2851,8363.
$B=$ 6936,1658,4759,4014.
$A=$ 6933,8182,9699,9493.

Dicatur ultimus numerorum A, penultimus B, & sic retro, atque Logarithmus quæsitus erit $A+\frac{A-B}{1}+\frac{2A-3B+C}{1\cdot3}+\frac{8A-14B+7C-D}{1\cdot3\cdot7}+\frac{64A-120B+70C-15D+E}{1\cdot3\cdot7\cdot15}$ + &c. Primi quinque termini dant 6931,4718,0559, 9457 pro Logarithmo Hyperbolico Binarii. Et quomodo hæc Series procedit in infinitum facile colligitur ex eo quod de priore diximus: estque etiam universalis, proprietates Hyperbolæ minime respiciens.

Extenditur quoque Methodus hæcce Differentialis ad Resolutionem Æquationum & alia quamplurima quorum hic non fit mentio. Continetque fundamenta Serierum generalissima; ut in Reductione Æquationum Irrationalium & Fluxionalium brevi forsan monstrabo.

IV. *An*

IV. *An account of some Experiments made on the 27th day of* April, 1719. *to find how much the Resistance of the Air retards falling Bodies.* By J. T. Desaguliers, LL. D. & F. R. S.

I Took 12 Balls (six of which were solid Leaden Globes of about 2 Inches Diameter; three hollow Glass Balls of about 5 Inches Diameter; and three light Pastboard hollow Globes of about the same Diameter) and having carried them to the upper Gallery in the Lanthorn, on the Dome of St. *Paul's* Church, I caused them to fall down by two at a time, in the following manner;

First, a Leaden Ball and a Glass Ball.

Secondly, a Leaden Ball and a Glass Ball.

Thirdly, a Leaden Ball and a Glass Ball.

Then I let fall in the same manner the three other Leaden Balls, each with a Pastboard Ball.

After that, having the Leaden and Pastboard Balls brought up again, I repeated the Experiment twice more with a Leaden and Pastboard Ball: then I made the Experiment twice more with a Pastboard Ball alone, to see how long it would be in falling.

Upon the whole it appeared that the Leaden Balls were a very little longer than $4\frac{1}{2}$ Seconds in falling; the two largest of the Glass Balls 6 Seconds, and the Pastboard Balls $6\frac{1}{2}$ Seconds.

The height of the Gallery, from whence the Bodies fell, was 272 Foot above the Pavement of the Church (then cover'd with Boards) upon which they fell.

The times of the Falls were taken two ways above, *viz.* with a Wheel-Chronometer, which measures a

ſmall part of Time accurately, nearer than to a quarter of a Second (made and contriv'd by Mr. *George Graham*, an ingenious Clock-maker) and with an ½ Second Pendulum: And the differences of Time between the fall of the Leaden Balls and the other Balls were taken below, by the Preſident, *Martin Folkes* Eſq; *F.R.S.* and another Perſon, who all agreed in their Obſervations of the Time, which they made each with an half Second Pendulum.

The following Table gives the Marks, Weights, and Diameters of the ſeveral Balls, in three Columns.

Leaden Balls	*Troy Weight.* *l. oz. d.*	*Diameters in Inches and Decimals.*
1c	2 : 1 : ½	2 , 1
2c	1 : 11 : 4	1 , 99
3c	1 : 11 : 12	2 , 0
4c	1 : 11 : 12	2 , 0
5c	1 : 11 : 12	2 , 0
6c	1 : 10 : 0	1 ,98
Paſtboard Balls.		
A	0 : 3 : 6	5 , 5
B	0 : 1 : 14	5 , 1
C	0 : 1 : 17	5 , 1
Glaſs Balls.		
D	0 : 3 : 13½	3 , 9
E	0 : 5 : 3½	5 ,42
F	0 : 6 : 0½	5 ,55

N.B. The Polar and Equatorial Diameters of the Glaſs Balls being different, I have ſet down a Mean Diameter for each of them; the true Diameters are thus, of D 4 & 3,8. of E 5, 6 and 5,25. of F 5, 7 & 5,4 Inches.

The

The particular Experiments are as follows.

Experiment I. Fall of 1*c* and D.
c fell by the Pendulum in $4\frac{1}{2}''$.
The Fall of D was so near it, that the Difference was not taken either above or below.

Experiments II. Fall of 2*c* and E.
2*c* fell by the Chronometer in $5''$, by the Pendulum in $4\frac{1}{2}''$.
Time of the fall of E not taken above.
The Difference taken below $1\frac{3}{4}''$.

Experiment III. Fall of 3*c* and F.
3*c* fell by Chronometer in $4\frac{1}{2}''$, by Pendulum in $4\frac{1}{2}''$.
F fell in Six Seconds.
Difference taken below was $1\frac{1}{2}''$.

Experiment IV. Fall of 4*c* and A.
4*c* fell by Chronometer in $4\frac{3}{4}$, by Pendulum in $4\frac{1}{2}$.
A fell in $6\frac{1}{2}$ Seconds.
Difference taken below $= 2''$.

Experiment V. Fall of 5*c* and B.
We made no Observation above nor below.

Experiment VI. Fall of 6*c* and C.
6*c* fell by Chronometer in $4\frac{3}{4}''$, by Pendulum in $4\frac{3}{4}''$.
C not taken above.
Difference below $= 2\frac{1}{4}''$.

Experiment VII. Fall 1 *c* and B.
1 *c* fell by Chronometer in $4\frac{3}{4}''$, by Pendulum in $4\frac{3}{4}''$.
B not taken above.
Difference taken below $2\frac{3}{8}''$.

Experiment VIII. Fall of *5c* and A.
5c fell by Pendulum in $4\frac{1}{2}''$.
A fell foul and so was not observ'd at all.
Difference taken below $2''$.

Experiment IX. Fall of B alone.
by the Chronometer in $6\frac{1}{2}''$, by the Pendulum in $6\frac{1}{2}''$.

Experiment X. Fall of C alone
by the Chronometer in $6\frac{1}{2}''$ by the Pendulum in $6\frac{1}{2}''$.

By *Galileo*'s Theory the Lead, which was $4\frac{1}{2}''$ in falling, must fall 4 Foot the first $\frac{1}{2}''$, or 16 Feet the first Second, which amounts to 324 Feet in $4\frac{1}{2}''$. But as the Sound of the Ball (as it struck the Bottom) by which we reckon'd our Time, had 272 Feet to move, we must abate a $\frac{1}{4}$ of a Second nearly, (supposing Sound to move one Mile in $4\frac{1}{2}''$) which will take away 35 Feet, that the Body must have fallen in the last $\frac{1}{4}$ of a Second, and reduce the number of Feet to 289: so that the Lead will have only fallen 17 Feet short of the Theory, which must be attributed to the Resistance of the Air.

The large Glass Ball in the 6 Seconds of its Fall, wou'd in a *Vacuum* go thro' 576 Feet: but taking away the last $\frac{1}{4}$ of a Second or 47 Feet, for motion of Sound, it must only fall 529 Feet in *Vacuo*. Now since it fell but 272, there have been 257 Feet taken off from the Fall by the Air's Resistance.

Likewise the Pastboard Ball in $6\frac{1}{2}$ Seconds must have fallen 676 Feet: but deducting the last quarter of a Second or 51 Feet for the motion of the Sound, there remains only 625 Feet for its fall in *Vacuo*. But as it fell only 272 Feet, we must allow a Retardment of 353 Feet for the Resistance of the Air.

At

At a mean we may call the weight of the Glass Ball 5 *oz. Troy*, and its Diameter 5 Inches and $\frac{1}{5}$; and the weight of the Pastboard Ball 2 Ounces *Troy*, and a little more than 5 Inches Diameter.

The Lead Balls all fell within near a Foot of one another, and made an impression in the Boards of about $\frac{1}{3}$ of their Depth.

The Barometer stood at 30, 1 Inches, and the Mercury was very Convex, and therefore inclined to rise still.

A further Account of Experiments made for the same purpose, upon the 27th Day of July *last. By the same.*

HAving found by our former Experiments, that thin Glass Balls, and even Balls of pasted Paper, were too heavy to make so considerable a Difference between the time of their Fall and the fall of Leaden Balls, that it might be easily Observ'd; I contrived a way to make dryed Hogs Bladders perfectly round, by blowing them (when moist) within a strong Spherical Box of *Lignum Vitæ*, and letting them dry in the said Box before I took them out: which I did by opening the Box that screw'd in the middle, and had a hole in the Pole of one of its Hemispheres to let the Bladder pass thro', in order to tye it after blowing; and some few small holes all over the Box, that in blowing no Air might be confin'd between the inside of the Box and the Bladder, so as to hinder it from putting on a Spherical Figure. Besides I took off the ends of the *Ureters*, the Fat and a great deal of the upper

upper Coats of the Bladders, before I blowed them in the Box, to render them ſtill lighter.

The Bladders I uſed were ſome of the thinneſt I cou'd find ready blown at a *Druggiſts*, which I moiſtned in Water, taking care to leave none in the inſide. I choſe thoſe rather than Green ones, which in drying wou'd have ſtuck ſo faſt to the inſide of the Box, that it wou'd ſcarce have been poſſible to have got them out without tearing.

Having prepared five Bladders in the manner aforeſaid, (which I have deſcribed the more fully to direct any body elſe that ſhou'd be willing to try the like Experiments) I took them up to the upper Gallery in the Lantern on the Top of the *Cupola* in St. *Pauls* Church; and there by a Contrivance, which I ſhall juſt now deſcribe, I let them fall by one at a time, together with a Leaden Ball of about 2 Inches Diameter, and weighing 2 *l. Troy*: and I took notice of the time of the Fall of each Bladder, knowing by former Experiments that the Balls are about 4 $\frac{1}{4}$ Seconds, or a little longer time, in falling the ſame Height, which is 272 Feet.

The following Table, conſiſting of five Columns, gives in the firſt, the Marks of the Bladders; in the next their Diameters; in the third their Weights in Grains *Troy*; in the fourth the times of their Fall in Second Minutes of time; and in the fifth, the difference of Time between the Falls of the Leads and of each Bladder, taken below by the *Preſident*, Dr. *Halley*, Dr. *Jurin*, *Martin Folkes* Eſq; and Mr. *George Graham* the Clock-maker. The Time was taken above with Mr. *Graham*'s Chronometer, (formerly deſcribed); and below with the ſame Inſtrument, and three half Second Pendulums, all which agreed very well together.

The

The Experiments having been made twice over, the Table is twice set down; and those Experiments in which the Bladders fell streight down, and the most regularly, have this Mark before them (*).

Marks.	*Diameters in Inches*	*Weight in Grains Troy*	*Time of the whole Fall*	*Diff. between the Lead and Bladder.*
A	5,3	128	$19\frac{3}{8}''$	$14\frac{7}{8}$ Seconds.
*B	5,193	156	$17\frac{1}{4}$	$12\frac{3}{4}$
C	5,33	$137\frac{1}{2}$	$18\frac{3}{4}$	$14\frac{1}{8}$
D	5,26	$97\frac{1}{2}$	$22\frac{1}{8}$	$17\frac{6}{8}$
*E	5,02	$99\frac{1}{8}$	$21\frac{5}{8}$	17
*A			$19''$	$14\frac{1}{2}$
B			$18\frac{5}{8}$	$14\frac{1}{4}$
*C			$18\frac{3}{8}$	14
D			24	$19\frac{1}{8}$
E			$21\frac{1}{4}$	$16\frac{6}{8}$

The Diameters and Weights may be relyed upon, being taken the Day that the Experiments were made, and the Day after; but the Diameters and Weights taken 10 Days before, not agreeing with these, I have left them out. For the Bladders by drying had lost of their Weight, and altered their Diameters.

As the Necks of the Bladders in drying shrink, so as to open a little, they must be blown before each Experiment. And for the manner of letting them fall exactly in the same Instant of time, it is described by Figure II, in which

A, A, A A, is the Hole through which the Bodies fell: 1, 2, is a Board laid over the Hole. G, D, D is another Board fixt to the first Board by the two Wood-Screws D, D, with a Pulley G at the other end of it, over the Hole. W is a two Pound Ball of Lead fastned

faſtned to a ſtrong Thread, which going over the Pulley is ſtretched horizontally from G to the Nails F; to which it is faſtned, ſo as to be about a quarter of an Inch above the Board.

B is one of the Bladders, hanging with the Neck or heavieſt part downwards, by means of a Loop of fine Thread as E H, which goes over the Horizontal Thread G E F. Now when with a pair of Sciſſars the Thread of the Lead (which in all is but one Foot long) is cut juſt at E, before the Loop of the Bladder, the Lead pulling away the String the Loop of the Bladder ſlips off the remaining Thread F E, and begins to fall exactly in the ſame Inſtant as the Lead: But if the Thread ſhould be cut between E and F, as the Lead falls its Thread might give the Bladder an oblique Direction.

He that obſerves the time either with a Pendulum or Chronometer may take it very exactly, by ſeeing the motion of the Sciſſars as they cut the Thread.

N. B. As the Diameters of the Bladders were taken by wrapping a Thread twice round them, and ſomething muſt be allowed for the thickneſs of the Thread; I have here under ſet down the Diameters of the Bladders, as corrected by that Allowance. *Viz.* A 5,28 Inches; B 5,19; C 5,30; D 5 $\frac{1}{4}$; and E juſt 5 Inches in Diameter.

The Bladder E was rough, with ſeveral Wrinkles and inequalities, which made it be longer in falling than it ought to have been, according to its Diameter and Weight.

A Pail of Water thrown down met with ſuch a Reſiſtance in falling 272 Foot thro' the Air, that it was all turn'd into Drops like Rain.

F I N I S.

ERRATA. *Phil. Tranſ.* N°. 357. Page 848. l. 22. lege ab *b* 11′ 32″. N°. 359. p. 932. l. 17. lege $t = 0''$,1. p. 937. l. 5, 6. lege reſtituuntur. Et Syſtole Arteriarum cum Cordis Diaſtole duratione convenit. N°. 362. p. 1005. l. 16. read, *proof of the falſity of the Opinion.*

VIII. *Nuperæ Observationes Astronomicæ cum Regia Societate communicatæ.*

Fig. 2.

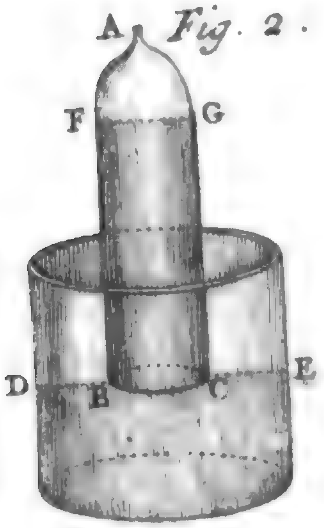


Fig. 3.

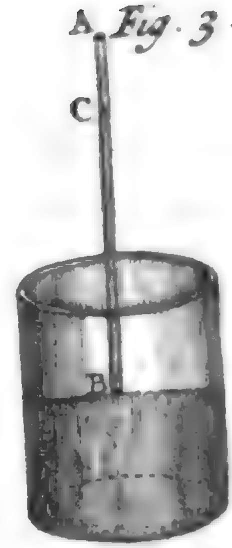


Fig. 5.

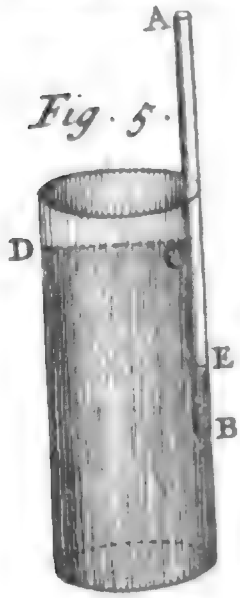


Fig. 6.

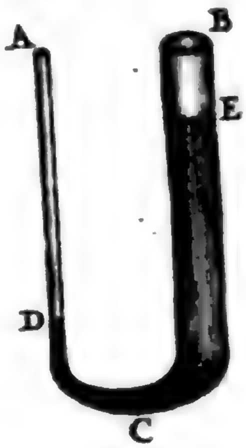


Fig. 8

N°. 362. p. 1005. l. 16. read, *proof of the falsity of the Opinion.*

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Novemb.* and *Decemb.* 1719.

The CONTENTS.

I. *A Letter of Mr.* Joſeph Williamſon *Watchmaker, to the Publiſher, wherein he aſſerts his Right to the curious and uſeful Invention of making Clocks to keep Time with the Suns Apparent Motion.*

HAving been inform'd of a *French* Book lately publiſhed, wherein the Author ſpeaks of making Clocks to agree with the Sun's apparent Motion; and ſupposeth that it was a thing never thought of by any before himſelf: I was therefore willing by the advice of ſome of my Friends, to write this ſhort Account of what I have performed in that matter my ſelf.

And in the firſt place I muſt take notice of the Copy of a Letter in this Book, wrote by one *P. Kreſa* a *Jeſuit*, to one Mr *Williamſon*, Clockmaker to his *Imperial Majeſty*; of a Clock found in the late King *Charles* the *Second* of *Spain*'s Cabinet, about the Year 1699 or 1700: which ſheweth both equal and apparent Time according to the Tables of Equation; and which went 400 Days without winding up. This I am well ſatisfied is a Clock of my own making; for about ſix Years before that time, I made one for Mr. *Daniel Quare*, for whom I then wrought moſtly, which agrees with the Deſcription he gives of it, and went 400 Days as he ſaith. This Clock Mr. *Daniel Quare* ſold, ſoon after it was made, to go to the ſaid King *Charles* the *Second* of *Spain*: and it was made ſo that if the *Pendulum* was adjuſted to the Sun's mean Motion, the Hands would ſhew Equal Time on two fixed Circles, on one the Hour, and on the other the Minute. But there were other two moveable Circles of the ſame kind, that

moved

moved forwards and backwards, as the time of the Year required; on which the same Hands shewed Apparent Time likewise, according to the Equation Tables. This Method the Author owns he knew of, and applyed the same Motion to Pocket Watches 12 or 14 Years ago, which I confess I never did; being well satisfied that Watches with Springs and Ballances are very unfit to shew the minute difference, as it increaseth and decreaseth, between equal and apparent Time.

Soon after this Clock was sent to *Spain*, I made others for Mr. *Quare* which shewed Apparent Time by lengthning and shortning the *Pendulum*, in lifting it up and letting it down again, by a Rowler somewhat in the form of an *Ellipsis*, through a slit in a piece of Brass, which the Spring at the Top of the Pendulum went through. By this means every vibration of the *Pendulum* would agree to a Second of Time of the Suns apparent Motion; that Rowler which lifted up the *Pendulum*, and let it down again, being continually moving about all the Year; so that it may seem very strange that this Author never heard of it, so many Years after they were made: For one of those, and not the first, made with the rising and setting of the Sun, Mr. *Quare* sold to the late King *William*, and it was set up at *Hampton-Court* in his Life time, where it hath been ever since. This contrivance of lengthning and shortning the *Pendulum*, I thought of several Years before I made any of them. Since then I have made others for Mr. *Quare* likewise, which shewed the difference between equal and apparent Time according to the Equation Tables, by a Hand moving both ways from the top of a Circle; on one side shewing how much a Clock keeping equal Time ought to be faster than the Sun, on the other side how much slower.

But

But these Clocks that I then made to agree with the Sun's Apparent Time, were done according to the Equation Tables, which I found not to agree very exactly with the Sun's apparent Motion: neither can any other be made to keep equal Time that will gain and lose all the Year agreeable to the said Tables: for though the Tables themselves may be true, yet some difference in Motion does proceed, in both sorts of Clocks, from Cold and Heat altering the length of their *Pendulums*. This difference by some Observations I have made, I suppose to be about the $\frac{1}{100}$ part of an Inch in the length of a *Pendulum* vibrating Seconds, which will alter the Motion of the Clock about 12 Seconds in 24 Hours. But to make my Clocks made for keeping Apparent Time to go as exact as possible, I made a Table my self by Observation: For observing the Sun, as often as it was to be seen, when it came on the Meridian, for several Years together, always setting down the Difference between its coming to the Meridian and the Time by a Clock I had adjusted as well as I could to equal Time, and always taking notice how much my Equal-Time Clock gain'd or lost at the end of every Year, I compleated my Table in the Year 1711. Since then I have made a considerable many of these Clocks, several of which I sold to Persons of great Note and Ingenuity; and in particular one I made about five or six Years since for the Right Honourable the Lord *Parker*, at present Lord High Chancellor of *Great Britain*; and all of them have given good content to those that bought them. So that I think I may justly claim the greatest right to this contrivance of making Clocks to go with Apparent Time; and I have never yet heard of any such Clock sold in *England*, but what was of my own making, though I have made of them so long.

II. *An*

II. *An Account of some new Experiments, relating to the Action of Glass Tubes upon Water and Quicksilver.* By James Jurin, M. D. Reg. Soc. & Coll. Med. Lond. Soc.

IN a * Discourse formerly presented to the *Royal Society*, I maintain'd, that the Suspension of Water in a Capillary Tube was owing to the Attraction of a small annular surface on the inside of the Tube, which touch'd the upper part of the Water. Among the several Experiments made use of to prove this Assertion, was that of a Glass Funnel of several Inches Diameter, having its small end drawn out into a very fine Tube, which Funnel being inverted and fill'd with Water, the whole quantity of Water therein contain'd was sustain'd above the Level by the Attraction of that narrow *Annulus* of Glass, with which the upper surface of the Water was in contact.

Soon after that Discourse was printed, came out a Book publish'd by a very Learned and Ingenious Member of this Society, in which that Experiment was accounted for in the following Manner.

If there be a Funnel, as A B C, Fig. 1. full of Water, and whose wide end stands in a Vessel of Water as B C; and the Top of the Funnel A ends in a Capillary Tube open at A, the whole Water will be sustain'd: the Pillar A a by the Attraction of the Circle of Glass within the Tube immediately above it; and all the rest of the Pillars of Water, as F f, D d, E e, G g, &c. in some measure by the Attraction of the parts of the Glass above them, as F, D, E, G;

* *Philos. Transact.* N°. 355.

D, E, G: And that the small Pillars or Threads of Water Dd, and Ee, do not slide down to Ff, and Gg, and so go quite down, seems to be owing to their Cohesion with the Pillar Aa, which is sustain'd by the Capillary Tube A: For if you break off the said Tube at DE, the whole Water will presently sink down.

As this Solution was very different from what I had before given, and the Reputation of that Gentleman, whose great Knowledge in Experimental Philosophy is generally known, was sufficient to give weight to any of his Opinions, I thought my self under an Obligation to examine his account of the Experiment, in order either to demonstrate its insufficiency, or to retract my own Solution. Accordingly at the next meeting of the Society, I produced the following Experiment.

The Funnel, AFGBC, *Fig* 2. whose lower part BCFG, was Cylindrical to a considerable height, and whose top was drawn out into a fine Tube at A, being fill'd with Water to the height BF, so that the surface of the Water FG, did not reach to the arched part of the Funnel, I touch'd the end A with a wetted Finger, whereby a small quantity of Water being insinuated into the Capillary Tube at A, the Water contain'd in the Funnel was suspended above the Level of the Water in the Cistern DE, as in the former Experiment.

In this Experiment it is manifest, that the little Columns, into which we may suppose the Cylinder of Water, FGBC, to be divided, are no way sustain'd by the Attraction of the arched part of the Glass above them, since they have no contact with it. Nor is there any such middle Pillar of Water, which, by its contact with the Tube at top, is both sustain'd it self, and helps to support the Pillars about it. Upon the

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ſuppoſition of which two Particulars that Gentleman's Solution was founded.

This Experiment may be thus accounted for. The Cylinder of Water F G B C, by its weight balances a part of the preſſure of the Atmoſphere, which is incumbent on the Water in the Ciſtern, and endeavours to force that Cylinder upwards. The reſt of that preſſure is balanced by the Spring of the Air, A F G, which is included between the Cylinder of Water F G B C, and the little Column of Water in the Capillary A. But, as this Air by its Spring preſſes equally every way, it muſt balance as much of the preſſure of the Atmoſphere upon the little Column of Water at A, as it does of that upon the Water in the Ciſtern. The remainder of the preſſure of the Atmoſphere upon the Column of Water at A is ſuſtain'd by the force with which that Column adheres to the Capillary Tube, which therefore does exactly balance the weight of the Cylinder of Water F G B C, and is the real, though not the immediate, cauſe of its Suſpenſion.

The experiment ſucceeds in the ſame manner when a Column of Quickſilver is raiſed into the Funnel, inſtead of the Column of Water F G B C, the top of the Tube being touch'd with a wet Finger as before. But then the height of the Quickſilver in the Funnel muſt be as much leſs than that of the Water, as its Specifick Gravity is greater.

I proceed now to acquit my ſelf of a Promiſe I made in the Diſcourſe abovemention'd, of examining whether the Experiments therein contain'd would ſucceed *in Vacuo*; and whether Water could be ſuſpended in a wide Tube by means of a Capillary at Top, at a greater height, than what it can be rais'd to by the Preſſure of the Atmoſphere.

In order to this, I boil'd some Water, and afterwards purged it of its Air by means of the Air-pump; which being done, those Experiments all succeeded in the exhausted Receiver, in the same manner as in the open Air.

The 13*th* Experiment in particular was made with a Tube of about 35 Inches in length, and a quarter of an Inch Diameter, the top of it being drawn out into a fine Capillary. Which being fill'd with Water purged of its Air, as before mention'd, the whole quantity continued suspended in the exhausted Receiver.

This plainly shews, that the success of that Experiment does not depend upon the Pressure of the Air, since the small quantity of Air left in the Receiver was by no means capable of sustaining the Water at so great a height, and consequently that the height, at which Water may be suspended in this manner, is not limited by that Pressure.

But here I must not omit taking notice of a considerable Difficulty, which presents it self to those who attentively consider this Experiment. In order to make which the better appear, it will be proper to observe what happens, when a simple Capillary Tube is fill'd with Water purged of Air, and inclos'd in the exhausted Receiver.

In this case the whole Column of Water contained in the Tube A C B, *Fig.* 3*d.* is suspended by the Attraction of the *Annulus* at the top of the Tube, A. And though that *Annulus* does not immediately act upon any part of the Water, except what is either contiguous to it, or so near as to be within the Sphere of its Attraction, which extends but to a very small distance; yet it is impossible, that any other part of the Water, as for instance that at C, should part from the Water above it and sink down, because its descent is oppos'd

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by the attraction of the contiguous *Annulus* at C. For this, being equal to the upper *Annulus* at A, is capable of sustaining a Column of Water of the length A B, and consequently is more than sufficient for supporting the Column of Water below it, C B From which it is plain, that no part of the Water contain'd in the Tube can possibly descend, unless the upper part, assisted by the weight of the Water below it, be sufficient to overcome the Attraction of the *Annulus* of Glass at A.

But in such a compound Tube as that made use of in our Experiment, *Fig.* 4*th* A C B, the case is very different, and it does not easily appear, why in a *Vacuum* any part of the Water in the wider part of the Tube, as for Example at C, should not leave that which is above it, and descend; since the *Annulus* at C is by much too wide to sustain a Column of Water of so great a length as C B.

The best answer I can give to this difficulty is, that the Cohesion between the Water contain'd in the Capillary and that below it, is sufficient to balance the weight of the Column suspended. But how far this Cohesion may depend upon the Pressure of a Medium subtile enough to penetrate the Receiver, is worthy of Consideration. For though such a Medium will pervade the Pores of the Water, as well as those of the Glass, yet it will act with its intire Pressure upon all the solid Particles, if I may so call them, of the surface of the Water in the Cistern; whereas so many of the solid Particles of the Water in the Tube, which happen to lie directly under the solid Particles of the Water above them, will thereby be secur'd from this Pressure; and consequently there will be a less Pressure of this Medium upon any surface of the Water in the Tube below the Capillary, than upon an equal surface of the Water in the Cistern. So that the Column of Water suspended

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in the Tube may be sustain'd by the difference between those two Pressures. This Explication seems to be favour'd by the following Experiments, which may all be accounted for in the same manner, though I shall anon mention another Cause, which contributes to the Success of the first and second.

The first I shall mention, is the famous Experiment of the suspension of Mercury purged of Air, to the height of 70 or 75 Inches, in the *Torricellian Tube*, in the open Air. To which we may add the sustaining of Mercury likewise purged of Air within the exhausted Receiver, as related by that Learned and Successful Promoter of Natural Knowledge, Monf. *Papin*, in his *Continuation du Digesteur*. I forbear to mention the suspension of Water purged of Air, in the *Vacuum*, which he describes in the same Book; because there is little difference between that Experiment and our own abovementioned, the very top of the arched part of his Tube, which top we may suppose as small as we please, supplying the place of the fine Capillary at the top of our Tube. But we must not omit the Experiments made by the famous Monf *Huygens*, and described by him in *Phil. Transact.* No 86. of the cohering of polish'd Plates with a considerable force in the exhausted Receiver; as likewise of the running of Water and Mercury, when purged of Air, through a Siphon of unequal Legs in the *Vacuum*: All which he accounts for from the same Principle, and much in the same manner, as we have used for explaining the Experiment above.

As to the Existence of such a Medium, I shall content my self to refer to what has been said by our *Illustrious President* in the Queries at the latter end of the last Edition of his *Opticks*: and as I have lately had the Honour to entertain the Society with some Experiments upon Quicksilver, which were exactly the reverse

reverse of those made by Dr. *Taylor*, the late Mr. *Hawksbee* and my self, upon Water; by which I am now enabled to throw this whole affair into a little System by it self, I shall beg leave to lay it down in the following Propositions, the Proof of which is contain'd in the Experiments annext.

PROPOSITION I.

The Particles of Water attract one another.

This, I think, is now universally acknowledged, and therefore needs no Demonstration; the Sphericity of the drops of Rain, and the running of two drops of Water into one another upon their contact, manifestly proving it.

PROPOSITION II.

The Particles of Quicksilver attract one another.

This is likewise manifest from the Spherical Figure, into which a drop of Mercury forms it self upon a Table; and from two of them immediately running together, as soon as they come to touch.

PROPOSITION III.

Water is attracted by Glass.

This plainly appears from all the Experiments, that we have shewn upon this Subject.

PROPOSITION IV.

Quicksilver is attracted by Glass.

Experiment I. If a small Globule of Quicksilver be laid upon a clean Paper, and be touched with a piece of clean Glass; upon drawing the Glass gently away,

away, the Quickſilver will adhere to it, and be drawn away with it. And if the Glaſs be lifted up from the Paper, the Quickſilver will be taken up by it, in the ſame manner as a piece of Iron is drawn up by the Loadſtone, and will ſtick to the Glaſs by a plain Surface of a conſiderable breadth, in proportion to the bulk of the drop, as manifeſtly appears by an ordinary Microſcope. Then if the Glaſs be held a little obliquely, the drop of Mercury will roll ſlowly upon its Axis along the under ſide of the Glaſs, till it comes to the end, where it will be ſuſpended as before.

Exp. 2*d*. If a pretty large drop of Mercury be laid upon a Paper, and two pieces of Glaſs be made to touch it, one on each ſide; upon drawing the Glaſſes gently from each other, the drop of Mercury will adhere to them both, and will be viſibly drawn out from a globular to an oval Shape; the longer Axis paſſing through the middle of thoſe Surfaces, in which the drop touches the Glaſſes.

PROPOSITION V.

The Particles of Water are more ſtrongly attracted by Glaſs, than by one another.

This manifeſtly appears from the riſing of Water in ſmall Tubes above the Level For when the Water begins to riſe into a Capillary Tube, all the Particles of Water, which touch the ſmall *Annulus* at the bottom of the Tube, muſt have quitted the contact of the other Water, and have riſen contrary to their Gravity, to come into contact with the Glaſs. After the ſame manner the other Experiments of Dr. *Taylor*, Mr. *Hawksbee* and my ſelf, upon this Subject, are eaſily explicable. For upon a careful Examination, it will be found in

in them all, that ſome parts of the Water quit the contact of the other Water, and join themſelves to the Glaſs.

PROPOSITION VI.

The Particles of Quickſilver are more ſtrongly attracted by one another, than by Glaſs.

Exp I. *Fig.* 5. If a ſmall Tube as A B, open at both ends, be dipt into a Glaſs Veſſel fill'd with Mercury, and be held cloſe to the ſide of the Veſſel, that the riſe of the Mercury within it may appear; the Mercury will partly enter into the Tube, but will ſtand within it at ſome depth, as C E, below the Surface of the Quickſilver in the Veſſel, C D; and this depth will always be reciprocally as the Diameter of the Tube.

In this Experiment a Column of Quickſilver of the height C E endeavours to force the Mercury higher into the Tube; and as Glaſs has been already prov'd to attract Quickſilver, the Attraction of the annular Surface on the inſide of the Tube, which is contiguous to the upper part of the Mercury, will likewiſe conſpire to farther its aſcent. What oppoſes the aſcent of the Quickſilver, is the Power, by which that part of it, which endeavours to riſe into the Glaſs, is drawn back by the Attraction of the other Mercury, with which it is in contact laterally, and this does not only balance the Attraction of the Glaſs, but likewiſe the weight of the Column of Mercury of the height C E, and conſequently this Attraction is conſiderably ſtronger than the Attraction of the Glaſs.

The cauſe therefore that ſuſpends the weight of the Column of Mercury C E, being the difference between the Attraction of the annular Surface of the Tube at E, and that of an equal Surface of the Quickſilver in the Ciſtern, from which the Mercury, that endeavours to

 riſe

rise into the Tube, must recede, in order to unite it self to such an Annulus of the Glass, will always be proportional to that annular Surface, or to the Diameter of the Tube. And since the Column sustain'd must be proportional to the Cause that suspends it, that Column must likewise be as the Diameter of the Tube. But the Column suspended is as the Square of the Diameter of the Tube and the height C E conjointly; from which it follows that the height C E must be as the Diameter of the Tube reciprocally, as it is found to be by Experiment.

The Experiment of the Ascent of Water above the Level in a Capillary Tube, is just the Reverse of this.

Exp. II. *Fig.* 6. Quicksilver being poured into the inverted Siphon A C B, one of whose Legs A C is narrower than the other C B; the height C E, at which the Mercury stands in the wider Leg C B, is greater than the height C D, at which it stands in the narrower Leg C A.

On the contrary, Water stands higher in the narrower Leg, than in the wider.

Exp. III. *Fig.* 7. A B C D represents a rectangular plane of Glass, which makes one side of a wooden Box. On the inside of this is another Glass plane of the same size, which at the end A C is prest close to the former, and opens to a small Angle at the opposite end B D. When Mercury is pour'd into this Box to any height as C E, it insinuates it self between the two Glass planes, and rising to different heights between the Glasses where the opening is greater or less, it forms the common Hyperbola C G F; one of whose Asymptotes E F is the line on which the Surface of the Mercury in the Box touches the inner Glass; the other is the line A C, in which the Planes are join'd. This Hyperbola being carefully examined by Mr. *Hawksbee* and

and my ſelf, the Rectangle EHG, whereſoever taken, proved always equal to it ſelf, to as great an accuracy as could be expected, when the Planes were opened to any conſiderable Angle: But when the opening was very ſmall, the inequalities of the Planes, though the beſt I could procure, bearing a greater proportion than before to the diſtance between them, occaſion'd a ſenſible variation. Which, by the way, I take to be the reaſon, why the Ordinates found by the late Mr *Hawksbee*, in examining the Curve produced in a contrary ſituation, upon dipping two Glaſs Planes ſo join'd into Spirit of Wine, do not anſwer to thoſe of the Hyperbola.

Exp. IV. *Fig.* 8. A B is a perpendicular Section through two Glaſs Planes join'd at A, and open'd to a ſmall Angle at B. C repreſents a pretty large drop of Mercury, the larger the better, which being made to deſcend as far as C, by holding the Planes in an erect poſture, with the end A downwards, retires from the contact of the Planes to D, upon inclining the Planes towards an horizontal Situation; and the diſtance C D becomes greater or leſs, as the Planes are more or leſs inclin'd towards the Horizon.

A drop of any Oily or Watery Liquor moves the contrary way, as has been ſhewn by the late Mr. *Hawksbee*.

Exp. V. *Fig.* 9. A B is a Tube open at both ends, and a Foot or two in length, whoſe lower part is drawn out into a fine Capillary at B. This Tube being fill'd with Mercury, the whole Column of Quickſilver will be ſuſtain'd in it, provided the Capillary Tube at B be ſufficiently ſmall. But if the Mercury in the end B be ſuffer'd to touch any other Mercury, it runs all out of the Tube If, without letting it touch any other Mercury, a ſmall part of the end B be broken off,

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the Mercury will run out, till it comes to some lesser height as B C, at which it will again stop, the height B C being nearly in a reciprocal proportion to the Diameter of the small end of the Tube.

The Seventh Experiment in *Phil. Transf.* N°. 355 is the Reverse of this.

Exp. VI. *Fig.* 10. Is the same in substance with the former, but made with a large Glass Funnel A B, instead of a Tube.

The Reverse of this in Water is the thirteenth Experiment in the same Transaction.

In all these Experiments it is easily seen, that the Effect is owing to the difference between the two Attractions, by which Mercury tends to Glass and to its own body; they being always opposed to one another, so that a particular Explication is no way necessary. But perhaps it may save some little trouble to the Reader, to remove the following Objection, which will readily occur to him.

In the Experiments brought to demonstrate the fourth Proposition, the Globule of Mercury adheres to the Glass in a plane Surface, which cannot be done without encreasing the Surface of the Globule, and consequently removing some of its Particles from the contact of one another. If therefore they tend more strongly to one another than to the Glass, why do they not recede from the Glass, and assume a figure perfectly Spherical, that they may all have the greatest possible contact with each other?

To this we may answer, that the Power, by which Mercury is attracted either by Glass, or by other Mercury, is proportional to the attracting Surface; and therefore, though, *cæteris paribus*, the tendency of Mercury to Glass is not so strong as its tendency to other Mercury, yet in this case a much greater number of Mer-

Mercurial Particles coming into contact with the Glaſs, than what recede from the contact of one another, it is no wonder, that the Attraction of the Glaſs prevails; and cauſes the Globule to adhere to it. For the number of Mercurial Particles which loſe their contact with the other Mercury, is no more than what makes up the difference of Surface, which ariſes from changing the figure of the Drop: whereas the Particles, which by this means come to adhere to the Glaſs, are all thoſe that conſtitute the plane Surface, in which the Globule touches it.

Which Conſideration ought likewiſe to be apply'd to the Suſpenſion of Quickſilver in Glaſs Tubes, either at extraordinary heights in the open Air, or at leſſer heights in a *Vacuum*, as above mention'd. For the top of the Tube being Spherical, or nearly ſo, it will be found, that the contact of the Mercury with the extremity of the Tube, is to the contact with other Mercury, which would be gain'd by its leaving the Top of the Tube and deſcending a very ſmall ſpace, in a *Ratio* infinitely great; and conſequently that the contact of the Mercury with the top of the Tube is one cauſe of its Suſpenſion.

Coroll. 1ſt. From this Propoſition it appears, that in a Barometer made with a narrow Tube, the Quickſilver will never ſtand at ſo great a height as in a wider. Which accounts for the *Phænomenon* ſo often mention'd in the Yearly Hiſtory of the Royal Academy of Sciences at *Paris* by Monſ. *De la Hire*; that in the Barometer, which he conſtantly made uſe of for his annual Obſervations, the Quickſilver did not riſe ſo high, as in another he kept by him, by about three Lines and a half, which is near a third of an Inch our Meaſure: For he tells us, that the Tube of his Barometer is very ſmall. So that there is no need to have recourſe to any

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peculiarity either in the Quickſilver or the Glaſs of which that Tube was made; or to an unperceived remnant of Air left in the Tube, from ſome of which cauſes that Effect and ſome others of the ſame kind were imagined to proceed.

Cor. 2d. In a Barometer made with a ſmall Tube, the Mercury will riſe and fall irregularly. For, as the height of the Mercury depends partly upon the Diameter of that part of the Tube that touches the upper Surface of the Mercury, it is plain, that the unavoidable inequalities in the Diameter of the Tube will be more conſiderable, in reſpect to the whole Diameter; and conſequently will affect the height of the Mercury more in a ſmall Tube than in a wider And this I take to be the reaſon, why it is ſo very difficult, not to ſay impoſſible, to make two Barometers, which ſhall exactly agree in the height of the Quickſilver in all conſtitutions of the Air, eſpecially if the Tubes be very narrow. This irregularity is ſtill more conſiderable in the Pendent Barometer, in which the Quickſilver moves through a large ſpace, in order to make a ſmall alteration in the length of the Column ſuſpended: The ſame conſideration is eaſily extended to thoſe Levels, that depend upon the riſing of Mercury to the ſame height in the oppoſite Legs of a bent Tube; an Inſtrument of which kind has been lately offer'd for the ſervice of the Publick. And as the effect is juſt contrary in Levels made with Water or Spirit of Wine, due regard ought to be had to this Property in the conſtruction of thoſe Inſtruments, by making the Tubes ſufficiently wide, in order to diminiſh the Error as much as poſſible.

III. *Part*

III. *Part of a Letter from Dr.* Rich. Richardson, *to* Will. Sherard, LL. D. *&* R. S. S. *giving a relation of a wonderful Fall of Water from a Spout, upon the Mores in* Lancashire.

I Had an opportunity when in *Lancashire* of visiting a second time a vast breach in the Ground, which was made by a Spout, which fell upon *Emott-more*. The account I took of it when I first saw it, I put into Writing; and upon a second Inspection, finding it to be pretty exact, I thought a Transcript of it, would not be ungrateful to you, which you may communicate to your Friends, and make what use of it you please. You may depend on it as true.

Tho' our printed Voyages of several parts of the World furnish us with frequent accounts of damage done at Sea by Spouts of Water, yet such rarely happening at Land, induc'd me to take the following Relation of a remarkable one, which fell on *Emott-more*, nigh *Coln* in *Lancashire*, on *Tuesday* the 3*d* of *June*, 1718. about ten in the Morning: when several Persons who were imploy'd in digging *Peat* nigh the place where this Accident happen'd, upon a sudden were so terrify'd with an unusual noise in the Air, that they left their Work and ran Home, which was about a Mile from the Place: But to their great surprise they were intercepted by Water; for a small brook in the Way was risen above Six Foot Perpendicular in a few Minutes time, and had overflown the Bridge.

It is to be observ'd, that there was no Rain at that time on *Emott more*, only a Mist, which is very fre-

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quent upon thoſe high Mountains in Summer time. There was a great Darkneſs in the Place where the Water fell, without either Thunder or Lightning (as I had my Information from an Eye Witneſs) The Meadows at *Wicolæ* were ſo much floated that the like had not been ſeen in ſeveral Years before, tho' there it was a very bright Day.

Upon this account, I went to view the Place where the Water fell; tho' I believ'd this Inundation might proceed from an eruption of Water out of the ſide of the Mountain; ſuch being not unfrequent, where Lead or Coal have been Dug, but neither have ever been ſought for here. Upon approaching the Place, I was ſtruck with unſpeakable Horror, the Ground was torn up to the very Rock, where the Water fell, which was above Seven Foot deep, and a deep Gulf made for above half a Mile, and vaſt heaps of Earth caſt up on each ſide of it, ſome pieces remaining yet above twenty Foot over, and ſix or ſeven Foot thick. About ten Acres of Ground were deſtroy'd by this Flood. The firſt Breach where the Water fell is about ſixty Foot over, and no appearance of any Eruption, the Ground being firm about it, and no Cavity appearing. I muſt not forget to mention, that the Ground on each ſide the Gulf was ſo ſhaken, that large *Chaſmes* appear'd at above 30 Foot diſtance, which a few Days after I obſerv'd the Shepherds were filling up, leaſt their Sheep ſhould fall into them.

IV. *An*

IV. *An Account of the* Phænomena *of a very extraordinary* Aurora Borealis, *seen at* London *on* November 10. 1719. *both Morning and Evening. By Dr.* Edmond Halley. R. S. Secr.

UPON *Tuesday, November* 10. 1719. in the Morning, *Jupiter* applying to the Second in the Wing of *Virgo*, I got up about 5 of the Clock to obſerve him, and having had the Satisfaction to ſee my Calculus perfectly well anſwer the Heavens, I found certain white Streaks in the Sky, ſeeming nearly Perpendicular; which whilſt I conſidered them ſeemed inſtantly to vaniſh, and ſoon after others came as inſtantaneouſly in their room. I began to imagine that this was likely to be ſome part of the *Phænomena* of the *Aurora Borealis*. But there appearing nothing like that luminous Arch which we have of late ſo often ſeen in the *North*, I knew not what to think; till looking up towards the *Zenith*, I perceived an entire *Canopy* of ſuch kind of white *Striæ*, ſeeming to deſcend from a white Circle of faint Clouds, about 7 or 8 degrees in Diameter, which Circle ſometimes would vaniſh on a ſudden, and as ſuddenly be renewed. I obſerved that the Center of this place of Concourſe was not preciſely in the *Zenith*, but rather 14 degrees to the Southwards thereof; which I was well enabled to eſtimate by a Star, which on each return thereof ſhewed its ſelf about the Center of the Circle. This Star is the 33*th* Star of the *Great Bear* in *Tycho*'s Catalogue, whoſe diſtance from the Pole at this time is 52$\frac{1}{2}$ degrees, and which about half an hour paſt Five that Morning paſt the Meridian, ſo that thoſe Rays centred very nearly on the Meridian it ſelf. It

10 G was

a very entertaining Sight, till such time as the Day-break began to obscure these Lights, which were but faint, though sufficiently distinguishable. They came none of them lower than to about 30 or 40 degrees of Altitude, and seem'd not to have ascended from the Horizon. The Sky was perfectly Serene and Calm, which seems to be one of the concomitant Circumstances attending the *Aurora Borealis*, of which this was certainly a Species. For the Night following a Neighbour gave me notice of a strange streaming of Lights seen in the Air, which thereupon I attended from the Hours of 9½ to 11, when a Fog came so thick as to put an end to my Prospect. But during that whole time there ascended out of the *E.N.E.* and *N.E.* a continued succession of whitish *Striæ*, arising from below; and after changing as it were into a sort of luminous Smoke, past over head with an incredible Swiftness, not inferiour to that of Lightning; and as it past, in some part of its Passage seemed as it were guilded, or rather as if the smoke had been strongly illuminated by a blaze of Fire below. Some of the *Striæ* would begin high in the Air, and a whole set of them subordinate to one another, like Organ Pipes, would present themselves with more rapidity than if a Curtain had been drawn from before them; some of which would die away where they first appeared, and others change into a luminous Smoke, and pass on to the Westwards with an immense Swiftness. And I am of opinion that had it not been for the Moon, then ten Days old and very bright, this for the time would have been reckoned as considerable an Appearance as that of the *6th* of *March*, 1716.

V. *A*

V. *A Relation of the same Appearance, seen at* Cruwys Morchard *in* Devonshire. *Being part of a Letter to* Sam. Cruwys, *Esq;* R. S. S. *and by him Communicated to the* Royal Society.

YOU have doubtless been surprized again afresh by the wonderful Lights which have been seen several times of late like those described in the *Philosophical Transactions*; seen on *March* 6. 1716.

Monday the 26 of *October*, between 7 and 8 in the Evening, I saw some small appearance of it, *viz.* 3 or 4 large Coruscations in form of Pyramids, of reddish Colour inclining to Yellow, which rose about 50 degrees above the Horizon, and continued but few Minutes. But the North part of the Hemisphere was very bright and red all the Evening both before and after, till Ten, if not longer.

Tuesday, Novemb. 10. These Lights were seen again about 4 in the Morning, of which some say (to use their own Expressions) that the Element opened sometime at one place, then at other; from whence came great shining Lights that continued a while and then went away by degrees, and the Holes closed up again. This continued till Day break.

The Evening following coming from *Tyverton* about half an hour after Eight, I saw the North part of the Horizon very light and reddish (notwithstanding the Moon being about 10 Days old, was then in or past the Meridian, and shone very bright) in a short time the streaming luminous Rays began to appear very plain, some in one shape, some another; many of them

like Cones or Pyramids, but moſt of them badly terminated; ſome of which mounted very high, almoſt to the Zenith, to which place, or near, they all or moſt, ſeemed to point. Shortly after there appeared a long Streak of about 30 Degrees, parallel to the Horizon and about 15 or 20 diſtant from it, and about 2 or 3 broad, but badly terminated and of a fiery red Colour: which ſent out ſome of the ſame ſtreaming Beams towards the Zenith. About 6 or 7 Minutes after there appeared (ſomewhat ſudden) a Circular Figure like an Iris, but twice as broad, of a pale Colour. The Eaſt part was terminated by the Horizon at full Eaſt, if not ſomething to the South, and the Weſt End about North Weſt; the upper part of its Arch being 50 or 60 Degrees high, great numbers of luminous Rays darted from it upward and downwards, (or elſe paſſing croſs it from the Horizon) at oblique Angles pointing to the Zenith, eſpecially from the North Eaſt part. This continued, as near as I can gueſs (by the diſtance I rode) about 8 or 9 Minutes, when it divided and diſappeared. After an Interval of 3 or 4 Minutes, another Iris-like Figure appeared, (of a Colour (as it ſeemed) paler than any of the ſtreaming Lights had been) whoſe Diameter was leſs than that of the former, and ſhewed more than its Semicircle above the Horizon, the upper part of its Arch approaching near the Zenith. I could not obſerve any Rays to paſs from, (or a-croſs) this as from the other. The Centre of this laſt was much more to the Weſt than that of the firſt. After the continuance of a Minute or two, it began to break in the upper part of its Arch and ſhining Particles being ſent out from both its broken Ends towards the Zenith, (to which they were near before) or rather a little beyond it to the South or South Weſt, they there formed a ſort of *Corona*, curving and bending ſomewhat like Flames reverberated on the Arch

of

of an Oven: tho' this expresseth it but badly, yet I know not how to describe it better. It seemed to me and others to be finely tinged with various Colours, Red-Yellow and Blueish, *&c.* and sent out every way from it (except South and South-west) long flame-coloured Rays. After this had continued about two Minutes, its shining Light abated, and it left behind it for some Minutes, something like a whitish Cloud (like in Colour to what the Light on the 19*th* of *March* last left behind it, after the fiery Particles were extinguished, but thinner).

N. B. All this while the Moon shone very bright, from which this *Corona* was not very far distant, perhaps not twenty Degrees, to the North East. After this there continued to be sent up many fiery Coloured or Yellowish streaming Lights, sometimes more, sometimes less, now here, now there, all along the North part of the Hemisphere, but mostly from the North North East All this while something like small whitish Clouds (which to me seemed to move towards the Zenith, or to point a little more Southward, but disappear'd as they approached the Moon) were carried very swiftly, and at very short Intervals, mostly coming from the East and North East, but many also from North and North West. We took but little notice of this at first, supposing it had been nothing but the reflection of the other Lights, or the shadows of the Clouds (whereof the North parts were pretty full) as the streams of Light past behind them: But at last we observed that when the Lights at any time abated, these kind of Clouds continued to fly as swift and frequent as ever. This I saw till Twelve or One next Morning: many others saw it next Morning till almost break of Day, when it appeared much more red and fiery than it was in the Evening; the Moon perhaps

being

being then ſet. Some People obſerved tall Cones to ariſe in the Eaſt, and to be carryed to the Weſt pretty ſwiftly in an erect Poſition, but I ſaw them not. It has been repreſented here in all ſorts of Appearances, Armies, Battles, *&c.* and has put abundance of People in diſmal Frights: But I had not an Imagination ſtrong enough for it, *&c.*

Novemb. 16. *Will. Maunder.*

VI. *A further relation of the ſame Appearance as ſeen at* Dublin, *communicated to the Publiſher by an unknown Hand.*

IT is with pleaſure that I now give you the trouble of reading the enſuing Account of the ſurprizing Lights which on *Tueſday* the tenth of *November* we ſaw in the Northern Semicircle of our *Horizon.*

The Afternoon was very Calm and Serene; about ſix in the Evening the Sky was ting'd with a ſtrange kind of Light, and ſome Streams began to project from the North and North Eaſt. One of them aroſe about N by E. and was nearly a Subtenſe of an Arc between that and S. W. by Weſt; it was a little curvated toward the Sun, and what I ſaw of it (for the North part of the *Horizon* was conceal'd by Houſes) very much reſembled the tail of a Comet: About the ſame time there was one or two which aroſe in the Eaſt, aſcending obliquely ſo as to leave the *Zenith* ſeveral Degrees to the Northward,

Theſe *Striæ* continu'd to appear and diſappear alternately till toward Eight in the Evening; they were *Pyramidal,* and their *Vertices* frequently projected ſeveral Degrees to the South of our *Zenith.*

Between

Between nine and ten I was agreeably ſurpriz'd with a kind of Coruſcation, or Flaſhing, that ſhew'd it ſelf between twenty and ſixty Degrees from the *Zenith*, in the South or South by Weſt; and which from four or five, ſometimes from more places at once, darted with a Velocity not much inferior to that of Lightning; and by interfering with each other produc'd a beautiful Tremour or Undulation in that ſubtile *Vapour*, which I cannot better illuſtrate, than by comparing it to the Beams of the Sun, reflected on a Ceiling from the Surfaces of two or three Baſons of Water: Theſe *Waves* of Light were only viſible at the inſtant of Coruſcation, and were of a pale whitiſh Colour, ſomewhat reſembling the flaſhes produced by the violent agitation of Quickſilver in an Exhauſted Receiver; but ſo ſtrong that a Gentleman who about that time was in a Room by himſelf, without a Candle, aſſur'd me he took it for common Lightning: Thus it continued inceſſantly for more than an Hour, during which time ſeveral lucid *Areas*, like little Clouds, diſcover'd themſelves in the pure Sky, and after they had continu'd about five or ſix ſecond Minutes, as near as I could gueſs would inſtantaneouſly diſappear; moſt of them pretty much reſembl'd a very thin white Smoke or Vapour illuminated by the Full Moon.

About three quarters paſt Ten, this Vapour was almoſt ſpent, or by a briſk Gale at South by Weſt diſpers'd and driven to the Northward; at which time, between the Weſt and North, a vaſt body of it, like a very bright Flame-colour'd *Crepuſculum*, ſeem'd to be fix'd: From this Baſis ſeveral Beams or *Striæ* of ſhining matter were at uncertain intervals, emitted; and tho' it was not ſo ſenſible to the Eaſtward of the North, yet ſeveral mighty Pillars were alſo ejected from thence; One, which if I miſtake not, aroſe directly under the

the Pole, was, above all others that had preceded it, both as to its Magnitude and Denſity ſo ſurprizing, that I'm perſuaded the ſmalleſt Print might have been read by the Light thereof, had not that of the Moon, which ſhone very bright, pretty much effac'd it: 'twas ting'd with a kind of Yellow and Violet Colour. In about two or three Minutes it died away, and was ſucceeded by others of an inferior Order: It was now about a quarter paſt Eleven of the Clock, and nothing but repeated *Phaſes* of the ſame Spectacle offering themſelves to View; the Vibrating Motion had ceaſed; the Vapour ſhewed it ſelf no longer in lucid *Areas*; the ſtreams of Light were not ſo frequent, and thoſe more languid than before; and the bright *Aurora* having ſetled nearer the *Horizon*, I concluded the *Scene* was at an End, and accordingly gave over the queſt of new *Phænomena*, with only obſerving that about N. E. there appear'd ſome Clouds that reflected an unuſual kind of reddiſh Light. Others, who thro' a Principle of Fear ſat up longer than I did, repreſent the End with very ſurprizing Circumſtances; but as it eſcap'd the Eyes of thoſe who were beſt qualified to oblige the World with an Hiſtory of it, ſo I deſpair of adding any thing that may be ſatisfactory: and there were no doubt many Circumſtances of Weight that I did not obſerve; for the wonderful Variety this *Phænomenon* afforded, and the frequency and ſuddenneſs of its Alterations, made it impoſſible for the Eye of any ſingle Perſon to trace it.

On *Tueſday* the 24*th* of *November* we had the ſame *Phænomena* repeated, tho' not with the ſame Variety: About a quarter paſt ten at Night, a vaſt Body of ſhining matter was collected between N. W. by Weſt, and N. by E. in the form of the Segment of a Circle, whoſe Center was about 25 or 30 Degrees below the *Horizon*;

Horizon; from its *Periphery* a few short *Pyramidal* Streams, of the same luminous Vapour, ascended by a slow and nearly uniform Motion, and were exceeding rare so as not to efface the smallest of the fix'd Stars; and in a Minute or two vanish'd: It was very remarkable that the Light which that Collection of Vapour emitted was so great, that in the otherwise very dark Night, I cou'd thereby (at three quarters past Ten) read the Title of the last *Philos. Transact.* which then happen'd to lye on my Desk; and at four or five Yards distance see the smallest Books in my Study.

VII. *An Account of another very considerable* Aurora Borealis *observed at* Streatham *in* Surrey, *by Mr.* Thomas Hearne; *and Communicated by Coll.* Francis Nicholson, R. S. S.

HAving seen Dr. *Halley*'s Account of the Coruscations in the Morning and Evening of *Novemb. 10th.* (and the Letter annexed to it from *Devonshire*) I had the Pleasure to find the Observations made upon that Appearance very agreeable to what I had my self observed the Evening of that Day; and to what I did not at that time observe, but had an opportunity of observing in the Night of *Dec.* 11. I believe much more plainly than Dr. *Halley* had in the Night of *November* 10.

Dec. 11th. About one a Clock at Night (or rather in the Morning of *Dec. 12th)* I was called to observe Coruscations which appeared of a much different Colour, and in a very different manner from any I had before seen.

The ſtreams of Light that darted upwards from the Horizon ſeemed to be at conſiderably a greater diſtance, but not at all in leſs quantity than thoſe of *Nov* 10*th*. But their meeting in a Point near the *Zenith*, and there forming a kind of *Canopy*, was what was particularly remarkable in the *Manner* of the Coruſcations now different from thoſe of *Nov*. 10.

The ſtreams of Light roſe from the Horizon only towards the North, and on each hand towards N. Eaſt and N. Weſt: But near the *Zenith* a Canopy was formed of ſtreams of Light meeting in a Point, not only from thoſe Quarters, but alſo from the South, *&c.* Only to thoſe Points they extended downwards from the *Zenith* but a little Way, and were neither in ſo great quantity nor quite ſo bright as thoſe Northwards. At firſt I thought the Point in which the Streams met was exactly the *Zenith*, but upon obſerving it ſomething longer, I found it was not ſo, but a few Degrees to the South of the *Zenith*. The ſtreams of Light near the *Zenith*, which formed this Canopy, were of a pretty bright Colour, and in great Quantity, and darted very ſwiftly.

On each ſide of the N. towards E. and W. but not exactly in the N. it ſelf (at leaſt when I ſaw it) from about 10 or 15° to 40 or 50° above the Horizon, the Streams were of a glowing red Colour, whereas all that I had ever ſeen before were very pale. The redneſs was like that of a burnt Brick, and neareſt of any thing I have ſeen to the Colour, which remained for a few Minutes, like that tract through which the Meteor paſſed in the Spring.

The Streams appeared of this fierce Colour when I firſt ſaw the Coruſcations, and continued ſo for ſome time, till the redneſs by degrees wearing off, in about ¼ of an Hour they appeared of the uſual Paleneſs,

when

when I left them ſtill forming a Canopy near the *Zenith*, as is above deſcribed.

The Air was very Calm and Serene, not a breath of Wind ſtirring; as I remember it was alſo *Nov.* 10*th*.

The Moon was now a Day or two older than it was on *Nov.* 10*th*. and a good deal further to the W. than when I ſaw the Coruſcations that Night being then near full South. She had now round her what is commonly called a Burr larger than ordinary, and ſeveral very lucid Clouds at a little diſtance.

VIII. *Nuperæ Obſervationes Aſtronomicæ cum Regia Societate communicatæ.*

CUM in Num. harum *Tranſactionum* 357mo. Obſerſervationes nonnullas Planetarum ac Lunæ conſervari digniſſimas in unum congeſſimus, ac probante Societate noſtra edidimus; liceat paucula ſolia hujuſmodi collectionibus in ſequentibus quotannis conſignari. Nuperæ autem quas habemus Obſervationes hæ ſunt.

1718. *October* 10°. mane, applicabatur *Jupiter* ad Fixas Teleſcopicas, quarum loca, occaſione primæ apparitionis Cometæ anni 1680, (de quâ vide *Phil. Tranſ.* N°. 342) ſedulo inquiſivit Rev. D. *Pound*, ac nuper verificata nobiſcum communicavit, una cum accuratâ obſervatione tranſitûs Jovis juxta eas hac vice, ac deinde alterâ *Febr.* 11°. ſtatim ab oppoſitione Solis & Jovis. Incunte autem *Januario* 1719. loca ſtellarum ſic ſe habuere.

	Long.	Lat. Bor.		Long.	Lat. Bor.
d ♌	29°.59′.43″	1. 7.50	*a* ♍	0°.25′.41″	1°.28′.54″
e ♍	0. 6.13	1.10.18	*x* ♍	0. 5.43	0.51.56
c ♍	0. 3.13	0.32.50			

Ubi notandum stellas *d* & *e* eandem præcisè hoc seculo sortiri declinationem, *x* vero exiguam esse stellulam in priore descriptione ob parvitatem omissam.

Jam *Octob.* 9°. 17^{h}. 50′ T. æq. Jovis limbus orientalis attigit lineam stellas *e* & *c* jungentem, simul centrum ejus distabat ab *e* 21′. 20″ & à *c* 16′. 25″. statimque aberat à *d* 19′. 35″. Parvula *x* Jovi proxima latuit, luce ejus obumbrata.

Decemb. 11°. 18^{h}. 30. T. æq *Saturni* centrum distabat à *μ Libræ Bayero*, 28′. 32″, & Fixâ Borealius erat 4′. 31″. Hinc conclusit D. *Pound* Observator *Saturni* locum ♍ 10°. 41′. 10″, cum Lat. Boreal. 2°. 16′. 43″.

1719 *Feb.* 11°. 6^{h}. 56′$\frac{1}{4}$ T. æq. *Jovis* retrogradi centrum distabat à stella *d* superius descriptâ —— 10′. 42″

6. 58$\frac{1}{4}$ Idem centrum distabat ab *e* —— 6. 7

9. 37$\frac{1}{2}$ Iterum distantia capta à *d* —— 10. 9

9. 43$\frac{1}{2}$ Iterum ab *e* —— 6. 11

9. 49$\frac{1}{2}$ Jovis centrum distabat ab *a* —— 25. 21

9. 58$\frac{1}{2}$ Idem centrum à parvula *x* —— 24. 38

Circa Horam septimam *Jovis* limbus orientalis attigit lineam per *x* & *e* productam; *Jupiter* itaque tunc habuit ♍ 0°. 6 cum Latitudine Boreal. 1°. 16′. 30″. Deinde,

Feb. 13°. 8^{h}. 0′. T. æq. Declinatio centri Jovis, Micrometro mensurata, Borealior erat eâ stellæ utriusque *d* & *e* 11′. 37″; & 8^{h}. 20′ eadem differentia inventa est 11′. 36″. Horâ vero 8^{h}. 48′ centrum Jovis distabat ab *e* 17′. 40″.

Apr. 22°. 10^{h}. 45′. T. æq. *Saturni* centrum sequebatur *μ Libræ* 4″$\frac{1}{2}$ Temp. sive 1′. 8″ Asc. Rectæ. Micrometro autem Borealior inventus est Fixâ 35′. 25″. Stella autem in *Catalogo Britannico* tunc habuit, ♏ 10°. 16′. 8″. Lat. Bor. 2°. 3′. 54″.

Maii

Maii 16°. 8h. 00′ T. æq. ♃ sequebatur *Cor Leonis* 1°. 34′½ Ascensionis rectæ; Borealior autem erat stellâ illâ 0′. 41″½. Temporis, hoc est, 10′. 7″ Arcus cœlestis.

Eâdem nocte, 15h. 18′ T. app. Observavit D. *Stephanus Grey Martem*, ratione Ascensionis rectæ, sequi stellam *in Cauda Capricorni* orientalem 16′. 15″; simul non nisi 0′. 11″. australior erat quam Fixa.

Junii 7°. 10h. 15′. T. app. *Jupiter* directus iterum reversus est ad stellas Telescopicas prædictas, & tum sequebatur stellam *d.* 0′. 35″ Ascensionis rectæ, & 10h. 30′ distabat fixa à limbo *Jovis* proximo 4′. 18″.

Postridie *Junii* 8°. 10h. 20′, *Jupiter* sequebatur stellam alteram *e* 1′. 30″ Ascensionis rectæ, ac statim distantia limbi *Jovis* proximi à stella capta est Micrometro 5′. 30″.

Julii 5. 8h. 26′. T. app. Conjungebantur arctè *Jupiter* & *Venus*, quæ tum Borealior præcedebat *Jovem* secundum Ascensionem rectam 1′. 20″: Centrorum autem distantia ex decies repetitis media, capta est 13′. 36″. Hæc tria *Londini* observata communicavit harum Scientiarum eximius Cultor D. *Martinus Folkes*, R. S. Soc.

Aug. 3. 12h. 20′ T. æq. *Mars* pene Acronychus sequebatur stellam τ *Aquarii Bayero* 10′. 58″ Temporis, sive 2°. 44′. 57″ Ascensionis Rectæ. Erat autem fixâ *Mars* Borealior 0′. 36″ tantum; unde concesso loco stellæ *Britannico* fit locus *Martis* observatus ♓ 7°. 10′. 10″ cum latitudine Australi 6°. 38′. 10″.

Aug. 10°. 11h. 50 T. æq. *Mars* sequebatur fixam minorem quæ præcedit τ *Aquarii* 1°. 39′. 30″ ratione Ascensionis rectæ; Australior vero quam fixa 10′. 42″.

Aug. 16°. 7h. 18′. T. æq. *Spica Virginis* præcedebat *Veneris* centrum 5″¼ secundis temporis, sive 1′. 20″ Ascensionis rectæ, australior Planetâ 18″¼ temp. sive 4′, 35″.

Aug. 17°. *Mars* pridie Acronychus ac Terris proximus observatus est ad duas stellulas contiguas, Parallaxis ejus

ejus investigandi gratiâ, juxta methodum à D. *Cassino*, in libro de Cometa anni 1680, exhibitam, Unde *Martis* Parallaxin eruere in *Transact*. proximâ conabimur. Harum vero stellularum borea tum temporis locum habuit ♓ 3°. 5′. 50″ cum Latitudine australi 6°. 6′$\frac{1}{4}$: altera vero Australior habuit ♓ 3°. 5′. 30″, cum Lat. Aust. 6°. 10′$\frac{1}{4}$ proximè. Horâ vero 10^h. 40′. T. æq. Australem sequebatur *Mars* 41′ min. 40″ Ascensionis rectæ, eâque adhuc Australior erat 7′. 50″.

Sept. 18. 9^h. 20′. T. æq. *Mars* visus est præcedere stellam in *Catalogo Britannico* Aquarii 53^{iam}. 3′. 45″ Temporis, sive 56′. 24″. Ascen. Rectæ; simulque Stella Borealior erat limbo Martis boreo, non nisi unâ Planetæ diametro. Locus stellæ ♒ 29′. 57′$\frac{1}{2}$ Lat. Aust. 4°. 48$\frac{1}{2}$.

Octob. 30. Vesperi 5^h. 45′. T. app. *Mars* proximus stellis duabus contiguis ad *h* ♒ *Bayero*, quæ sunt ♒ 73^{ia} & 74ta *Catal. Brit*. Præterierat rectam per easdem ductam, eratque angulus ad *Martis* centrum ad sensum rectus: Borea vero stellarum eandem habuit declinationem cum limbo Planetæ austrino. 5^h. 53′ distantia stellæ à centro *Martis* 2′. 30″. 5^h. 56′ centrum *Martis* distabat à tertiâ & Australiore ad *h*, sive 75ta Aquarii, 17′. 04″. 6^h. 18′ distantia centri à Boreâ sive 73^{ia} erat 3′. 5″. Hinc concludere licet *Martem*, horâ 3^h. 30′ proximè, stellæ Boreæ conjunctum fuisse, eamque uno tantum minuto ad Boream reliquisse. Fixæ autem locus è *Catalogo Britannico* tunc erat ♓ 10°. 29′. 00″ cum Lat. Aust. 1°. 40′$\frac{1}{4}$. 74ta vero habuit ♓ 10°. 29′. 50″ cum Lat. Aust. 1°. 44′$\frac{1}{4}$.

Novemb. 16°. 19^h. 18′. T. æq. *Venus* præcedebat *Lancem Libræ* Austrinam 3′. 13″ Temp. sive 48′. 23″ Ascen. Rect. simulque fixâ borealius erat centrum *Veneris* 7′. 45″. *Venus* quasi Stationaria apud Nodum ejus Ascendentem.

Decemb.

Decemb. 3°. 19^h. T. æq. *Saturnus* præcedebat tertiam ad ζ *Libræ*, ſive *Libræ* 29^{am}. *Cat. Brit.* 0'. 46" Temp. ſive 11'. 32" Aſc. Rect. Erat autem fixâ Auſtralior 15'. 29". differentiâ per Micrometrum captâ. Unde Saturni locus ♏ 20.25$\frac{1}{4}$ cum Lat. Bor. 2°. 5' $\frac{1}{4}$.

Obſervationes Lunæ & Eclipſium.

In dicta *Tranſact.* N°. 357. *pag.* 852. Obſervationem dedimus Eclipſeos Lunaris anno 1717 *Martii* 15° P. M. St. vet. apud *Cambridg Nov-Anglorum* habitam; apud nos vero ob Nubes inconſpicuam. Deſiit autem Eclipſis ibidem 11^h. 42' $\frac{1}{2}$, neque alia ejus tum temporis ſuppetebat obſervatio. Poſtea vero arte Nauticâ & induſtriâ inter primos inſignis Dom. *Candler* Navarcha Regius, ex *America* attulit & nobiſcum communicavit ejuſdem Eclipſeos phaſes *Limæ Peruviæ* à *Dom. Petro Peralta*, Mathematico Regio multis titulis claro, obſervatas, Typiſque ibidem impreſſas. Initium autem Eclipſis ponit *Limæ* 8^h. 41'. 8". Finem vero 11^h. 19'. 55". Simul laudatus D. *Candler* propriam obſervationem, ad Inſulam quam *Virgine Gorda* vocant captam, conceſſit, Ibi deſiit Eclipſis 12^h. 13'. P. M. Fine per cælum ſudum diſtinctè viſo. Poſtremo inter Acta Regiæ Scientiarum Academiæ *Pariſienſis* iſtius anni, comperimus duas & quidem ſatis conformes hujus Eclipſeos obſervationes, alteram à D. *Caſſino*, alteram à D. *de la Hire* in Obſervatorio Regio captas: Hic Initium æſtimavit 13^h. 54'. Finem vero certius 16^h. 38'. 10". At ille Initium 13^h. 55' & Finem 16^h. 38'. 25". Maxima obſcuratio huic 7$\frac{1}{3}$ Dig. illi 7$\frac{1}{2}$ Dig.

Hinc ex Fine, in ſingulis locis ut videtur accuratius ſumpto, proveniunt Longitudinum differentiæ inter *Pariſios & Limam* 5^h. 18'. 20", Inter *Pariſios* & *Cambridg*

4^h. 55'.

$4^h. 55'. 50''$. Inter *Parisios* & Insulam *Virgine Gorda* $4^h. 25'. 20''$. E quibus si $9'. 40''$ subduxeris, provenient Longitudines ad occasum *Londini*, nempe *Limæ* $77°. 10'$. *Cambridg Nov Anglorum* $71°\frac{1}{2}$, ac denique Insulæ *Virgine Gorda* $65°. 55'$; unde Insularum adjacentium situs Geographici certo corrigi poterint.

Altera Lunæ Eclipsis ejusdem anni *Septembris* nono Vesperi, ab iisdem observatoribus & D. *Maraldo Parisiis* conspecta est. Finem *Londini* observavimus in ædibus Societatis Regiæ $7^h. 26'$. *Parisiis* vero D. *Cassino* Finis $7^h. 34'. 50''$, D. *Maraldo* $7^h. 35'. 30''$, & D^no^ *De la Hire* $7^h. 34'. 15''$. Simul D. *Wurtzelbaur Noribergæ* eundem Finem vidit $8^h. 10'. 45''$ Hinc confirmantur Meridianorum differentiæ *Londinum* inter & *Parisios*, præsertim ex observatione D. *Maraldi*, nempe $9'. 30''$; uti & inter *Londinum* & *Noribergam* $44'. 45''$, quantam sæpius olim experti sumus. Porro quinto die post Eclipsin, *Septembris* 14^to^ vesperi, Luna occultavit *Palilicium Parisiis*, observantibus sigillatim DD. *Maraldo* & *Delisle* Juniore. Evanuit autem stella è regione Maculæ *Grimaldi* sive *Paludis Mareotidis*, Hora $9^h. 11'. 35''$. Emersit autem è limbo Lunæ obscuro $10^h. 3'. 55''$. Hujus Occultationis observatio *Londini* habetur *pag*. 853. *Phil. Transact*.

Observationes illas, in quibus Temp. æq. *adhibetur, Rev. D.* Pound *acceptas debemus. Tubo autem quindecimpedali captæ pro certissimis habendæ sunt.*

AN

INDEX

To the *Thirtieth* Volume of the *Philosophical Transactions.*

A.

B.

C.

H. *Heart*,

Spouts,

FINIS.

ERRATUM. *P*. 1110. *l*. 11. *pro* m *l*. m.